University of Anbar
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CHAPTER TWO
LATERAL EARTH PRESSURE THEORY

LECTURES
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2024-2025

2.1 Introduction

Vertical or near-vertical slopes of soil are supported by retaining walls, cantilever sheet-pile walls, sheet-pile bulkheads, braced cuts, and other similar structures. The proper design of those structures requires an estimation of lateral earth pressure, which is a function of several factors, such as **(a) the type and amount of wall movement, (b) the shear strength parameters of the soil, (c) the unit weight of the soil, and (d) the drainage conditions in the backfill**. Figure 2.1 shows a retaining wall of height H. For similar types of backfill,

a. The wall may be restrained from moving (Figure 2.1a). The lateral earth pressure on the wall at any depth is called the at-rest earth pressure.
b. The wall may tilt away from the soil that is retained (Figure 2.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as active earth pressure.
c. The wall may be pushed into the soil that is retained (Figure 6.1c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as passive earth pressure.

Figure 2.2 shows the nature of variation of the lateral pressure, σ'_h, at a certain depth of the wall with the magnitude of wall movement. The wall movement required to mobilize the active state is less than that required to mobilize the passive state.

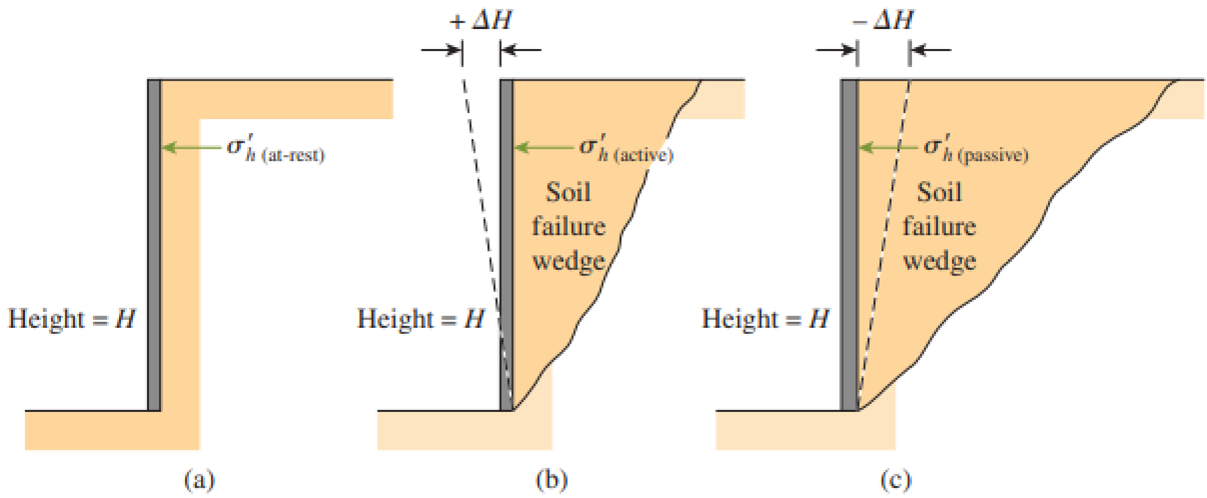


Figure 2.1 Nature of lateral earth pressure on a retaining wall

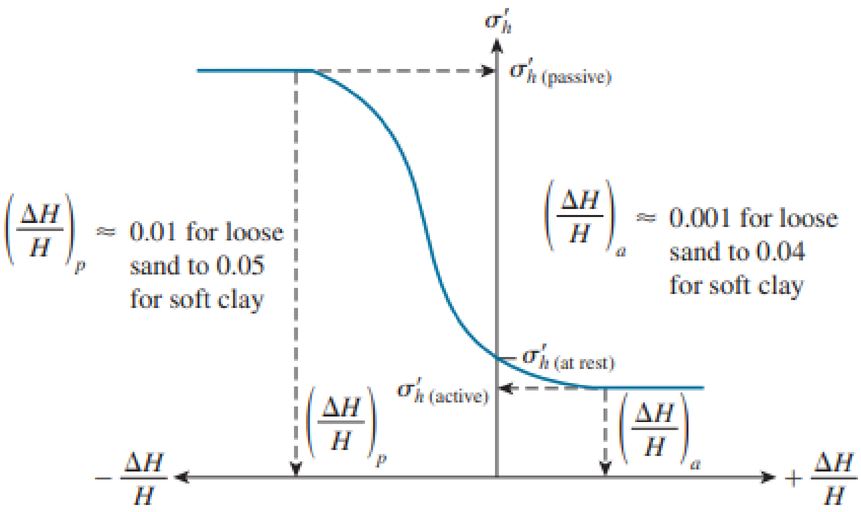


Figure 2.2 Nature of variation of lateral earth pressure at a certain depth

In this chapter, we will focus on the three special cases of lateral earth pressures:

- At-rest earth pressure
- Active earth pressure
- Passive earth pressure

Here, active and passive pressures are the two extreme loadings where the soil is at failure. The principles and procedures for determining the lateral earth pressures under these special cases are discussed in this chapter. It is assumed that the reader has studied lateral earth pressure in the past, so this chapter will serve as a review. Figure 2.3 shows some examples of earth retaining structures.

