University of Anbar
Engineering College
Civil Engineering Department

CHAPTER THREE

RETAINING WALLS

LECTURES

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3.1 Introduction

A retaining wall is a wall that provides lateral support for a vertical or near-vertical slope of soil. It is a common structure used in many construction projects. The most common types of retaining wall may be classified as follows:

1. Gravity retaining walls
2. Semigravity retaining walls
3. Cantilever retaining walls
4. Counterfort retaining walls

- ***Gravity retaining walls*** **(Figure 3.1a)** are constructed with plain concrete or stone masonry. They depend for stability on their own weight and any soil resting on the masonry. This type of construction is not economical for high walls.
- In many cases, a small amount of steel may be used for the construction of gravity walls, thereby minimizing the size of wall sections. Such walls are generally referred to as ***semigravity walls*** **(Figure 3.1b)**.
- ***Cantilever retaining walls*** **(Figure 3.1c)** are made of reinforced concrete that consists of a thin stem and a base slab. This type of wall is economical to a height of about 8 m as Figure (3.2).
- ***Counterfort retaining walls*** **(Figure 3.1d)** are similar to cantilever walls. At regular intervals, however, they have thin vertical concrete slabs known as *counterforts* that tie the wall and the base slab together. The purpose of the counterforts is to reduce the shear and the bending moments.

- To design retaining walls properly, an engineer must know the basic parameters— the *unit weight, angle of friction,* and *cohesion*—of the soil retained behind the wall and the soil below the base slab. Knowing the properties of the soil behind the wall enables the engineer to determine the lateral pressure distribution that has to be designed for.

- There are two phases in the design of a conventional retaining wall. First, with the lateral earth pressure known, the structure as a whole is checked for *stability*. The structure is examined for possible *overturning*, *sliding*, and *bearing capacity* failures. Second, each component of the structure is checked for *strength*, and the *steel reinforcement* of each component is determined.

- This chapter presents the procedures for determination of lateral earth pressure and retaining-wall stability.

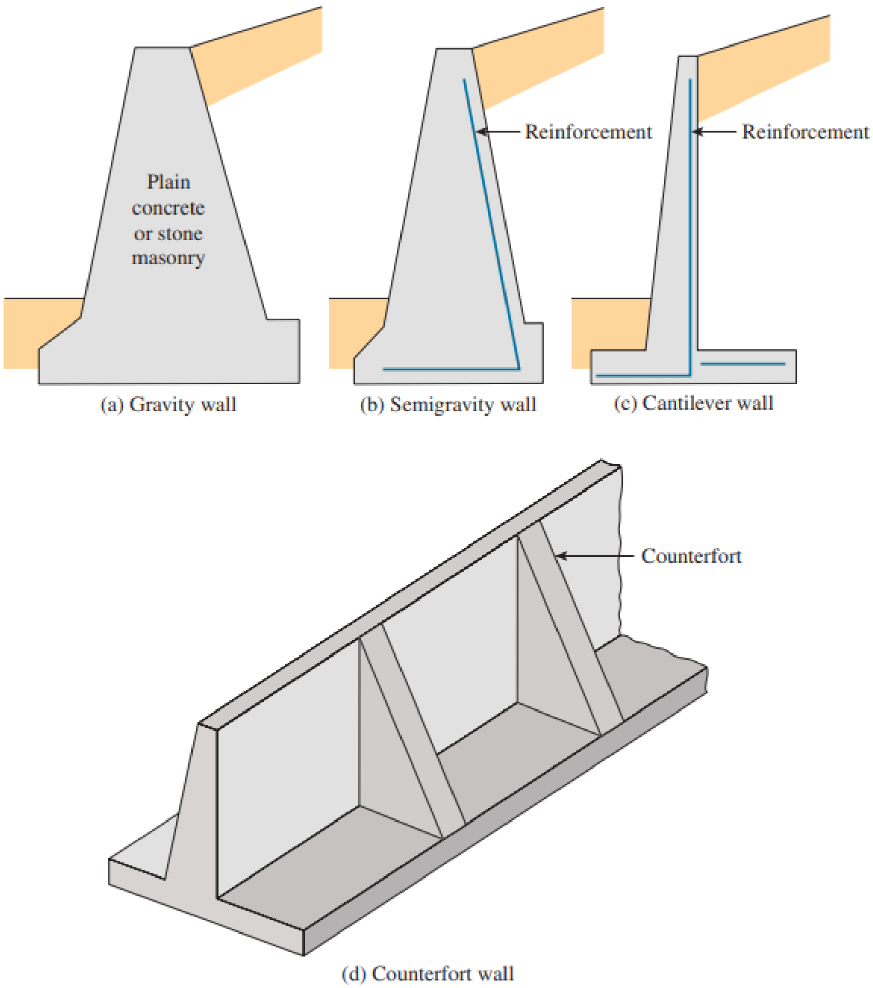


Figure 3.1 Types of retaining wal

Figure 3.2 A cantilever retaining wall under construction

3.2 Gravity and Cantilever Walls

3.2.1 Proportioning Retaining Walls

- In designing retaining walls, an engineer must assume some of their dimensions. Called *proportioning,* such assumptions allow the engineer to check trial sections of the walls for stability. If the stability checks yield undesirable results, the sections can be changed and rechecked. Figure 3.3 shows the general proportions of various retaining-wall components that can be used for initial checks.
- Note that the top of the stem of any retaining wall should not be less than about 0.3 m. for proper placement of concrete. The depth, *D,* to the bottom of the base slab should be a minimum of 0.6m. However, the bottom of the base slab should be positioned below the seasonal frost line.
- For counterfort retaining walls, the general proportion of the stem and the base slab is the same as for cantilever walls. However, the counterfort slabs may be about 0.3 m thick and spaced at center-to-center distances of 0.3*H* to 0.7*H.*

