University of Anbar
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CHAPTER FOUR

SHEET PILES

LECTURE
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4.1. Introduction

Connected or semi-connected sheet piles are often used to build continuous walls for waterfront structures that range from small waterfront pleasure boat launching facilities to large dock facilities. (See Figure 4.1). In contrast to the construction of other types of retaining wall, the building of sheet-pile walls does not usually require dewatering of the site. Sheet piles are also used for some temporary structures, such as braced cuts. The principles of sheet-pile wall design are discussed in the current chapter.

Several types of sheet pile are commonly used in construction:

(a) wooden sheet piles,

(b) precast concrete sheet piles, and

(c) steel sheet piles.

Aluminum sheet piles are also marketed.

1- Wooden sheet piles are used only for temporary, light structures that are above the water table. The most common types are ordinary wooden planks and ***Wakefield piles***. The wooden planks are about 50 mm × 300 mm in cross section and are driven edge to edge (Figure 4.2a). Wakefield piles are made by nailing three planks together, with the middle plank offset by 50 to 75 mm (Figure 4.2b). Wooden planks can also be milled to form *tongue-and-groove piles*, as shown in Figure 4.2c. Figure 4.2d shows another type of wooden sheet pile that has precut grooves. Metal ***splines*** are driven into the grooves of the adjacent sheetings to hold them together after they are sunk into the ground.

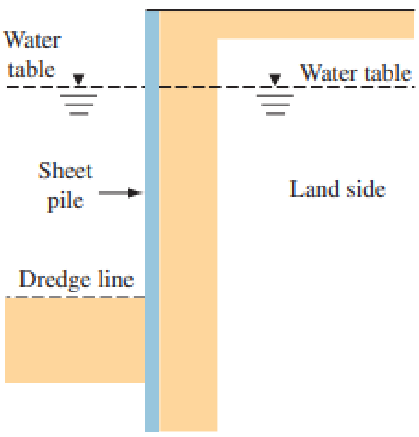


Figure 4.1 Example of waterfront sheet-pile wall.

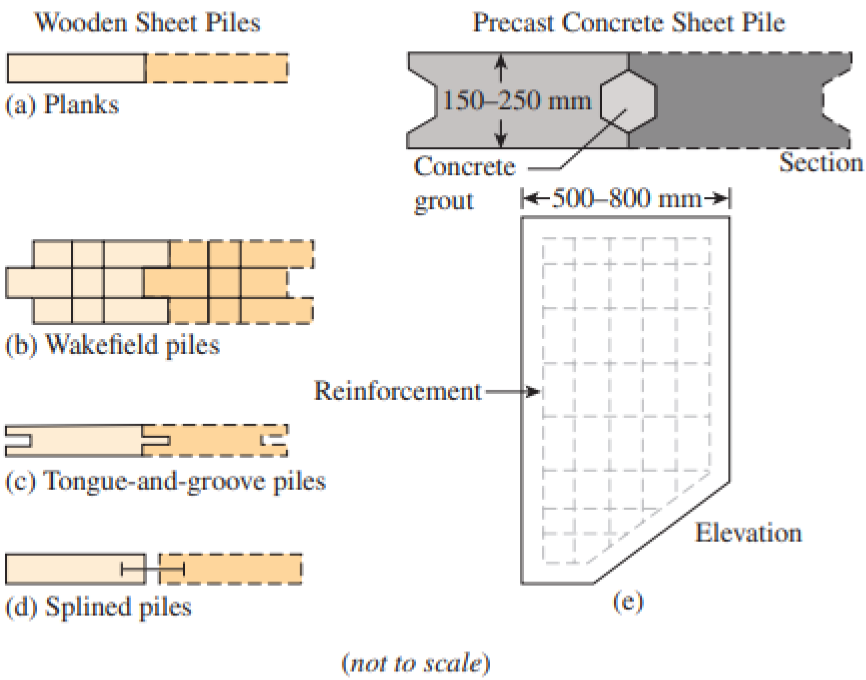
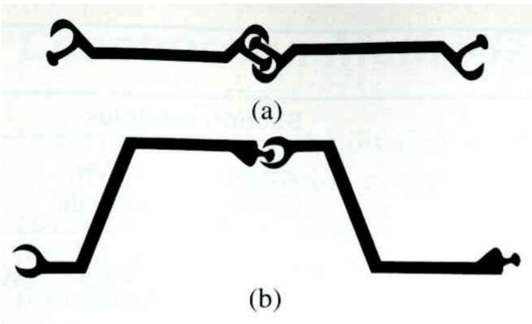


Figure 4.2 Various types of wooden and concrete sheet pile

2- ***Precast concrete sheet piles*** are heavy and are designed with reinforcements to withstand the permanent stresses to which the structure will be subjected after construction and also to handle the stresses produced during construction. In cross section, these piles are about 500 to 800 mm wide and 150 to 250 mm thick. Figure 4.2e is a schematic diagram of the elevation and the cross section of a reinforced concrete sheet pile.

3- ***Steel sheet piles*** in the United States are about 10 to 13 mm (0.4 to 0.5 in.) thick. European sections may be thinner and wider. Sheet-pile sections may be ***Z, deep arch, low arch,* or *straight web*** sections. The interlocks of the sheet-pile sections are shaped like a ***thumb-and-finger*** or ***ball-and-socket*** joint for watertight connections. Figure 4.3a is a schematic diagram of the thumb-and-finger type of interlocking for straight web sections. The ball-and-socket type of interlocking for *Z* section piles is shown in Figure 4.3b. Figure 4.4 shows some sheet piles at a construction site. Figure 4.5 shows a small enclosure with steel sheet piles for an excavation work. Table 4.1 lists the properties of the steel sheet-pile sections produced by Hammer & Steel, Inc. of

Hazelwood, Missouri. The allowable design flexural stress for the steel sheet piles is as follows:



Type of steel	Allowable stress	
ASTM A-328	170 MN/m^2	(25,000 lb/in^2)
ASTM A-572	210 MN/m^2	(30,000 lb/in^2)
ASTM A-690	210 MN/m^2	(30,000 lb/in^2)