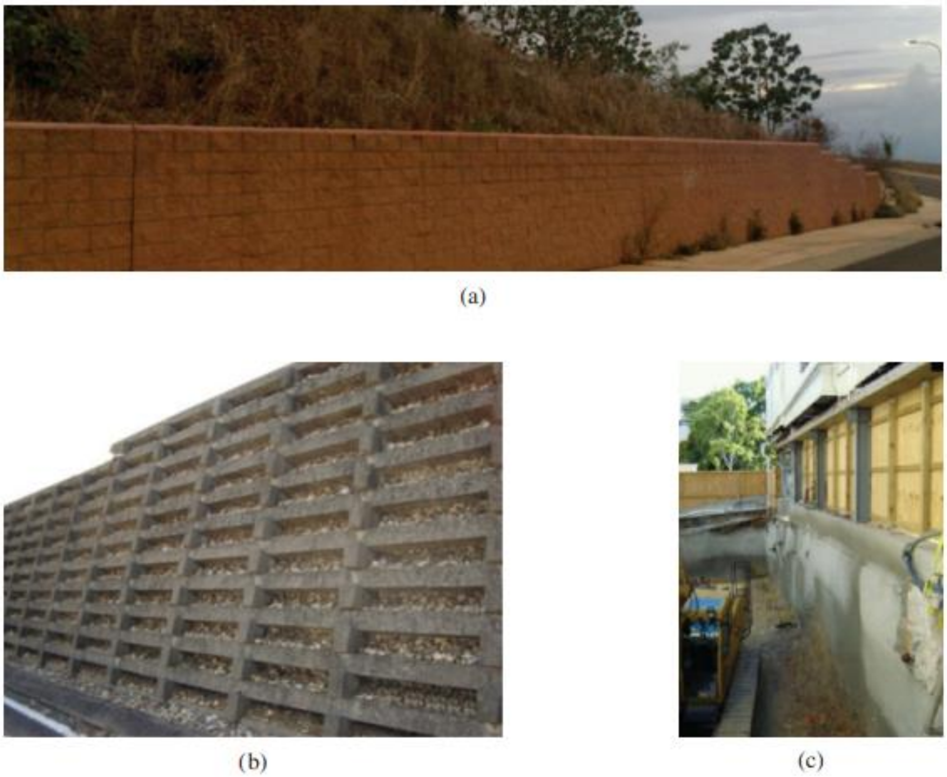


Figure 2.3 Earth retaining structures: (a) retaining wall; (b) crib wall; (c) basement wall

2.2 Lateral Earth Pressure at Rest

In a homogeneous soil mass where the ground level is horizontal, the ratio of the effective horizontal stress (σ'_h) to the effective vertical stress (σ'_v) at any point within the soil is a constant, specific to the soil. The soil is known to be "at rest" and the constant is called the coefficient of earth pressure at rest, denoted by K_o. When the soil is at rest, there is no lateral strain within the soil. A clay element subjected to one-dimensional consolidation does not undergo any lateral strains and hence is in the K_o state (at rest).

Consider a vertical wall of height *H*, as shown in Figure 2.2, retaining a soil having a unit weight of γ. A uniformly distributed load, q/unit area, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

Where
c' = cohesion
ϕ' = effective angle of friction
σ' = effective normal stress

At any depth z below the ground surface, the vertical subsurface stress is

$$\sigma'_o = q + \gamma z \qquad (2.1)$$

If the *wall is at rest and is not allowed to move at all,* either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth *z* is

$$\sigma_h = K_o \sigma'_o + u \qquad (2.2)$$

Where
u= pore water pressure
K_o = coefficient of at-rest earth pressure

For normally consolidated soil, the relation for K_o (Jaky, 1944) is

$$K_o \approx 1 - \sin\phi' \qquad (2.3)$$

Equation (2.3) is an empirical approximation.
For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin\phi')OCR^{\sin\phi'} \qquad (2.4)$$

where OCR = overconsolidation ratio.
Alpan (1967) suggested that for normally consolidated clays

$$K_o = 0.19 + 0.233 \log PI \qquad (2.5a)$$

Massarsch (1979) suggested that for normally consolidated clays,

$$K_o = 0.44 + 0.0042\, PI \qquad (2.6a)$$

- K_o of a normally consolidated soil is typically in the range of 0.4–0.6, and when soil are overconsolidated it can be larger and sometimes even exceed 1.0. When the soil mass is treated as a linear elastic continuum, the coefficient of earth pressure at rest is expressed as