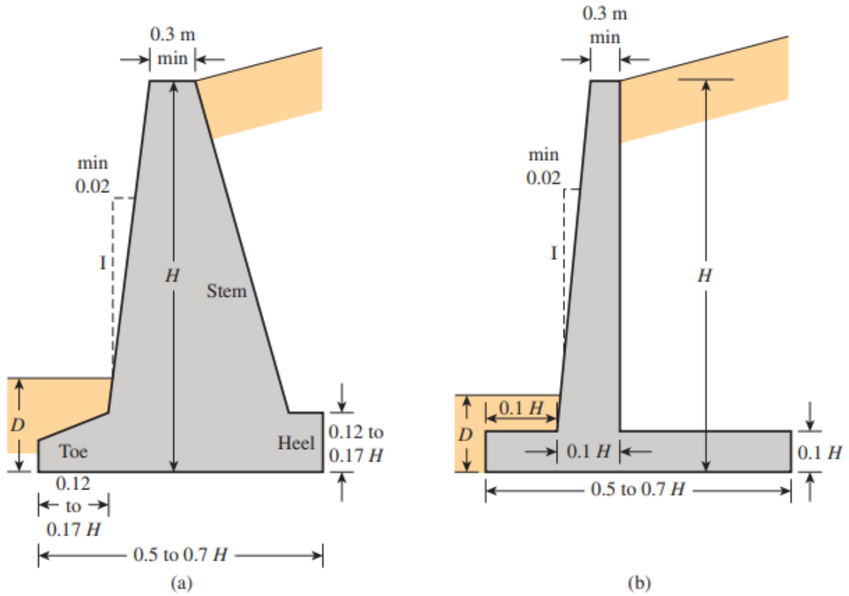


Figure 3.3 Approximate dimensions for various components of retaining wall for initial stability checks: (a) gravity wall; (b) cantilever wall

3.3 Application of Lateral Earth Pressure Theories to Design

The fundamental theories for calculating lateral earth pressure were presented in Chapter 2. To use these theories in design, an engineer must make several simple assumptions. In the case of cantilever walls, the use of the Rankine earth pressure theory for stability checks involves drawing a vertical line *AB* through point *A*, located at the edge of the heel of the base slab in Figure 3.4a. The Rankine active condition is assumed to exist along the vertical plane *AB*. Rankine active earth pressure equations may then be used to calculate the lateral pressure on the face *AB* of the wall. In the analysis of the wall's stability, the force $P_{a(\mathrm{Rankine})}$, the weight of soil above the heel, and the weight W_c of the concrete all should be taken into consideration. The assumption for the development of Rankine active pressure along the soil face *AB* is theoretically correct if the shear zone bounded by the line *AC* is not obstructed by the stem of the wall. The angle, *h*, that the line *AC* makes with the vertical is

$$\eta = 45 + \frac{\alpha}{2} - \frac{\phi'}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) \qquad (3\text{-}1)$$

A similar type of analysis may be used for gravity walls, as shown in Figure 3.4b. However, Coulomb's active earth pressure theory also may be used, as shown in Figure 3.4c. If it is used, the only forces to be considered are $P_{a(\mathrm{Coulomb})}$ and the weight of the wall, W_c.

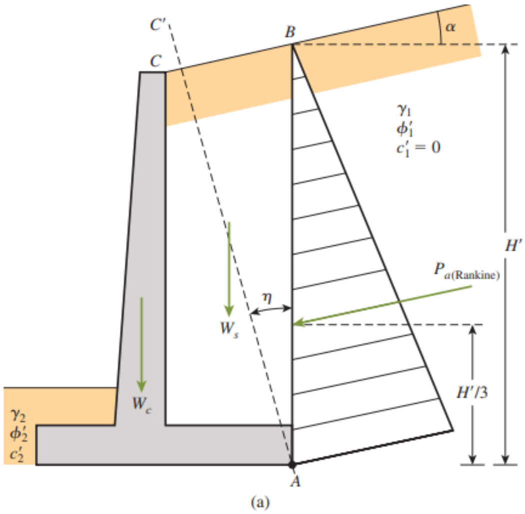


Figure 3.4 Assumption for the determination of lateral earth pressure: (a) cantilever wall; (b) and (c) gravity wall