Figure 4.3 (a) Thumb-and-finger type sheet-pile connection; (b) ball-and-socket type sheet-pile connection

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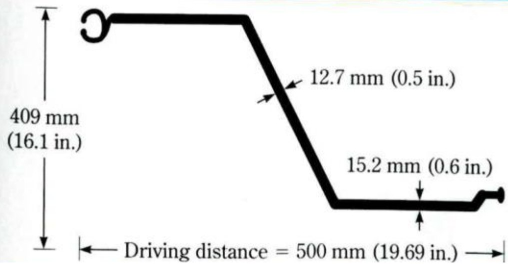
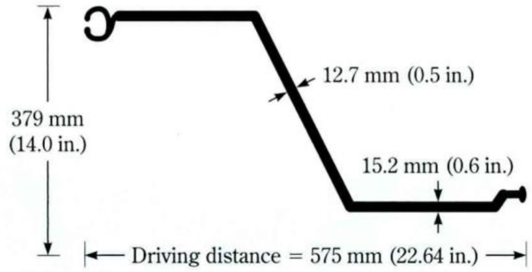
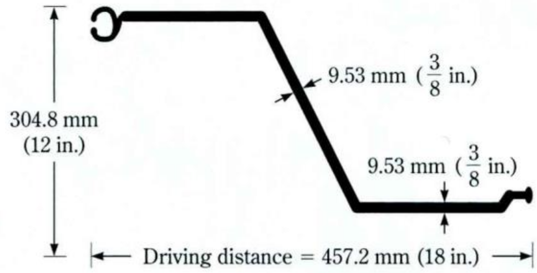
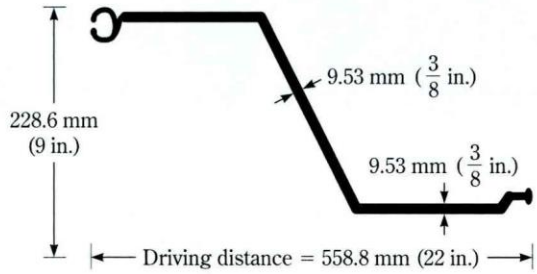
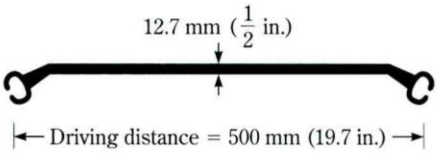
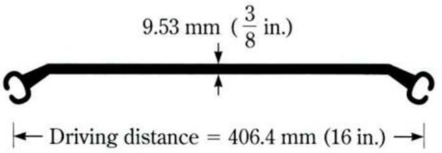
Figure 4.4 Some steel sheet piles at a construction site

Figure 4.5 A small enclosure with steel sheet piles for an excavation work

Table 4.1 Properties of Some Sheet-Pile Sections Production by Bethlehem Steel Corporation

Section designation	Sketch of section	Section modulus m^3/m of wall	Section modulus in^3/ft of wall	Moment of inertia m^4/m of wall	Moment of inertia in^4/ft of wall
PZ-40		326.4×10^{-5}	60.7	670.5×10^{-6}	490.8
PZ-35		260.5×10^{-5}	48.5	493.4×10^{-6}	361.2
PZ-27		162.3×10^{-5}	30.2	251.5×10^{-6}	184.2
PZ-22		97×10^{-5}	18.1	115.2×10^{-6}	84.4
PSA-31		10.8×10^{-5}	2.01	4.41×10^{-6}	3.23
PSA-23		12.8×10^{-5}	2.4	5.63×10^{-6}	4.13

4.2 Construction Methods

Sheet-pile walls may be divided into two basic categories:

(a) cantilever and

(b) anchored.

- In the construction of sheet-pile walls, the sheet pile may be driven into the ground and then the backfill placed on the land side, or the sheet pile may first be driven into the ground and the soil in front of the sheet pile dredged. In either case, the soil used for backfill behind the sheet-pile wall is usually granular. The soil below the dredge line may be sandy or clayey. The surface of soil on the water side is referred to as the *mud line* or *dredge line.*
- Thus, construction methods generally can be divided into two categories (Tsinker, 1983):

1. Backfilled structure

2. Dredged structure

The sequence of construction for a *backfilled structure* is as follows (see Figure 4.6):

Step 1. Dredge the *in situ* soil in front and back of the proposed structure.

Step 2. Drive the sheet piles.

Step 3. Backfill up to the level of the anchor, and place the anchor system.

Step 4. Backfill up to the top of the wall.

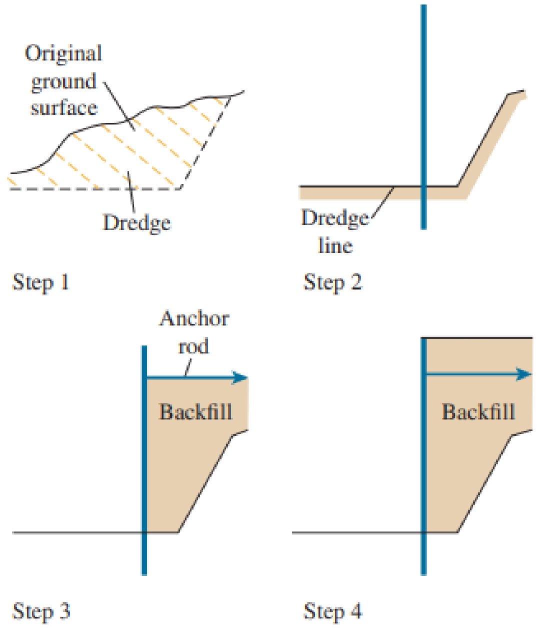


Figure 4.6 Sequence of construction for a backfilled structure

- **For a cantilever type of wall, only Steps 1, 2, and 4 apply. The sequence of construction for a *dredged structure* is as follows (see Figure 4.7):**

Step 1. Drive the sheet piles.

Step 2. Backfill up to the anchor level, and place the anchor system.

Step 3. Backfill up to the top of the wall.