$$K_o = \frac{\mu}{1-\mu} \qquad (2.7a)$$

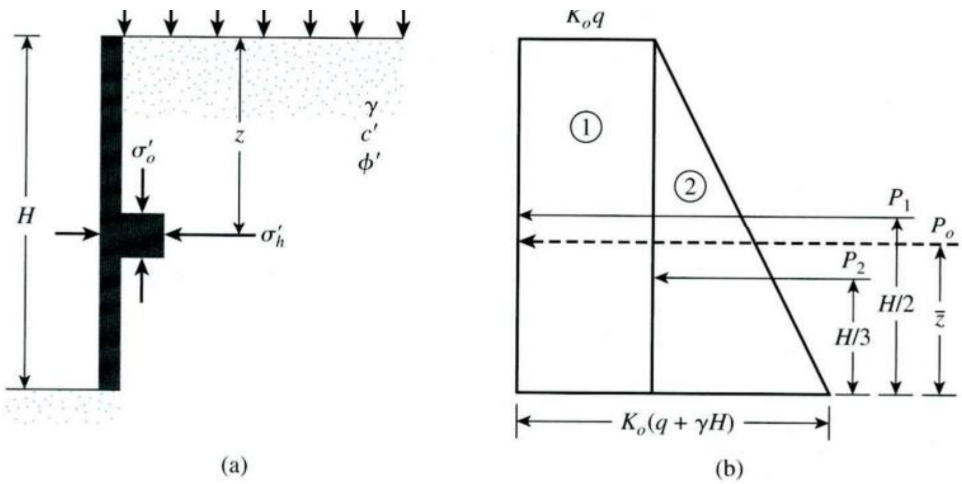
where μ is the Poisson's ratio of the elastic medium.

With a properly selected value of the at-rest earth pressure coefficient, Eq. (2.2) can be used to determine the variation of lateral earth pressure with depth *z*. Figure 2.2b shows the variation of σ_h' with depth for the wall depicted in Figure 2.2a. Note that if the surcharge q=0 and the pore water pressure the pressure u=0, diagram will be a triangle. The total force, P_o, *per unit length* of the wall given in Figure 2.2a can now be obtained from the area of the pressure diagram given in Figure 2.2b and is

$$P_o = P_1 + P_2 = qK_0H + \frac{1}{2}\gamma H^2 K_o \qquad (2.5)$$

where
P_1= area of rectangle 1
P_2= area of triangle 2
The location of the line of action of the resultant force, P_o, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right)+P_2\left(\frac{H}{3}\right)}{P_o} \qquad (2.6)$$



Figure(2.2)* At-rest earth pressure

If the water table is located at a depth z< H, the at-rest pressure diagram shown in Figure 2.2b will have to be somewhat modified, as shown in Figure 2.3. If the effective unit weight of soil below the water table equals γ (i.e., $\gamma_{sat} - \gamma_w$), then

At z = 0 : $\sigma_h' = K_o \sigma_o' = K_o q$

At z= H_1 : $\sigma_h' = K_o \sigma_o' = K_o (q + \gamma H_1)$

And

At z =H_2 , $\sigma_h' = K_o \sigma_o' = K_o (q + \gamma H_1 + \gamma' H_2)$

Note that in the preceding equations, σ_o' and $\sigma_h'$are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure, *u,* which is zero from z=0 to z= H_1 and is $\gamma_w H_2$ at z= H_2. The variation of σ_h' and *u* with depth is shown in Figure 2.3b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where A= area of the pressure diagram.
So,

$$P_o = K_o q H_1 + \frac{1}{2} K_o \gamma H_1^2 + K_o (q + \gamma H_1) H_2 + \frac{1}{2} K_o \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

