# Computer Graphics 2D

***3rd class***

# Lecture 8: 2D Transformation

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## 8.1 Rotation

**Rotation** in 2D is a transformation that turns an object around a fixed point (often the origin) by a certain angle, without changing its shape or size. The rotation is defined by the angle of rotation and the center point about which the object rotates. An object with a set of coordinates (**x, y**) on a 2D plane. The angle $\boldsymbol{\theta}$ (in degrees or radians) by which the object is rotated. A positive angle usually means a counterclockwise rotation, while a negative angle means a clockwise rotation. The point around which the rotation occurs. Typically, this is the origin (**0,0**).

- **Applying Rotation**

  To rotate a point around the origin by an angle θ the new coordinates ($\mathbf{x'}, \mathbf{y'}$) of a point (**x, y**) after rotation are given by:

$$\mathbf{x'} = \mathbf{x} \cdot \cos(\boldsymbol{\theta}) - \mathbf{y} \cdot \sin(\boldsymbol{\theta})$$

$$\mathbf{y'} = \mathbf{x} \cdot \sin(\boldsymbol{\theta}) + \mathbf{y} \cdot \cos(\boldsymbol{\theta})$$

- **Matrix Representation:**

  In matrix form, the rotation transformation can be represented using a **2x2** matrix:

$$R(\boldsymbol{\theta}) = \begin{pmatrix} \cos(\boldsymbol{\theta}) & -sin(\boldsymbol{\theta}) \\ sin(\boldsymbol{\theta}) & cos(\boldsymbol{\theta}) \end{pmatrix}$$

  When using homogeneous coordinates (which also include translation), the 3x3 rotation matrix becomes:

$$R(\boldsymbol{\theta}) = \begin{pmatrix} cos(\boldsymbol{\theta}) & -sin(\boldsymbol{\theta}) & 0 \\ sin(\boldsymbol{\theta}) & cos(\boldsymbol{\theta}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

✓**Example 1:** Suppose you have a point **P** with coordinates (**3**,**4**) and you want to rotate it by **90** degrees counterclockwise around the origin:

- **Rotation Angle:** $\theta = 90^\circ$
- **Cosine and Sine Values**: cos (**90**°) = **0**, sin (**90**°) = **1**
- **New Coordinates**:
  - $\mathbf{x'} = 3 \cdot \cos(\mathbf{90^\circ}) - 4 \cdot \sin(\mathbf{90^\circ}) = 3 \cdot 0 - 4 \cdot 1 = \mathbf{-4}$
  - $\mathbf{y'} = 3 \cdot \sin(\mathbf{90^\circ}) + 4 \cdot \cos(\mathbf{90^\circ}) = 3 \cdot 1 + 4 \cdot 0 = \mathbf{3}$

So, the new coordinates of P after a **90**-degree rotation are (**−4**,**3**)

✓**Example 2:** Consider a triangle with vertices (**1**,**1**), (**4**,**1**), and (**1**,**4**). If we rotate it by **45** degrees counterclockwise about the origin:

- **Original Vertices**:
  - **A** = (**1**,**1**)
  - **B** = (**4**,**1**)
  - **C** = (**1**,**4**)
- **New Vertices after Rotation by 45°:**
  - $\mathbf{A'} = (1 \cdot \cos(45^\circ) - 1 \cdot \sin(45^\circ),\ 1 \cdot \sin(45^\circ) + 1 \cdot \cos(45^\circ)) = (\mathbf{0}, \sqrt{\mathbf{2}})$
  - $\mathbf{B'} = (4 \cdot \cos(45^\circ) - 1 \cdot \sin(45^\circ),\ 4 \cdot \sin(45^\circ) + 1 \cdot \cos(45^\circ)) = (\mathbf{2}\sqrt{\mathbf{2}}, \mathbf{5}\sqrt{\mathbf{2}})$
  - $\mathbf{C'} = (1 \cdot \cos(45^\circ) - 4 \cdot \sin(45^\circ),\ 1 \cdot \sin(45^\circ) + 4 \cdot \cos(45^\circ)) = (\sqrt{\mathbf{5}}, \mathbf{-1})$