Step 4. Dredge the front side of the wall.

With cantilever sheet-pile walls, Step 2 is not required.

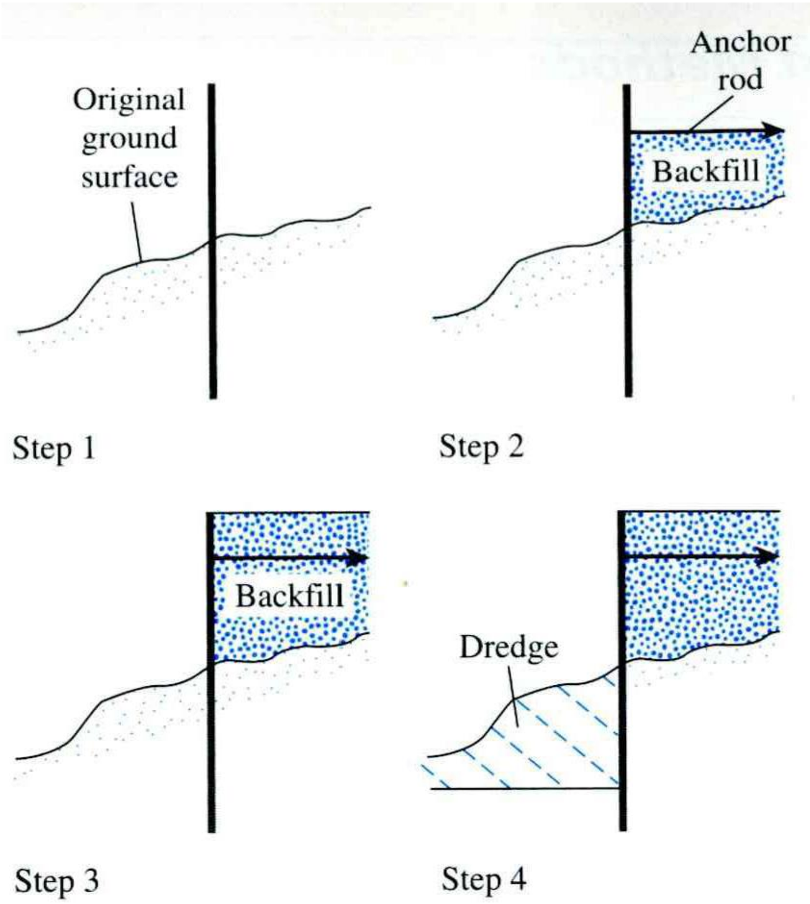


Figure 4.7 Sequence of construction for a dredged structure

4.3 Cantilever Sheet-Pile Walls

Cantilever sheet-pile walls are usually recommended for walls of moderate height—about 6 m or less, measured above the dredge line. In such walls, the sheet piles act as a wide cantilever beam above the dredge line. The basic principles for estimating net lateral pressure distribution on a cantilever sheet-pile wall can be explained with the aid of Figure 4.8. The figure shows the nature of lateral yielding of a cantilever wall penetrating sand layer below the dredge line. The wall rotates about point *O* (Figure 4.8a). Because the hydrostatic pressures at any depth from both sides of the wall will cancel each other, we consider only the effective lateral soil pressures. In zone *A*, the lateral pressure is just the active pressure from the land side. In zone *B*, because of the nature of yielding of the wall, there will be active pressure from the land side and passive pressure from the water side. The condition is reversed in zone *C*—that is, below the point of rotation, *O*. The net actual pressure distribution on the wall is like that shown in Figure 4.8b. However, for design purposes, Figure 4.8c shows a simplified version.

Sections 4.4 through 4.7 present the mathematical formulation of the analysis of cantilever sheet-pile walls. Note that, in some waterfront structures, the water level may fluctuate as the result of tidal effects. Care should be taken in determining the water level that will affect the net pressure diagram.

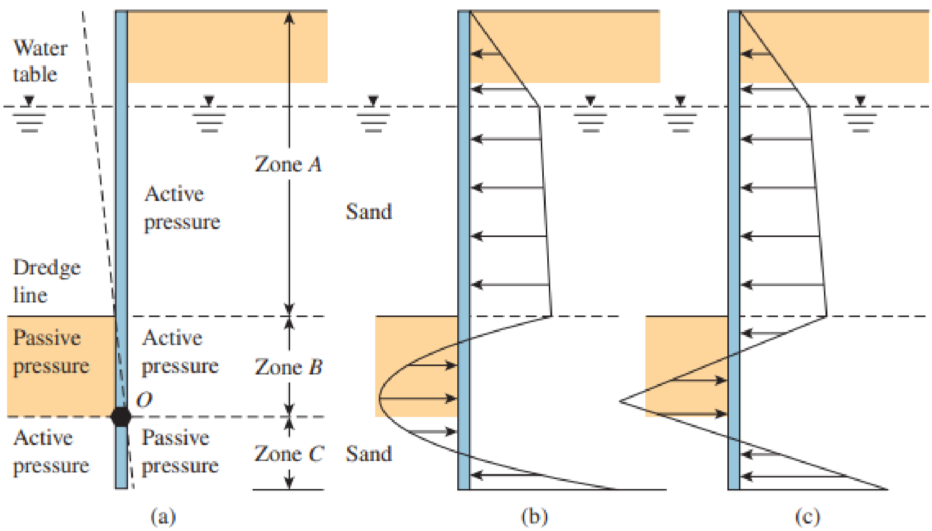


Figure 4.8 Cantilever sheet pile penetrating sand

4.4 Cantilever Sheet Piling Penetrating Sandy Soils

To develop the relationships for the proper depth of embedment of sheet piles driven into a granular soil, examine Figure 4.9a. The soil retained by the sheet piling above the dredge line also is sand. The water table is at a depth L_1 below the top of the wall. Let the effective angle of friction of the sand be ϕ'. The intensity of the active pressure at a depth z = L_1 is