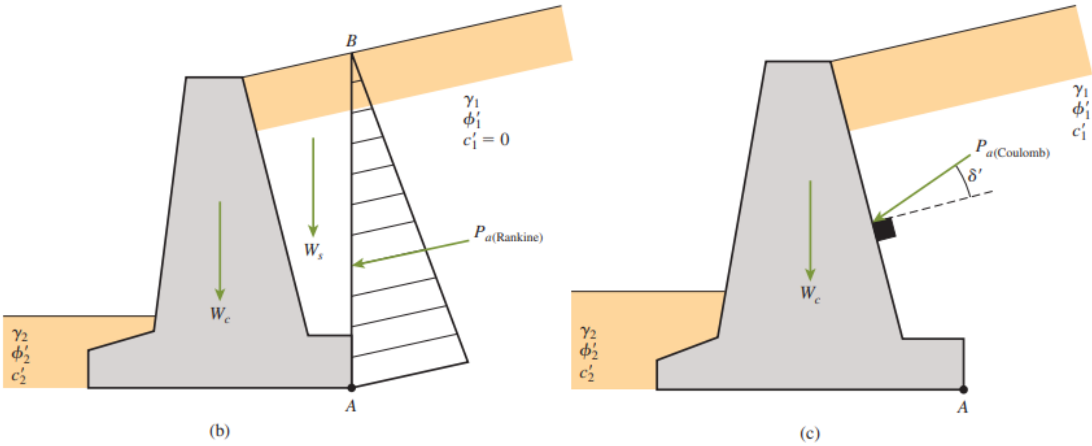


Figure 3.4 (*continued*)

- If Coulomb's theory is used, it will be necessary to know the range of the wall friction angle δ' with various types of backfill material. Following are some ranges of wall friction angle for masonry or mass concrete walls:

Backfill material	Range of δ' (deg)
Gravel	27–30
Coarse sand	20–28
Fine sand	15–25
Stiff clay	15–20
Silty clay	12–16

- In the case of ordinary retaining walls, water table problems and hence hydrostatic pressure are not encountered. Facilities for drainage from the soils that are retained are always provided.

3.4 Stability of Retaining Walls

A retaining wall may fail in any of the following ways:

- It may overturn about its toe. (See Figure 3.5a.)
- It may *slide* along its base. (See Figure 3.5b.)
- It may fail due to the loss of *bearing capacity* of the soil supporting the base. (See Figure 3.5c.)
- It may undergo deep-seated shear failure. (See Figure 3.5d.)
- It may go through excessive settlement.
    - The checks for stability against overturning, sliding, and bearing capacity failure will be described in Sections 3.5, 3.6, and 3.7. When a weak soil layer is located at a shallow depth—that is, within a depth of 1.5 times the width of the base slab of the retaining wall—the possibility of excessive settlement should be considered. In some cases, the use of lightweight backfill material behind the retaining wall may solve the problem.
    - *Deep shear failure* can occur along a cylindrical surface, such as *abc* shown in Figure 3.6, as a result of the existence of a weak layer of soil underneath the wall at a depth of about 1.5 times the width of the base slab of the retaining wall. In such cases, the critical cylindrical failure surface *abc* has to be determined by trial and error, using various centers such as *O*. The failure surface along which the minimum factor of safety is obtained is the *critical surface of sliding*. For the backfill slope with α less than about 10^o, the critical failure circle apparently passes through the edge of the heel slab (such as *def* in the figure). In this situation, the minimum factor of safety also has to be determined by trial and error by changing the center of the trial circle.

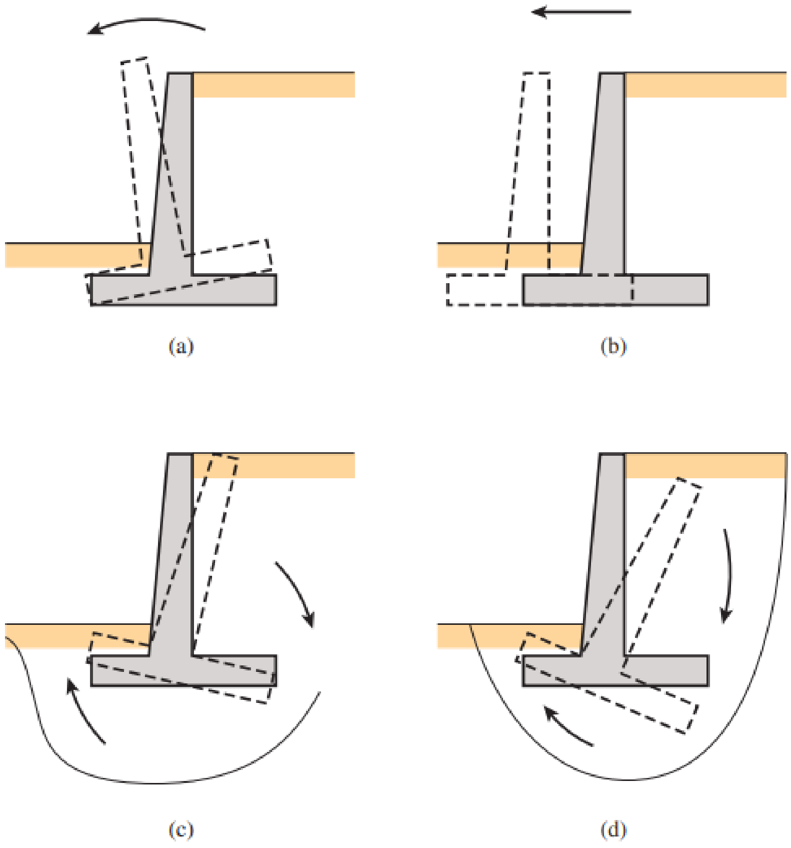


Figure 3.5 Failure of retaining wall: (a) by overturning; (b) by sliding; (c) by bearing capacity failure; (d) by deep-seated shear failure