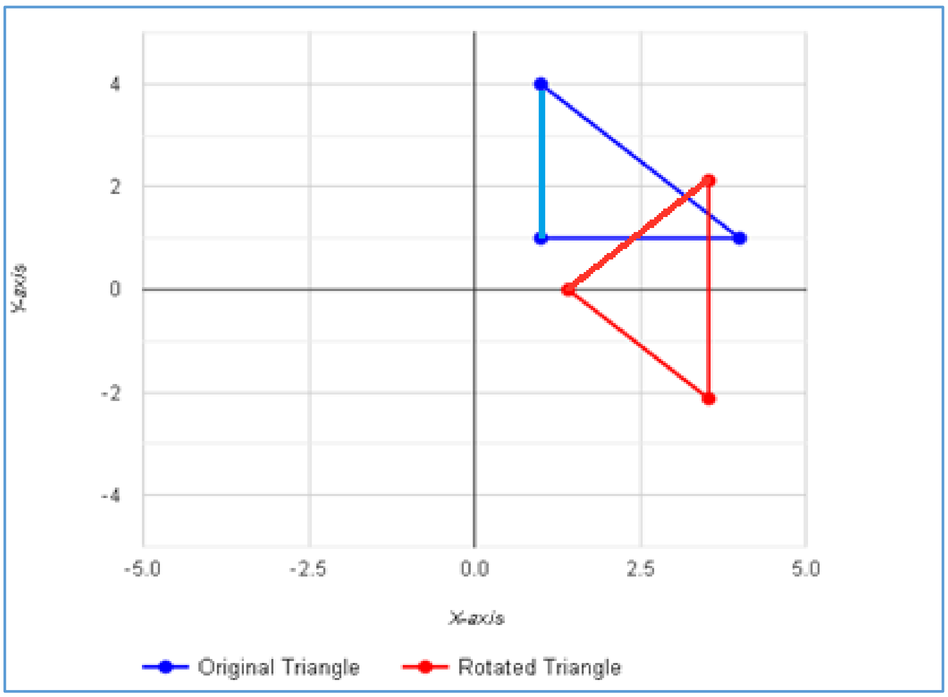


**Figure (4):** Rotation triangle on axes

✓**Example 3:** To rotate a triangle about the origin with vertices at original coordinates (**10,20**), (**10, 10**), (**20, 10**) by **30** degrees, we compute as followings:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Rotation of vertex (10, 20):**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 * 10 + (-0.5) * 20 + 0 * 1 \\ 0.5 * 10 + 0.866 * 20 + 0 * 1 \\ 0 * 10 + 0 * 20 + 1 * 1 \end{bmatrix} = \begin{bmatrix} -1.34 \\ 22.32 \\ 1 \end{bmatrix}$$

**Rotation of vertex (10, 10):**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 * 10 + (-0.5) * 10 + 0 * 1 \\ 0.5 * 10 + 0.866 * 10 + 0 * 1 \\ 0 * 10 + 0 * 10 + 1 * 1 \end{bmatrix} = \begin{bmatrix} 3.66 \\ 13.66 \\ 1 \end{bmatrix}$$

**Rotation of vertex (20, 10):**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 * 20 + (-0.5) * 10 + 0 * 1 \\ 0.5 * 20 + 0.866 * 10 + 0 * 1 \\ 0 * 20 + 0 * 10 + 1 * 1 \end{bmatrix} = \begin{bmatrix} 12.32 \\ 18.66 \\ 1 \end{bmatrix}$$

The resultant coordinates of the triangle vertices are (-1.34, 22.32), (3.6,13.66), and (12.32,18.66) respectively.