$$\sigma_1' = \gamma L_1 K_a \qquad (4.1)$$

where

K_a = Rankine active pressure coefficient = $tan^2(45 - \frac{\phi'}{2})$

γ = unit weight of soil above the water table

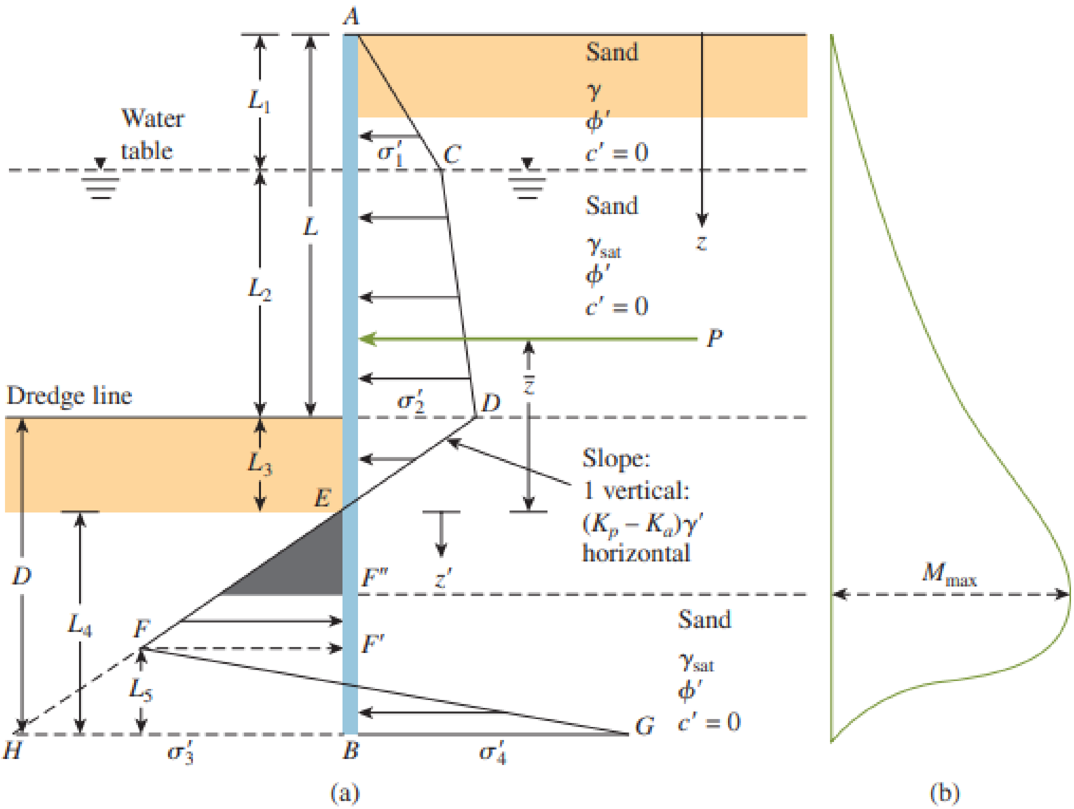


Figure 4.9 Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

Similarly, the active pressure at a depth z = $L_1 + L_2$ (i.e., at the level of the dredge line) is

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2) K_a \qquad (4.2)$$

where γ' = effective unit weight of soil $= \gamma_{sat} - \gamma_w$.

Note that, at the level of the dredge line, the hydrostatic pressures from both sides of the wall are the same magnitude and cancel each other.

To determine the net lateral pressure below the dredge line up to the point of rotation, *O*, as shown in Figure 4.8a, an engineer has to consider the passive pressure acting from the left side (the water side) toward the right side (the land side) of the wall and also the active pressure acting from the right side toward the left side of the wall. For such cases, ignoring the hydrostatic pressure from both sides of the wall, the active pressure at depth z is

$$\sigma'_a = [\gamma L_1 + \gamma' L_2 + \gamma'(z - L_1 - L_2)] K_a \qquad (4.3)$$

Also, the passive pressure at depth z is

$$\sigma'_p = \gamma'(z - L_1 - L_2) K_p \qquad (4.4)$$

where K_p = Rankine passive pressure coefficient = $tan^2(45 + \frac{\phi'}{2})$

Combining Eqs. (4.3) and (4.4) yields the net lateral pressure, namely,

$$\sigma' = \sigma'_a - \sigma'_p = (\gamma L_1 + \gamma' L_2) K_a - \gamma'(z - L_1 - L_2)(K_p - K_a) \qquad (4.5)$$

$$= \sigma'_2 - \gamma'(z - L)(K_p - K_a)$$

where $L = L_1 + L_2$. The net pressure, σ' equals zero at a depth L_3 below the dredge line, so

$$\sigma'_2 - \gamma'(z - L)(K_p - K_a) = 0$$

or

$$(z - L) = L_3 = \frac{\sigma'_2}{\gamma'(K_p - K_a)} \qquad (4.6)$$

Equation (4.6) indicates that the slope of the net pressure distribution line *DEF* is 1 vertical to $(K_p - K_a)\gamma'$ horizontal, so, in the pressure diagram,

$$\overline{HB} = \sigma'_3 = L_4 (K_p - K_a) \gamma' \qquad (4.7)$$

At the bottom of the sheet pile, passive pressure, σ'_p , acts from the right toward the left side, and active pressure acts from the left toward the right side of the sheet pile, so, at z = $L + D$,

$$\sigma'_p = (\gamma L_1 + \gamma' L_2 + \gamma' D) K_p \qquad (4.8)$$

At the same depth,

$$\sigma'_a = \gamma' D K_a \qquad (4.9)$$

Hence, the net lateral pressure at the bottom of the sheet pile is

$$\begin{aligned} \sigma'_p - \sigma'_a = \sigma'_4 &= (\gamma L_1 + \gamma' L_2) K_p + \gamma' D (K_p - K_a) \\ &= (\gamma L_1 + \gamma' L_2) K_p + \gamma' L_3 (K_p - K_a) + \gamma' L_4 (K_p - K_a) \\ &= \sigma'_5 + \gamma' L_4 (K_p - K_a) \end{aligned} \qquad (4.10)$$