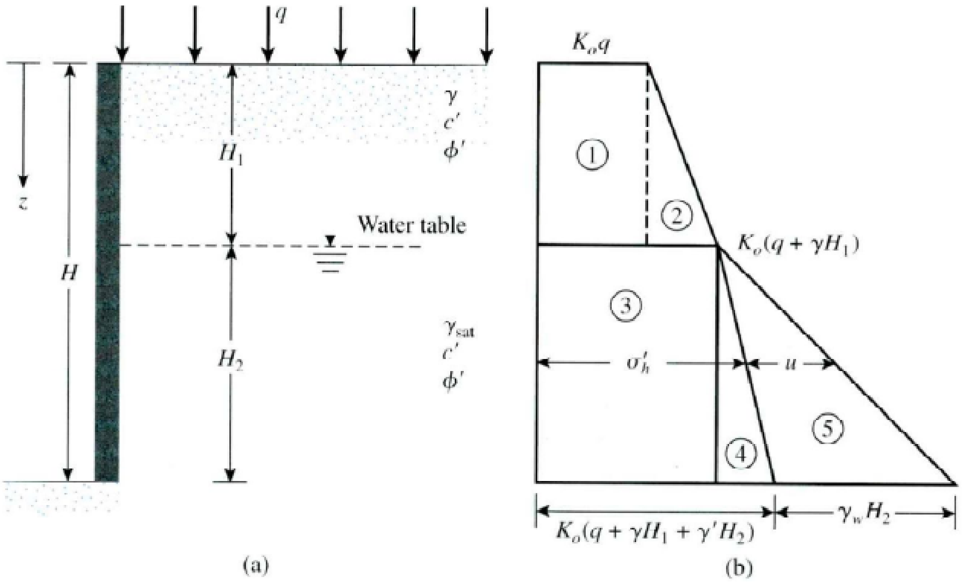


Figure 2.3 At-rest earth pressure with water table located at a depth z< H

Example 2.1

For the retaining wall shown in Figure 2.6, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR = 1.

SOLUTION

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma_o' = 0$; $\sigma_h' = 0$

At $z = 2.5$ m, $\sigma_o' = (16.5)(2.5) = 41.25$ kN/m^2;

$$\sigma_h' = K_o\sigma_o' = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

At $z = 5$ m, $\sigma_o' = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98$ kN/m^2;

$$\sigma_h' = K_o\sigma_o' = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m^2.

The pressure distribution for the wall is shown in Figure 16.6b.

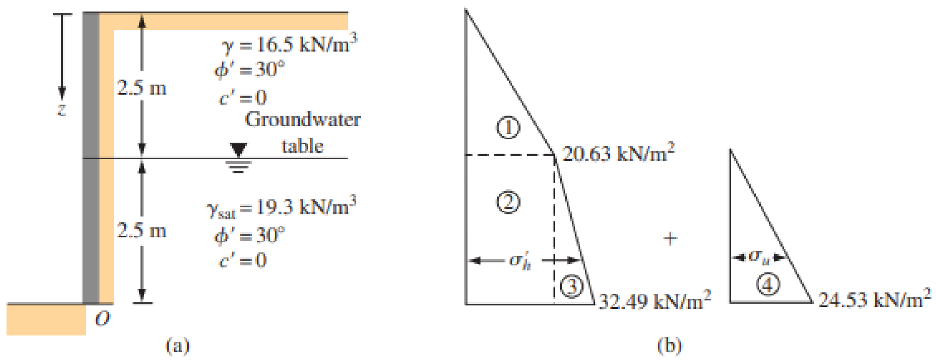


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Figure 2.6

The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$
$$= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63)$$
$$+ \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$\bar{z} = \frac{(\text{Area 1})\left(2.5 + \frac{2.5}{3}\right) + (\text{Area 2})\left(\frac{2.5}{2}\right) + (\text{Area 3} + \text{Area 4})\left(\frac{2.5}{3}\right)}{P_o}$$

$$= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85}$$

$$= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}$$

2.3 Active Pressure

2.3.1 Rankine Active Earth Pressure

The lateral earth pressure described in Section 2.2 involves walls that do not yield at all. However, if a wall tends to move away from the soil a distance Δx as shown in Figure 1.4a, the soil pressure on the wall at any depth will decrease. For a wall that is *frictionless*, the horizontal stress, σ'_h , at depth *z* will equal $K_o\sigma'_o(= K_o\gamma z)$ when Δx is zero. However, with Δx > 0, σ'_h will be less than $K_o\sigma'_o$.

The Mohr's circles corresponding to wall displacements of Δx =0 and Δx > 0 are shown as circles *a* and *b*, respectively, in Figure 2.4b. If the displacement of the wall, Δx , continues to increase, the corresponding Mohr's circle eventually will just touch the Mohr–Coulomb failure envelope defined by the equation

$$S = c' + \sigma' tan\emptyset'$$

This circle, marked *c* in the figure, represents the failure condition in the soil mass; the horizontal stress then equals σ'_a , referred to as the ***Rankine active pressure***. The *slip lines* (failure planes) in the soil mass will then make angles of $\pm\left(45 + \frac{\emptyset'}{2}\right)$ with the horizontal, as shown in Figure 2.4a.