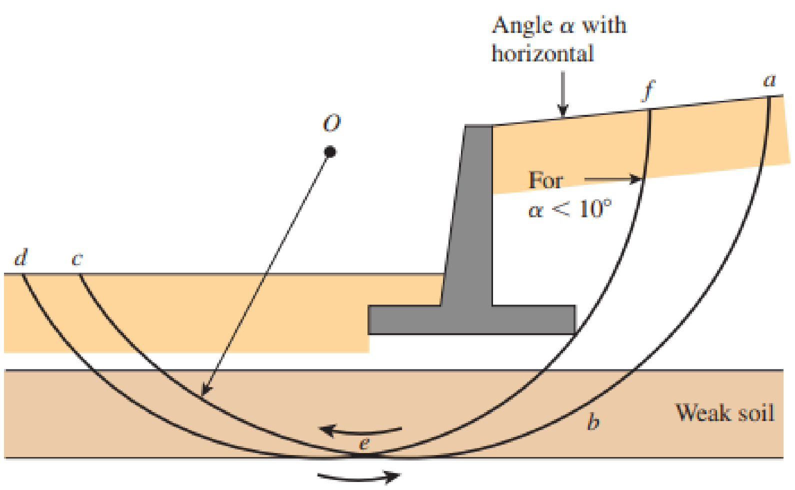


Figure 3.6 Deep-seated shear failure

3.5 Check for Overturning

Figure 3.7 shows the forces acting on a cantilever and a gravity retaining wall, based on the assumption that the Rankine active pressure is acting along a vertical plane *AB* drawn through the heel of the structure. P_p is the Rankine passive pressure; recall that its magnitude is

$$P_p = \frac{1}{2} K_p \gamma_2 D^2 + 2c_2' \sqrt{K_p} D$$

where

γ_2= unit weight of soil in front of the heel and under the base slab

K_p = Rankine passive earth pressure coefficient 5 tan2s45 1 *f*92y2d

c_2' , ϕ_2' = cohesion and effective soil friction angle, respectively

The factor of safety against overturning about the toe—that is, about point *C* in Figure 3.7—may be expressed as

$$FS_{(overturning)} = \frac{\sum M_R}{\sum M_o} \qquad (3\text{-}2)$$

where

$\sum M_R$= sum of the moments of forces tending to overturn about point *C*

$\sum M_o$ = sum of the moments of forces tending to resist overturning about point *C*

The overturning moment is

$$\Sigma M_o = P_h \left(\frac{H'}{3} \right) \qquad (3\text{-}3)$$

Where $P_h = P_a cos\alpha$

To calculate the resisting moment, $\sum M_R$ (neglecting P_p), a table such as Table 3.1 can be prepared. The weight of the soil above the heel and the weight of the concrete (or masonry) are both forces that contribute to the resisting moment. Note that the force P_v also contributes to the resisting moment. P_v is the vertical component of the active force P_a , or

$$P_v = P_a sin\alpha$$

The moment of the force P_v about *C* is

$M_v = P_v B = P_a sin\alpha B$ (3-4)

where B = width of the base slab.
Once $\sum M_R$ is known, the factor of safety can be calculated as

$$FS_{(overturning)} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v}{P_a \cos\alpha (H'/3)} \quad (3\text{-}5)$$