Where

$$\sigma'_5 = (\gamma L_1 + \gamma' L_2) K_p + \gamma' L_3 (K_p - K_a) \qquad (4.11)$$

$$D = L_3 + L_4 \qquad (4.12)$$

For the stability of the wall, the principles of statics can now be applied:

∑ horizontal forces per unit length of wall = 0

and

∑ moment of the forces per unit length of wall about point *B* = 0

For the summation of the horizontal forces, we have

Area of the pressure diagram *ACDE* - area of *EFHB* + area of *FHBG* = 0

Or

$$P - \tfrac{1}{2}\sigma'_3 L_4 + \tfrac{1}{2} L_5 (\sigma'_3 + \sigma'_4) = 0 \qquad (4.13)$$

where *P* = area of the pressure diagram *ACDE.*

Summing the moment of all the forces about point *B* yields

$$P(L_4 + \bar{z}) - \left(\frac{1}{2} L_4 \sigma'_3\right)\left(\frac{L_4}{3}\right) + \frac{1}{2} L_5 (\sigma'_3 + \sigma'_4)\left(\frac{L_5}{3}\right) = 0 \qquad (4.14)$$

From Eq.(4.13)

$$L_5 = \frac{\sigma'_3 L_4 - 2P}{\sigma'_3 + \sigma'_4} \qquad (4.15)$$

Combining Eqs. (4.7), (4.10), (4.14), and (4.15) and simplifying them further, we obtain the following fourth-degree equation in terms of L_4 :

$$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4 - A_4 = 0 \tag{4.16}$$

In this equation

$$A_1 = \frac{\sigma'_5}{\gamma'(K_p - K_a)} \tag{4.17}$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)} \tag{4.18}$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma'_5]}{\gamma'^2(K_p - K_a)^2} \tag{4.19}$$

$$A_4 = \frac{P(6\bar{z}\sigma'_5 + 4P)}{\gamma'^2(K_p - K_a)^2} \tag{4.20}$$

Step-by-Step Procedure for Obtaining the Pressure Diagram

Based on the preceding theory, a step-by-step procedure for obtaining the pressure diagram for a cantilever sheet-pile wall penetrating a granular soil is as follows:

Step 1. Calculate K_a and K_p .

Step 2. Calculate σ'_1 [Eq. (4.1)] and σ'_2 [Eq. (4.2)]. (*Note:* L_1 and L_2 will be given.)

Step 3. Calculate L_3 [Eq. (4.6)].

Step 4. Calculate P.

Step 5. Calculate z (i.e., the center of pressure for the area *ACDE*) by taking the moment about *E*.

Step 6. Calculate σ'_5 [Eq. (4.11)].

Step 7. Calculate A_1 , A_2 , A_3 , and A_4 [Eqs. (4.17) through (4.20)].

Step 8. Solve Eq. (4.16) by trial and error to determine L_4 .

Step 9. Calculate σ'_4 [Eq. (4.10)].

Step 10. Calculate σ'_3 [Eq. (4.7)].

Step 11. Obtain L_5 from Eq. (4.15).

Step 12. Draw a pressure distribution diagram like the one shown in Figure 4.9a.

Step 13. Obtain the theoretical depth [see Eq. (4.12)] of penetration as $L_3 + L_4$.The actual depth of penetration is increased by about 20 to 30%.

- Note that some designers prefer to use a factor of safety on the passive earth pressure coefficient at the beginning. In that case, in Step 1,

$$K_{p(\text{design})} = \frac{K_p}{\text{FS}}$$

where FS = factor of safety (usually between 1.5 and 2).

For this type of analysis, follow Steps 1 through 12 with the value of $K_a = tan^2(45 - \frac{\phi'}{2})$ and $K_{p(\text{design})}$ (instead of K_p). The actual depth of penetration can now be determined by adding L_3 , obtained from Step 3, and L_4 , obtained from Step 8.

Calculation of Maximum Bending Moment

The nature of the variation of the moment diagram for a cantilever sheet-pile wall is shown in Figure 4.9b. The maximum moment will occur between points E and F'. Obtaining the maximum moment (M_{max}) per unit length of the wall requires determining the point of zero shear. For a new axis z' (with origin at point E) for zero shear,

$$P = \tfrac{1}{2}(z')^2(K_p - K_a)\gamma'$$

Or

$$z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}} \qquad (4.21)$$

Once the point of zero shear force is determined (point F'' in Figure 4.9a), the magnitude of the maximum moment can be obtained as

$$M_{max} = P(\bar{z} + z') - [\tfrac{1}{2}\gamma' z'^2(K_p - K_a)](\tfrac{1}{3})z' \qquad (4.22)$$

The necessary profile of the sheet piling is then sized according to the allowable flexural stress of the sheet pile material, or

$$S = \frac{M_{max}}{\sigma_{all}} \quad (4.23)$$

where

S = section modulus of the sheet pile required per unit length of the structure

σ_{all} = allowable flexural stress of the sheet pile

Example 4.1

Figure 4.10 shows a cantilever sheet-pile wall penetrating a granular soil. Here, L_1= 2 m, L_2= 3 m, γ = 15.9 kN/m^3, γ_{sat} = 19.33 kN/m^3, and ϕ'= 32°.

a. What is the theoretical depth of embedment, D?

b. For a 30% increase in D, what should be the total length of the sheet piles?

c. What should be the minimum section modulus of the sheet piles? Use σ_{all}=172 MN/m^2.