Equation (2.7) relates the principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

$$\sigma'_1 = \sigma'_3 tan^2\left(45 + \frac{\emptyset'}{2}\right) + 2c'\tan\left(45 + \frac{\emptyset'}{2}\right) \qquad (2.7)$$

For the Mohr's circle *c* in Figure 2.4b,

Major principle stress: $\sigma'_1 = \sigma'_o$

and

Minor principle stress: $\sigma'_3 = \sigma'_a$

Thus,

$$\sigma'_o = \sigma'_a tan^2\left(45 + \frac{\emptyset'}{2}\right) + 2c'\tan\left(45 + \frac{\emptyset'}{2}\right)$$

$$\sigma'_a = \frac{\sigma'_o}{tan^2\left(45+\frac{\emptyset'}{2}\right)} - \frac{2c'}{\tan\left(45+\frac{\emptyset'}{2}\right)}$$

or

$$\sigma'_a = \sigma'_o tan^2\left(45 - \frac{\emptyset'}{2}\right) - 2c'\tan\left(45 - \frac{\emptyset'}{2}\right)$$

$$= \sigma'_o K_a - 2c'\sqrt{K_a} \qquad (2.8)$$

where $K_a = tan^2\left(45 - \frac{\emptyset'}{2}\right)$ = Rankine active pressure coefficient.

The variation of the active pressure with depth for the wall shown in Figure 2.4a is given in Figure 2.4c. Note that $\sigma'_o = 0$ at z= 0 and $\sigma'_o = \gamma H$ at z=H. the pressure distribution shows that at z=0 the active pressure equals $-2c'\sqrt{K_a}$, indicating a tensile stress that decreases with depth and becomes zero at a depth z= z_c , or

$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

And

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} \qquad (2.9)$$

The depth z_c is usually referred to as the depth of tensile crack, because the tensile stress in the soil will eventually cause a crak along the soil-wall interface.

Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is

$$P_a = \int_0^H \sigma'_a dz = \int_0^H \gamma z\, K_a dz - \int_0^H 2c'\sqrt{K_a} dz$$

$$= \frac{1}{2}\gamma H^2 K_a - 2c' H\sqrt{K_a} \qquad (2.10)$$

After the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths z= z_c and z= H as shown by the hatched area in Figure 2.4c. This force may be expressed as

$$P_a = \frac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) \qquad (2.11)$$

$$P_a = \frac{1}{2}\left(H - \frac{2c'}{\gamma\sqrt{K_a}}\right)(\gamma H K_a - 2c'\sqrt{K_a}) \qquad (2.12)$$

However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to "yield" sufficiently. The necessary amount of outward displacement of the wall is about $0.001H$ to $0.004H$ for granular soil backfills and about $0.01H$ to $0.04H$ for cohesive soil backfills.

Note further that if the *total stress* shear strength parameters (c, ϕ) were used, an equation similar to Eq. (2.9) could have been derived, namely

$$\sigma_a = \sigma_o \, tan^2\left(45 - \frac{\phi'}{2}\right) - 2c \ \tan\left(45 - \frac{\phi'}{2}\right)$$