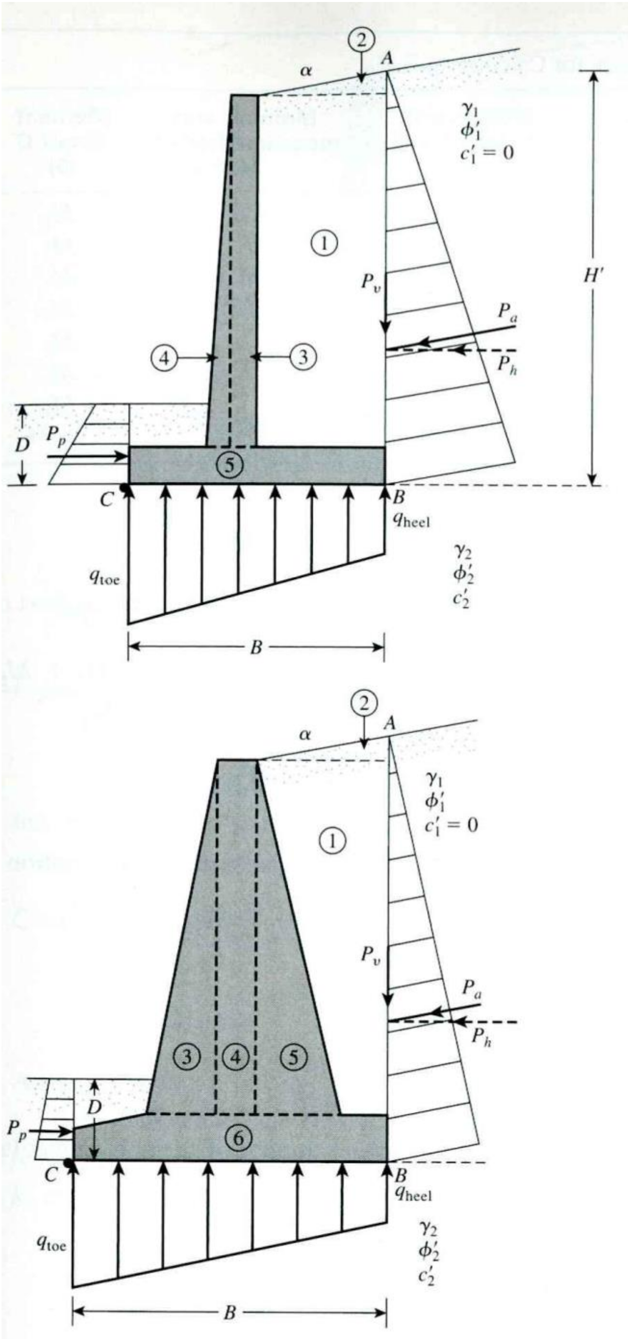


Figure 3.7 Check for overturning, assuming that the Rankine pressure is valid

Table 3.1 Procedure for Calculating $\sum M_R$

Section (1)	Area (2)	Weight/unit length of wall (3)	Moment arm measured from C (4)	Moment about C (5)
1	A_1	$W_1 = \gamma_1 \times A_1$	X_1	M_1
2	A_2	$W_2 = \gamma_1 \times A_2$	X_2	M_2
3	A_3	$W_3 = \gamma_c \times A_3$	X_3	M_3
4	A_4	$W_4 = \gamma_c \times A_4$	X_4	M_4
5	A_5	$W_5 = \gamma_c \times A_5$	X_5	M_5
6	A_6	$W_6 = \gamma_c \times A_6$	X_6	M_6
		P_v	B	M_v
		ΣV		ΣM_R

(*Note:* γ_l = unit weight of backfill
γ_c = unit weight of concrete)

The usual minimum desirable value of the factor of safety with respect to overturning is 2 to 3.

Some designers prefer to determine the factor of safety against overturning with the formula

$$\text{FS}_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6}{P_a \cos\alpha(H'/3) - M_v} \quad (3\text{-}6)$$

3.6 Check for Sliding along the Base

The factor of safety against sliding may be expressed by the equation

$$FS_{(sliding)} = \frac{\sum F_R}{\sum F_d} \quad (3\text{-}7)$$

where

$\sum F_R$ = sum of the horizontal resisting forces

$\sum F_d$ = sum of the horizontal driving forces

Figure 3.8 indicates that the shear strength of the soil immediately below the base slab may be represented as

$$s = \sigma' \tan\delta' + c'_a$$

where

δ' = angle of friction between the soil and the base slab

c_a' = adhesion between the soil and the base slab

Thus, the maximum resisting force that can be derived from the soil per unit length of the wall along the bottom of the base slab is

$$R' = s(\text{area of cross section}) = s(B \times 1) = B\sigma' \tan\delta' + Bc'_a$$

However,

$$B\sigma' = \text{sum of the vertical force} = \Sigma V \quad \text{(see Table 3.1)}$$

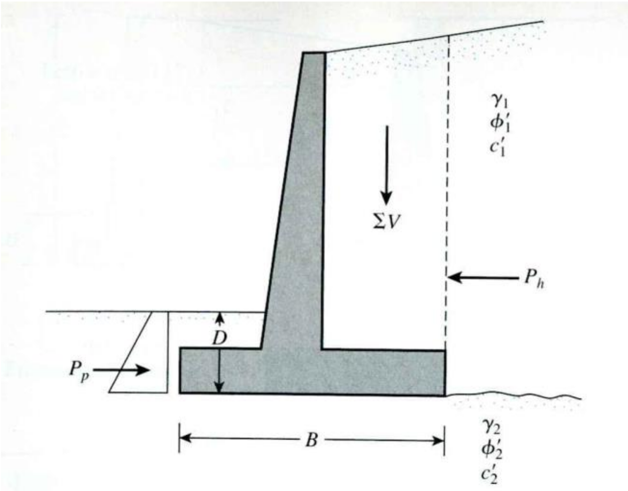


Figure 3.8 Check for sliding along the base

Figure 3.8 shows that the passive force P_p is also a horizontal resisting force. Hence,

$$\Sigma F_{R'} = (\Sigma V)\tan\delta' + Bc'_a + P_p \qquad (3\text{-}8)$$

The only horizontal force that will tend to cause the wall to slide (a *driving force*) is the horizontal component of the active force P_a , so

$$\Sigma F_d = P_a \cos\alpha \qquad (3\text{-}9)$$

Combining Eqs. (3.7), (3.8), and (3.9) yields

$$\text{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan\delta' + Bc'_a + P_p}{P_a \cos\alpha} \qquad (3\text{-}10)$$

A minimum factor of safety of 1.5 against sliding is generally required.