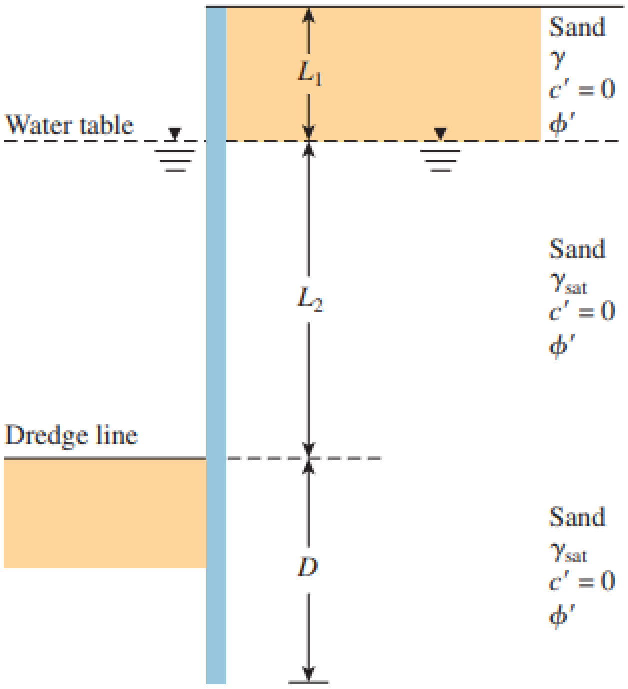


Figure 4.10 Cantilever sheet-pile wall

Part a

Using Figure 4.9a for the pressure distribution diagram, one can now prepare the following table for a step-by-step calculation.

Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45-\frac{\phi'}{2}\right)=\tan^2\left(45-\frac{32}{2}\right)=0.307$
K_p	—	$\tan^2\left(45+\frac{\phi'}{2}\right)=\tan^2\left(45+\frac{32}{2}\right)=3.25$
σ_1'	18.1	$\gamma L_1 K_a=(15.9)(2)(0.307)=9.763\text{ kN/m}^2$
σ_2'	18.2	$(\gamma L_1+\gamma' L_2)K_a=[(15.9)(2)+(19.33-9.81)(3)](0.307)=18.53\text{ kN/m}^2$
L_3	18.6	$\frac{\sigma_2'}{\gamma'(K_p-K_a)}=\frac{18.53}{(19.33-9.81)(3.25-0.307)}=0.66\text{ m}$
P	—	$\frac{1}{2}\sigma_1'L_1+\sigma_1'L_2+\frac{1}{2}(\sigma_2'-\sigma_1')L_2+\frac{1}{2}\sigma_2'L_3$ $=\left(\frac{1}{2}\right)(9.763)(2)+(9.763)(3)+\left(\frac{1}{2}\right)(18.53-9.763)(3)$ $+\left(\frac{1}{2}\right)(18.53)(0.66)$ $=9.763+29.289+13.151+6.115=58.32\text{ kN/m}$
$\bar{z}$	—	$\frac{\Sigma M_E}{P}=\frac{1}{58.32}\left[\begin{matrix}9.763(0.66+3+\frac{2}{3})+29.289(0.66+\frac{3}{2})\\+13.151(0.66+\frac{3}{3})+6.115(0.66\times\frac{2}{3})\end{matrix}\right]=2.23\text{ m}$
σ_5'	18.11	$(\gamma L_1+\gamma' L_2)K_p+\gamma' L_3(K_p-K_a)=[(15.9)(2)+(19.33-9.81)(3)](3.25)+(19.33-9.81)(0.66)(3.25-0.307)=214.66\text{ kN/m}^2$
A_1	18.17	$\frac{\sigma_5'}{\gamma'(K_p-K_a)}=\frac{214.66}{(19.33-9.81)(3.25-0.307)}=7.66\text{ m}$
A_2	18.18	$\frac{8P}{\gamma'(K_p-K_a)}=\frac{(8)(58.32)}{(19.33-9.81)(3.25-0.307)}=16.65\ \text{m}^2$
A_3	18.19	$\frac{6P[2\bar{z}\gamma'(K_p-K_a)+\sigma_5']}{\gamma'^2(K_p-K_a)^2}$ $=\frac{(6)(58.32)[(2)(2.23)(19.33-9.81)(3.25-0.307)+214.66]}{(19.33-9.81)^2(3.25-0.307)^2}$ $=151.93\ \text{m}^3$
A_4	18.20	$\frac{P(6\bar{z}\sigma_5'+4P)}{\gamma'^2(K_p-K_a)^2}=\frac{58.32[(6)(2.23)(214.66)+(4)(58.32)]}{(19.33-9.81)^2(3.25-0.307)^2}$ $=230.72\ \text{m}^4$
L_4	18.16	$L_4^4+A_1L_4^3-A_2L_4^2-A_3L_4-A_4=0$ $L_4^4+7.66L_4^3-16.65L_4^2-151.93L_4-230.72=0;\ L_4\approx 4.8\text{ m}$

Thus,

$$D_{theory} = L_3 + L_4 = 0.66 + 4.8 = \mathbf{5.46\ m}$$

Part b

The total length of the sheet piles is

$$L_1 + L_2 + 1.3(L_3 + L_4) = 2 + 3 + 1.3(5.46) = \mathbf{12.1\ m}$$

Part c

Finally, we have the following table.

Quantity required	Eq. no.	Equation and calculation
z'	18.21	$\sqrt{\dfrac{2P}{(K_p - K_a)\gamma'}} = \sqrt{\dfrac{(2)(58.32)}{(3.25 - 0.307)(19.33 - 9.81)}} = 2.04\text{ m}$
M_{max}	18.22	$P(\bar{z} + z') - \left[\dfrac{1}{2}\gamma' z'^2 (K_p - K_a)\right]\dfrac{z'}{3} = (58.32)(2.23 + 2.04)$
		$-\left[\left(\dfrac{1}{2}\right)(19.33 - 9.81)(2.04)^2(3.25 - 0.307)\right]\dfrac{2.04}{3}$
		$= 209.39\text{ kN}\cdot\text{m/m}$
S	18.29	$\dfrac{M_{max}}{\sigma_{all}} = \dfrac{209.39\text{ kN}\cdot\text{m}}{172 \times 10^3\text{ kN/m}^2} = \mathbf{1.217 \times 10^{-3}\ m^3/m\ of\ wall}$
From Table 18.1, use PS-31.		