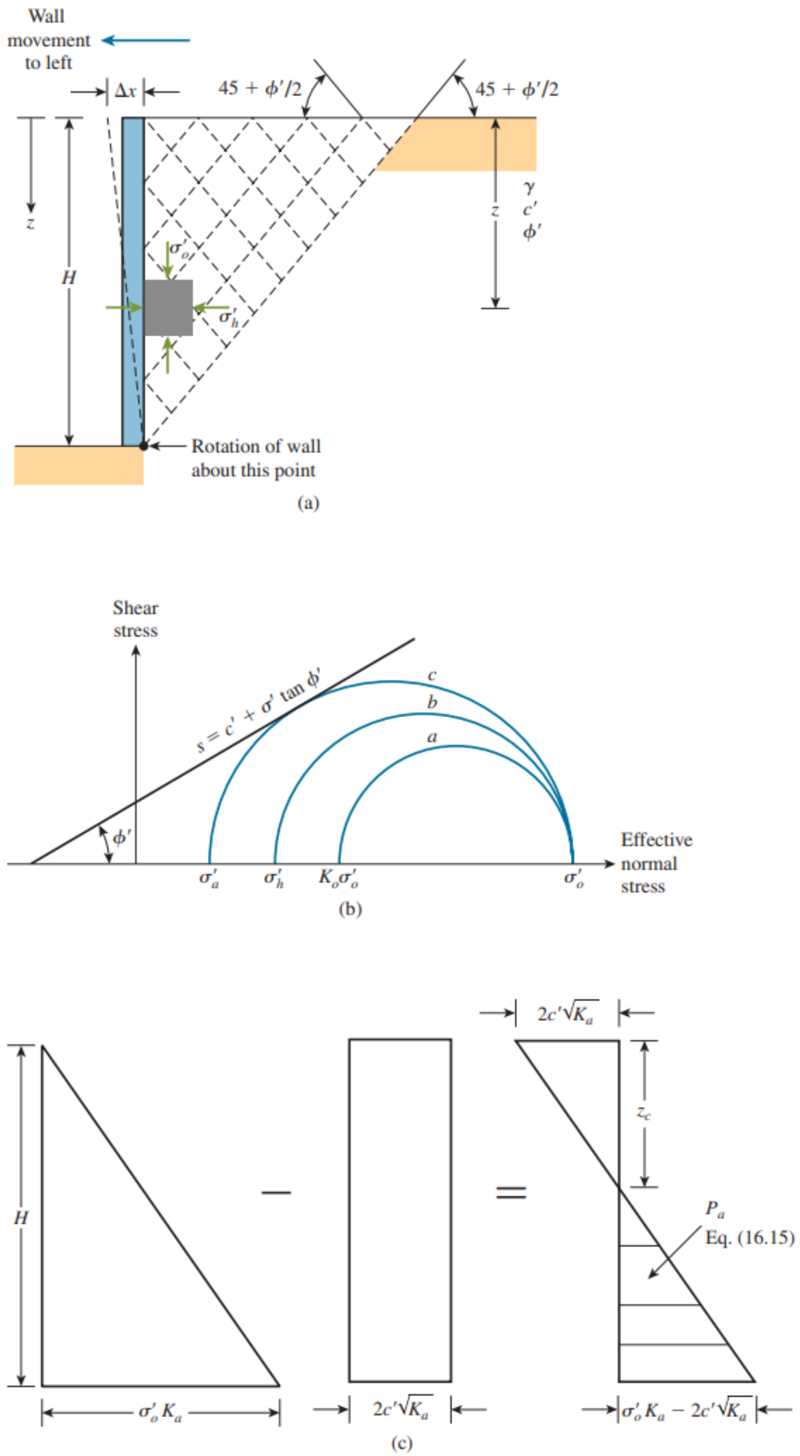


Figure 2.4 Rankine active pressure

Example 2.2

A 6 m high retaining wall is to support a soil with unit weight $\gamma = 17.4$ kN/m^3, soil friction angle $\phi' = 26°$, and cohesion $c' = 5$ kN/m^2. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

SOLUTION

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

From Figure 16.7c, at $z = 0$,

$$\sigma'_a = -2c'\sqrt{K_a} = -2(5)(0.625) = -6.25 \text{ kN/m}^2$$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(5)(0.625)$$
$$= 40.72 - 6.25 = 34.47 \text{ kN/m}^2$$

Active Force Before the Tensile Crack Appeared: Eq. (16.13)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(6.25) = 122.16 - 37.5 = \mathbf{84.66 \text{ kN/m}}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (37.5)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 112.5}{84.66} = \mathbf{1.56 \text{ m}}$$

Active Force After the Tensile Crack Appeared: Eq. (16.12)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(5.0)}{(17.4)(0.625)} = 0.92 \text{ m}$$

Using Eq. (16.14) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 0.92)(34.47) = \mathbf{87.55 \text{ kN/m}}$$

Figure 16.7c indicates that the force $P_a = 87.55$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 0.92}{3} = \mathbf{1.69 \text{ m}}$$

Example 2.3

Assume that the retaining wall shown in Figure 16.8a can yield sufficiently to develop an active state. Determine the Rankine active force per unit length of the wall and the location of the resultant line of action.

SOLUTION

If the cohesion, c', is zero, then

$$\sigma'_a = \sigma'_o K_a$$

For the top layer of soil, $\phi'_1 = 30°$, so

$$K_{a(1)} = \tan^2\left(45 - \frac{\phi'_1}{2}\right) = \tan^2(45 - 15) = \frac{1}{3}$$

Similarly, for the bottom layer of soil, $\phi'_2 = 36°$, and it follows that

$$K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$$

The following table shows the calculation of σ'_a and u at various depths below the ground surface.

Depth, z (m)	σ'_o (kN/m²)	K_a	$\sigma'_a = K_a \sigma'_o$ (kN/m²)	u (kN/m²)
0	0	1/3	0	0
3.05^-	(16)(3.05) = 48.8	1/3	16.27	0
3.05^+	48.8	0.26	12.69	0
6.1	(16)(3.05) + (19 − 9.81)(3.05) = 76.83	0.26	19.98	(9.81)(3.05) = 29.92

The pressure distribution diagram is plotted in Figure 16.8b. The force per unit length is

$$P_a = \text{area 1} + \text{area 2} + \text{area 3} + \text{area 4}$$
$$= \tfrac{1}{2}(3.05)(16.27) + (12.69)(3.05) + \tfrac{1}{2}(19.98 - 12.69)(3.05) + \tfrac{1}{2}(29.92)(3.05)$$
$$= 24.81 + 38.70 + 11.12 + 45.63 = \mathbf{120.26\ kN/m}$$