In many cases, the passive force P_p is ignored in calculating the factor of safety with respect to sliding. In general, we can write $\delta' = k_1\phi_2'$ and $c_a' = k_2 c_2'$. In most cases, k_1 and k_2 are in the range from 1/2 to 2/3. Thus,

$$\text{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a \cos\alpha} \qquad (3\text{-}11)$$

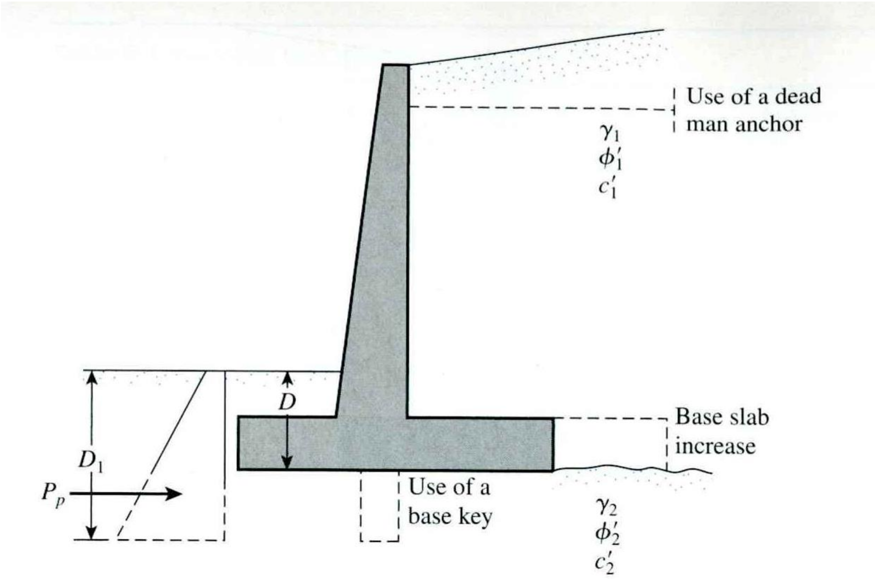


Figure 3.9 Alternatives for increasing the factor of safety with respect to sliding

If the desired value of $FS_{(sliding)}$ is not achieved, several alternatives may be investigated (see Figure 3.9):

- Increase the width of the base slab (i.e., the heel of the footing).
- Use a key to the base slab. If a key is included, the passive force per unit length of the wall becomes

$$P_p = \frac{1}{2}\gamma_2 D_1^2 K_p + 2c'_2 D_1 \sqrt{K_p}$$

where $K_p = \tan^2\left(45 + \frac{\phi'_2}{2}\right)$.

- Use a *deadman anchor* at the stem of the retaining wall.

Example 3.1

The cross section of a cantilever retaining wall is shown in Figure 3.10. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

SOLUTION

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

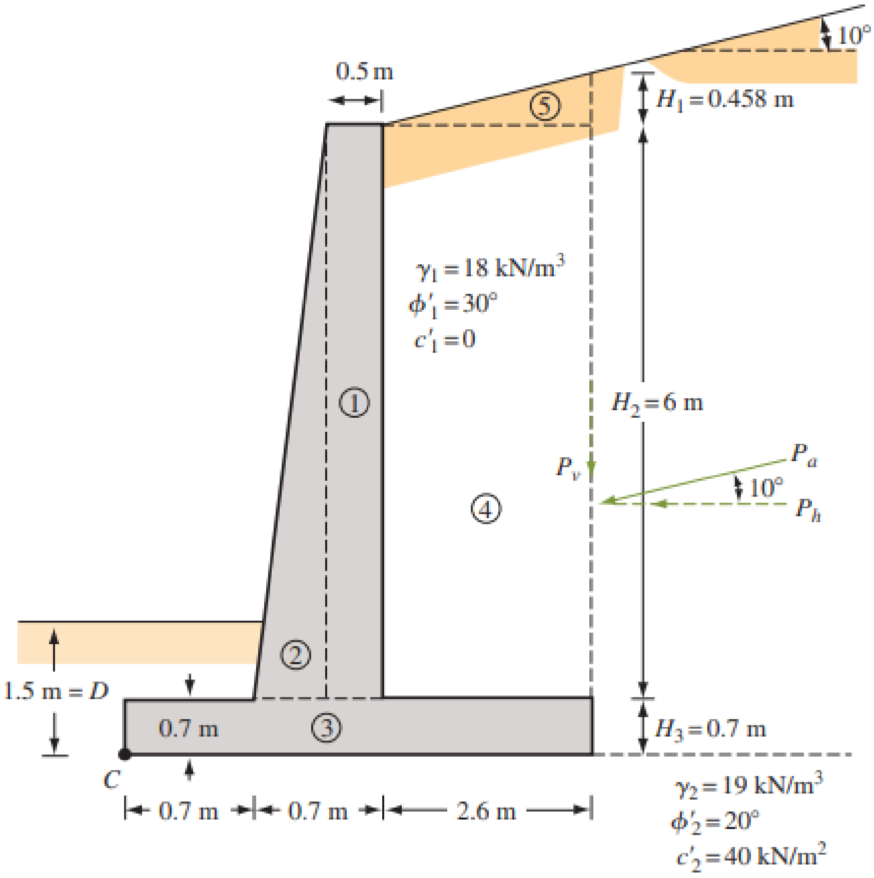


Figure 3.10 Calculation of stability of a retaining wall

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi'_1 = 30°$ and $\alpha = 10°$, K_a is equal to 0.3495. (See Table 16.1.) Thus,