4.5 Cantilever Sheet Piling Penetrating Clay

At times, cantilever sheet piles must be driven into a clay layer possessing an undrained cohesion (ϕ=0). The net pressure diagram will be somewhat different from that shown in Figure 4.9a. Figure 4.13 shows a cantilever sheet-pile wall driven into clay with a backfill of granular soil above the level of the dredge line. The water table is at a depth L_1 below the top of the wall. As before, Eqs. (4.1) and (4.2) give the intensity of the net pressures σ'_1 and σ'_2 , and the diagram for pressure distribution above the level of the dredge line can be drawn. The diagram for net pressure distribution below the dredge line can now be determined as follows.

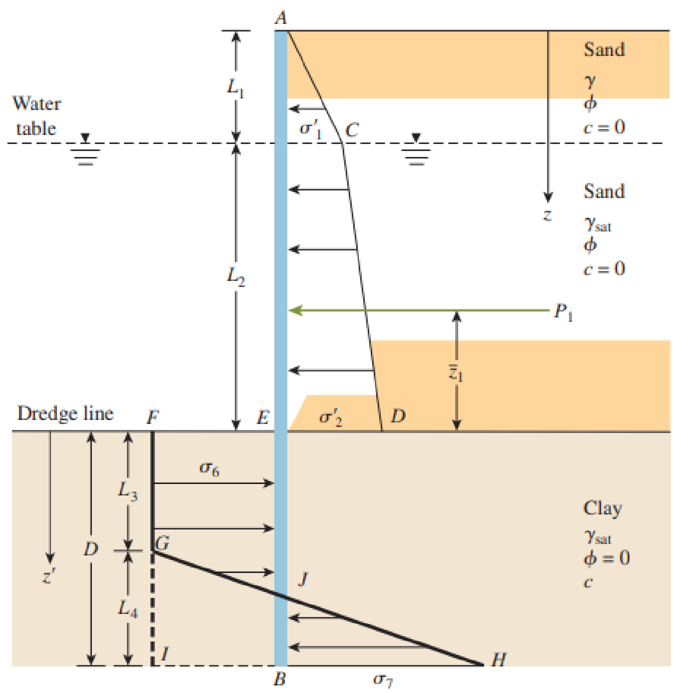


Figure 4.13 Cantilever sheet pile penetrating clay

At any depth greater than $L_1 + L_2$, for ϕ=0, the Rankine active earth-pressure coefficient K_a = 1. Similarly, for ϕ=0, the Rankine passive earth-pressure coefficient K_p=1. Thus, above the point of rotation (point O in Figure 4.8a), the active pressure, from right to left is

$$\sigma_a = [\gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2)] - 2c \qquad (4.24)$$

Similarly, the passive pressure from left to right may be expressed as

$$\sigma_p = \gamma_{sat}(z - L_1 - L_2) + 2c \qquad (4.25)$$

Thus, the net pressure is

$$\begin{aligned}\sigma_6 = \sigma_p - \sigma_a &= [\gamma_{sat}(z - L_1 - L_2) + 2c] \\ &\quad - [\gamma L_1 + \gamma' L_2 + \gamma_{sat}(z - L_1 - L_2)] + 2c \\ &= 4c - (\gamma L_1 + \gamma' L_2)\end{aligned} \qquad (4.26)$$

At the bottom of the sheet pile, the passive pressure from right to left is

$$\sigma_p = (\gamma L_1 + \gamma' L_2 + \gamma_{sat} D) + 2c \qquad (4.27)$$

Similarly, the active pressure from left to right is

$$\sigma_a = \gamma_{sat} D - 2c \qquad (4.28)$$

Hence, the net pressure is

$$\sigma_7 = \sigma_p - \sigma_a = 4c + (\gamma L_1 + \gamma' L_2) \qquad (4.29)$$

For equilibrium analysis, $\sum F_H = 0$; that is, the area of the pressure diagram *ACDE* minus the area of *EFIB* plus the area of *GIH* = 0, or

$$P_1 - [4c - (\gamma L_1 + \gamma' L_2)]D + \tfrac{1}{2}L_4[4c - (\gamma L_1 + \gamma' L_2) + 4c + (\gamma L_1 + \gamma' L_2)] = 0$$

where P_1 = area of the pressure diagram *ACDE.*

Simplifying the preceding equation produces

$$L_4 = \frac{D[4c - (\gamma L_1 + \gamma' L_2)] - P_1}{4c} \qquad (4.30)$$

Now, taking the moment about point *B* ($\sum M_B = 0$) yields

$$P_1(D + \bar{z}_1) - [4c - (\gamma L_1 + \gamma' L_2)]\frac{D^2}{2} + \frac{1}{2}L_4(8c)\left(\frac{L_4}{3}\right) = 0 \qquad (4.31)$$

where $\bar{z}_1$ = distance of the center of pressure of the pressure diagram *ACDE*, measured from the level of the dredge line.

Combining Eqs. (4.30) and (4.31) yields

$$D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c} = 0 \qquad (4.32)$$

Equation (4.32) may be solved to obtain *D*, the theoretical depth of penetration of the clay layer by the sheet pile.

Step-by-Step Procedure for Obtaining the Pressure Diagram

Step 1. Calculate K_a for the granular soil (backfill).
Step 2. Obtain σ'_1 and σ'_2 . [See Eqs. (4.1) and (4.2).]
Step 3. Calculate P_1 and z_1 .
Step 4. Use Eq. (4.32) to obtain the theoretical value of D.
Step 5. Using Eq. (4.30), calculate L_4.
Step 6. Calculate σ_6 and σ_7 . [See Eqs. (4.26) and (4.29).]
Step 7. Draw the pressure distribution diagram as shown in Figure 4.13.
Step 8. The actual depth of penetration is

$$D_{\text{actual}} = 1.4 \text{ to } 1.6(D_{\text{theoretical}})$$

Maximum Bending Moment

According to Figure 4.13, the maximum moment (zero shear) will be between $L_1 + L_2 < z < L_1 + L_2 + L_3$. Using a new coordinate system *z*9 (with $z' = 0$ at the dredge line) for zero shear gives

$$P_1 - \sigma_6 z' = 0$$

or

$$z' = \frac{P_1}{\sigma_6} \qquad (4.33)$$

The magnitude of the maximum moment may now be obtained:

$$M_{\max} = P_1(z' + \bar{z}_1) - \frac{\sigma_6 z'^2}{2} \qquad (4.34)$$

Knowing the maximum bending moment, we determine the section modulus of the sheet-pile section from Eq. (4.23).