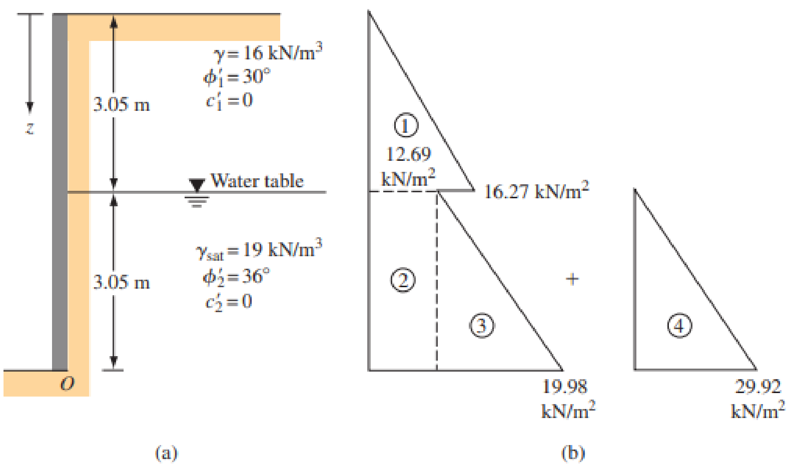


FIGURE 16.8 Rankine active force behind a retaining wall

The distance of the line of action of the resultant force from the bottom of the wall can be determined by taking the moments about the bottom of the wall (point O in Figure 16.8a) and is

$$\bar{z} = \frac{(24.81)\left(3.05 + \frac{3.05}{3}\right) + (38.7)\left(\frac{3.05}{2}\right) + (11.12 + 45.63)\left(\frac{10}{3}\right)}{120.26} = \mathbf{1.81\ m}$$

2.3.2 Rankine Active Earth Pressure for Inclined Backfill

If the backfill of a frictionless retaining wall is a granular soil ($c'=0$) and rises at an angle α with respect to the horizontal (see Figure 2.5), the active earth-pressure coefficient may be expressed in the form

$$K_a = cos\alpha \frac{cos\alpha - \sqrt{cos^2\alpha - cos^2\phi'}}{cos\alpha + \sqrt{cos^2\alpha - cos^2\phi'}} \qquad (2.13)$$

where ϕ'=angle of friction of soil.

At any depth z, the Rankine active pressure may be expressed as

$$\sigma_a' = \gamma H^2 K_a \qquad (2.14)$$

Also, the total force per unit length of the wall is

$$P_a = 1/2\, \gamma H^2 K_a \qquad (2.15)$$

Note that, in this case, the direction of the resultant force P_a is *inclined at an angle with the horizontal* and intersects the wall at a distance H/3 from the base of the wall. **Table 2.1** presents the values of K_a (active earth pressure) for various values of α and ϕ'.

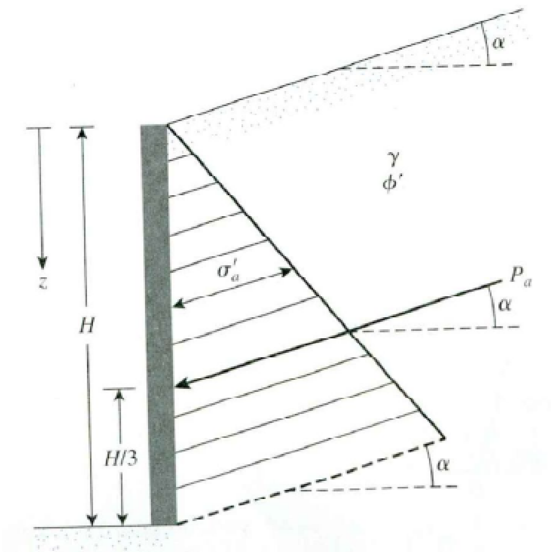


Figure 2.5 Notations for active pressure—Eqs. (2.13), (2.14), (2.15)

2.3.3 Coulomb's Active Earth Pressure

The Rankine active earth pressure calculations discussed in the preceding sections were based on the assumption that the wall is frictionless. In 1776, Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration.

To apply Coulomb's active earth pressure theory, let us consider a retaining wall with its back face inclined at an angle βwith the horizontal, as shown in Figure 2.6a. The backfill is a granular soil that slopes at an angle α a with the horizontal. Also, let δ' be the angle of friction between the soil and the wall (i.e., the angle of wall friction).

Under active pressure, the wall will move away from the soil mass (to the left in the figure). Coulomb assumed that, in such a case, the failure surface in the soil mass would be a plane (e.g., BC_1, BC_2, …). So, to find the active force, consider a possible soil failure wedge ABC_1. The forces acting on this wedge (per unit length at right angles to the cross section shown) are as follows:

1. The weight of the wedge, *W*.
2. The resultant, *R*, of the normal and resisting shear forces along the surface,BC_1.

The force *R* will be inclined at an angle to the normal drawn to BC_1.