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3495) = 161.2 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 161.2 (\sin 10°) = 28.0 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 161.2 (\cos 10°) = 158.75 \text{ kN/m}$$

Factor of Safety Against Overturning

The following table can now be prepared for determining the resisting moment:

Section no.[a]	Area (m^2)	Weight/unit length (kN/m)	Moment arm from point C (m)	Moment (kN·m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.0$	4.0	112.0
		$\Sigma V = 470.42$		$1128.86 = \Sigma M_R$

[a]For section numbers, refer to Figure 17.12.
Note: $\gamma_{concrete} = 23.58\ \text{kN/m}^3$

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 158.75\left(\frac{7.158}{3}\right) = 378.78\ \text{kN}\cdot\text{m/m}$$

and

$$FS_{(overturning)} = \frac{\Sigma M_R}{M_o} = \frac{1128.86}{378.78} = \mathbf{2.98 > 2,\ OK}$$

Factor of Safety Against Sliding

From Eq. (17.11),

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi_2') + Bk_2c_2' + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \tfrac{1}{2}K_p\gamma_2D^2 + 2c_2'\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5\ \text{m}$$

So

$$P_p = \tfrac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215\ \text{kN/m}$$

Hence,

$$FS_{(sliding)} = \frac{(470.42)\tan\left(\frac{2 \times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{158.75}$$

$$= \frac{111.49 + 106.67 + 215}{158.75} = \mathbf{2.73 > 1.5, OK}$$

Note: For some designs, the depth D in a passive pressure calculation may be taken to be *equal to the thickness of the base slab.*

Example 3.2

A gravity retaining wall is shown in Figure 3.11. Use δ' = 2/3ϕ' 1and Coulomb's active earth pressure theory. Determine:

a. The factor of safety against overturning

b. The factor of safety against sliding

c. The pressure on the soil at the toe and heel

SOLUTION

The height

$$H' = 5 + 1.5 = 6.5 \text{ m}$$

Coulomb's active force is

$$P_a = \tfrac{1}{2}\gamma_1 H'^2 K_a$$

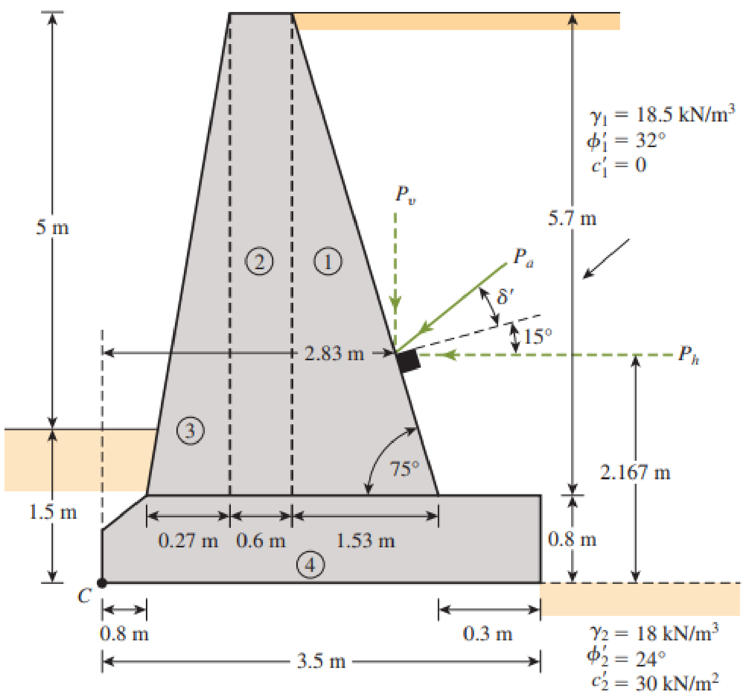


Figure 3.11 Gravity retaining wall

With $\alpha = 0°$, $\beta = 75°$, $\delta' = 2/3\phi_1'$, and $\phi_1' = 32°$, $K_a = 0.4023$. (See Table 16.6.) So,

$$P_a = \tfrac{1}{2}(18.5)(6.5)^2(0.4023) = 157.22 \text{ kN/m}$$

$$P_h = P_a \cos(15 + \tfrac{2}{3}\phi_1') = 157.22 \cos 36.33 = 126.65 \text{ kN/m}$$

and

$$P_v = P_a \sin(15 + \tfrac{2}{3}\phi_1') = 157.22 \sin 36.33 = 93.14 \text{ kN/m}$$

Part a: Factor of Safety Against Overturning

From Figure 17.13, we can prepare the following table:

Area no.	Area (m^2)	Weight* (kN/m)	Moment arm from C (m)	Moment (kN · m/m)
1	$\frac{1}{2}(5.7)(1.53) = 4.36$	102.81	2.18	224.13
2	$(0.6)(5.7) = 3.42$	80.64	1.37	110.48
3	$\frac{1}{2}(0.27)(5.7) = 0.77$	18.16	0.98	17.80
4	$\approx (3.5)(0.8) = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77$ kN/m		$\Sigma M_R = 731.54$ kN · m/m