Example 4.2:

In Figure 4.14, for the sheet-pile wall, determine

a. The theoretical and actual depth of penetration. Use $D_{actual} = 1.5D_{theory}$.

b. The minimum size of sheet-pile section necessary. Use σ_{all} = 172.5 MN/m^2.

SOLUTION

We will follow the step-by-step procedure given in Section 18.6.

Step 1.

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$$

Step 2.

$$\sigma_1' = \gamma L_1 K_a = (15.9)(2)(0.307) = 9.763 \text{ kN/m}^2$$
$$\sigma_2' = (\gamma L_1 + \gamma' L_2)K_a = [(15.9)(2) + (19.33 - 9.81)3]0.307$$
$$= 18.53 \text{ kN/m}^2$$

Step 3. From the net pressure distribution diagram given in Figure 18.13, we have

$$P_1 = \frac{1}{2}\sigma_1' L_1 + \sigma_1' L_2 + \frac{1}{2}(\sigma_2' - \sigma_1')L_2$$
$$= 9.763 + 29.289 + 13.151 = 52.2 \text{ kN/m}$$

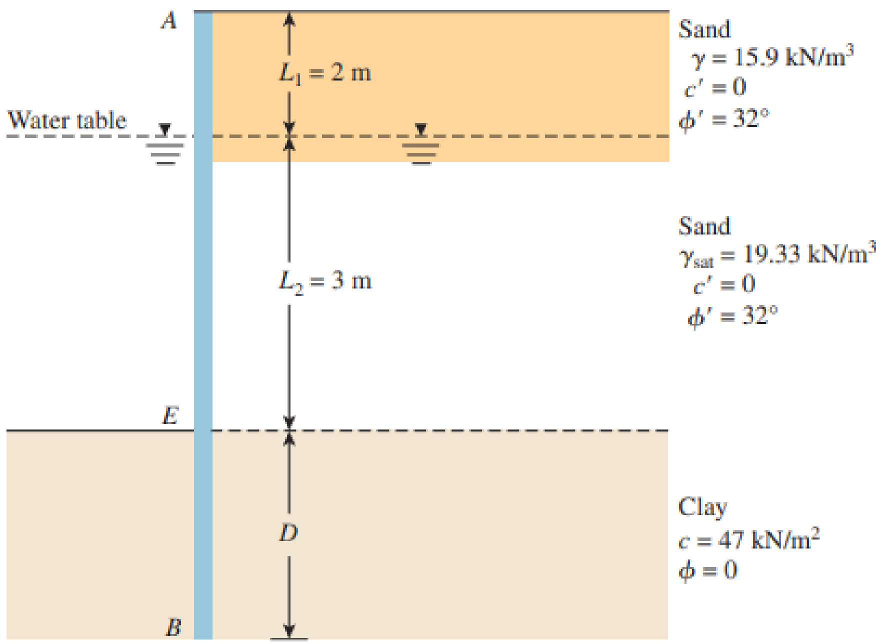


Figure 4.14 Cantilever sheet pile penetrating into saturated clay

and

$$\bar{z}_1 = \frac{1}{52.2}\left[9.763\left(3 + \frac{2}{3}\right) + 29.289\left(\frac{3}{2}\right) + 13.151\left(\frac{3}{3}\right)\right]$$
$$= 1.78 \text{ m}$$

Step 4. From Eq. (18.48),

$$D^2[4c - (\gamma L_1 + \gamma' L_2)] - 2DP_1 - \frac{P_1(P_1 + 12c\bar{z}_1)}{(\gamma L_1 + \gamma' L_2) + 2c} = 0$$

Substituting proper values yields

$$D^2\{(4)(47) - [(2)(15.9) + (19.33 - 9.81)3]\} - 2D(52.2)$$
$$- \frac{52.2[52.2 + (12)(47)(1.78)]}{[(15.9)(2) + (19.33 - 9.81)3] + (2)(47)} = 0$$

or

$$127.64D^2 - 104.4D - 357.15 = 0$$

Solving the preceding equation, we obtain $D = 2.13$ m.

Step 5. From Eq. (18.46),

$$L_4 = \frac{D[4c - (\gamma L_1 + \gamma' L_2)] - P_1}{4c}$$

and

$$4c - (\gamma L_1 + \gamma' L_2) = (4)(47) - [(15.9)(2) + (19.33 - 9.81)3]$$
$$= 127.64 \text{ kN/m}^2$$

So,

$$L_4 = \frac{2.13(127.64) - 52.2}{(4)(47)} = 1.17 \text{ m}$$

Step 6.

$$\sigma_6 = 4c - (\gamma L_1 + \gamma' L_2) = 127.64 \text{ kN/m}^2$$
$$\sigma_7 = 4c + (\gamma L_1 + \gamma' L_2) = 248.36 \text{ kN/m}^2$$

Step 7. The net pressure distribution diagram can now be drawn, as shown in Figure 18.13.

Step 8. $D_{\text{actual}} \approx 1.5\, D_{\text{theoretical}} = 1.5(2.13) \approx$ **3.2 m**

Maximum Moment Calculation

From Eq. (18.49),

$$z' = \frac{P_1}{\sigma_6} = \frac{52.2}{127.64} \approx 0.41 \text{ m}$$

Again, from Eq. (18.49),

$$M_{\max} = P_1(z' + \bar{z}_1) - \frac{\sigma_6 z'^2}{2}$$

So

$$M_{\max} = 52.2(0.41 + 1.78) - \frac{127.64(0.41)^2}{2}$$
$$= 114.32 - 10.73 = 103.59 \text{ kN·m/m}$$

The minimum required section modulus (assuming that $\sigma_{all} = 172.5\ MN/m^2$) is

$$S = \frac{103.59\ kN \cdot m/m}{172.5 \times 10^3\ kN/m^2} = \mathbf{0.6 \times 10^{-3}\ m^3/m\ of\ the\ wall}$$