*$\gamma_{concrete} = 23.58$ kN/m^3

Note that the weight of the soil above the backface of the wall is not taken into account in the preceding table. We have

$$\text{Overturning moment} = M_o = P_h\left(\frac{H'}{3}\right) = 126.65(2.167) = 274.45 \text{ kN} \cdot \text{m/m}$$

Hence,

$$FS_{(overturning)} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{731.54}{274.45} = \mathbf{2.67 > 2, OK}$$

Part b: Factor of Safety Against Sliding

We have

$$FS_{(sliding)} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi_2'\right) + \frac{2}{3}c_2'B + P_p}{P_h}$$

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

and

$$K_p = \tan^2\left(45 + \frac{24}{2}\right) = 2.37$$

Hence,

$$P_p = \tfrac{1}{2}(2.37)(18)(1.5)^2 + 2(30)(1.54)(1.5) = 186.59 \text{ kN/m}$$

So

$$FS_{(sliding)} = \frac{360.77 \tan\left(\frac{2}{3} \times 24\right) + \frac{2}{3}(30)(3.5) + 186.59}{126.65}$$

$$= \frac{103.45 + 70 + 186.59}{126.65} = \mathbf{2.84}$$

If P_p is ignored, the factor of safety is **1.37**.

Example 3.3

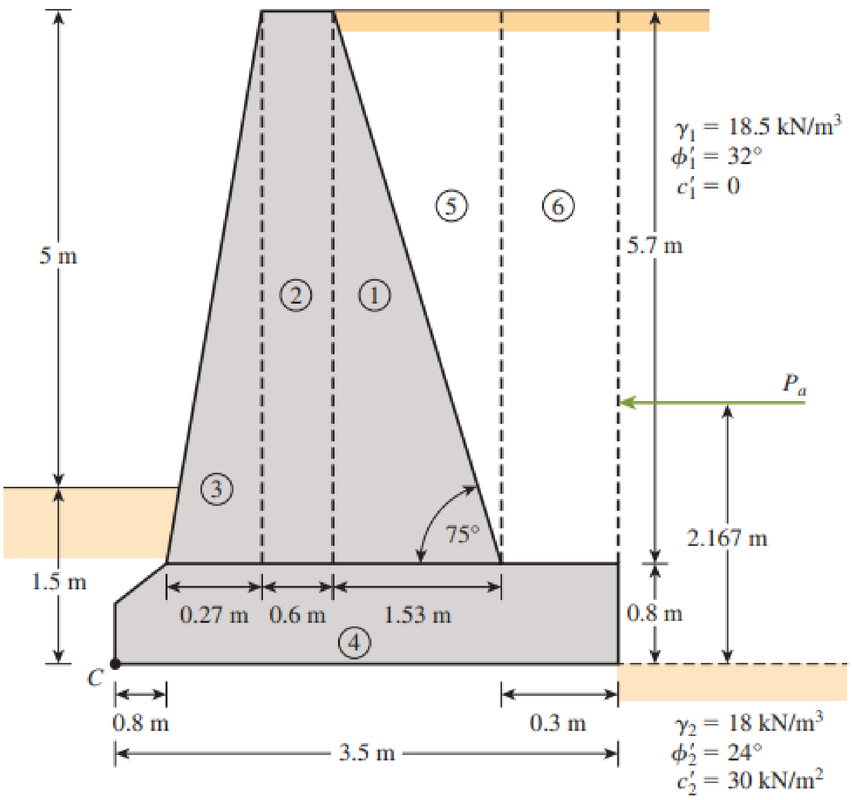
Refer to the gravity wall described in Example 3.2 and redo the problem using Rankine active pressure.

SOLUTION

The retaining wall is redrawn in Figure 17.14. From the figure, $H' = 5 + 1.5 = 6.5$ m.

$$K_a = \tan^2\left(45 - \frac{\phi'_1}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$$

$$P_a = \frac{1}{2}\gamma H'^2 K_a = \frac{1}{2}(18.5)(6.5)^2(0.307) = 119.98 \text{ kN/m} \approx 120 \text{ kN/m}$$



Part a: Factor of Safety Against Overturning

The following table can now be prepared.

Section no.	Area (m^2)	Weight per unit length (kN/m)	Moment arm from *C* (m)	Moment about *C* (kN · m/m)
*1	4.36	102.81	2.18	224.13
*2	3.42	80.64	1.37	110.48
*3	0.77	18.16	0.98	17.80
*4	2.8	66.02	1.75	115.54
5	(0.5)(1.53)(5.7) = 4.36	80.66	2.69	216.98
6	(5.7)(0.3) = 1.71	31.44	3.35	105.99
		$\Sigma V = 379.93$		$\Sigma M_R = 790.92$

Overturning moment, $M_o = P_a\left(\frac{H'}{3}\right) = (120)(2.167) \approx 260.0 \text{ kN}\cdot\text{m/m}.$

$$FS_{(overturning)} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{790.92}{260.0} = \mathbf{3.04}$$

Part b: Factor of Safety Against Sliding

$$FS_{(sliding)} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi'_2\right) + \frac{2}{3}c'_2 + P_p}{P_h}$$

From Example 17.2, $P_p = 186.59$ kN/m. So,

$$FS_{(sliding)} = \frac{(379.93)(\tan 16) + \left(\frac{2}{3}\right)(30)(3.5) + 186.59}{120} = \mathbf{3.05}$$