3. The active force per unit length of the wall, P_a, which will be inclined at an angle δ' to the normal drawn to the back face of the wall.

For equilibrium purposes, a force triangle can be drawn, as shown in Figure 2.6b. Note that θ_1 is the angle that BC_1 makes with the horizontal. Because the magnitude of *W*,as well as the directions of all three forces, are known, the value of P_a can now be determined. Similarly, the active forces of other trial wedges, such as ABC_2, ABC_3, …, can be determined. The maximum value of P_a thus determined is Coulomb's active force (see top part of Figure 2.7), which may be expressed as

$$P_a = 1/2\,\gamma H^2 K_a \qquad (2.16)$$

Where

K_a= Coulomb's active earth-pressure coefficient

$$= \frac{sin^2(\beta-\emptyset')}{sin^2\beta\,\sin(\beta-\delta')\left[1+\sqrt{\frac{\sin(\emptyset'+\delta')\sin(\emptyset'-\alpha)}{\sin(\beta+\delta')\sin(\alpha+\beta)}}\right]^2} \qquad (2.17)$$

and H= height of the wall.

The values of the active earth pressure coefficient, K_a ,for a vertical retaining wall (β = 90°) with horizontal backfill (α = 0°) are given in Table 2.2. Note that the line of action of the resultant force (P_a) will act at a distance H/3 above the base of the wall and will be inclined at an angle δ' to the normal drawn to the back of the wall.

In the actual design of retaining walls, the value of the wall friction angle δ' is assumed to be between $\emptyset'/2$ and $2/3\emptyset'$. The active earth pressure coefficients for various values of $\emptyset', \alpha,$ and β with $\emptyset'/2$ and $2/3\emptyset'$ are respectively given in Tables 2.3 and 2.4. These coefficients are very useful design considerations.

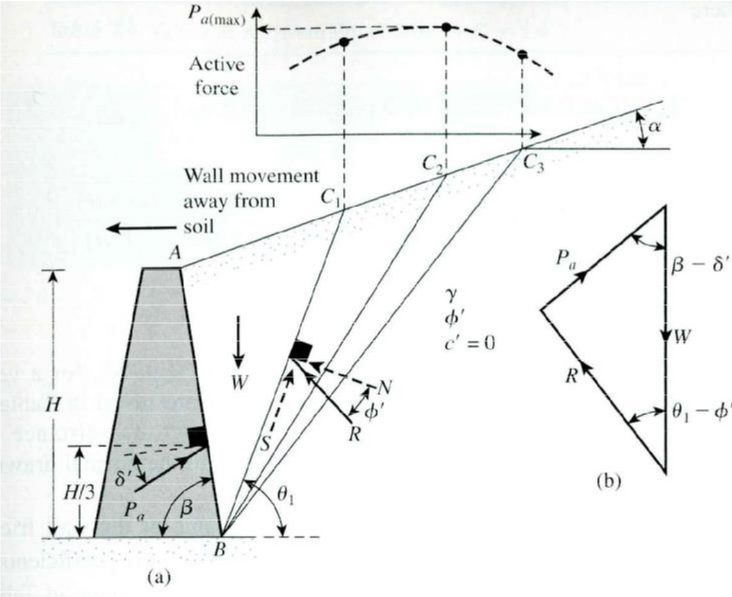


Figure 2.6 Coulomb's active pressure

Table 2.2 Values of K_a Eq(2.17) for β=90° and α=0°

	δ' (deg)					
ϕ' (deg)	**0**	**5**	**10**	**15**	**20**	**25**
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

Table 2.3 Values of K_a Eq(2.17) for $\delta' = 2/3$

α (deg)	φ′ (deg)	β (deg) 90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	29	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	31	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	32	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	33	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	35	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	37	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	38	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	39	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
	41	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	42	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

(continued)

Table 2.3 Values of K_a Eq(2.17) for $\delta' = 2/3$

α (deg)	φ′ (deg)	β (deg) 90	85	80	75	70	65
	29	0.3881	0.4397	0.4987	0.5672	0.6483	0.7463
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7265
	31	0.3541	0.4049	0.4629	0.5305	0.6106	0.7076
	32	0.3384	0.3887	0.4462	0.5133	0.5930	0.6895
	33	0.3234	0.3732	0.4303	0.4969	0.5761	0.6721
	34	0.3091	0.3583	0.4150	0.4811	0.5598	0.6554
	35	0.2954	0.3442	0.4003	0.4659	0.5442	0.6393
	36	0.2823	0.3306	0.3862	0.4513	0.5291	0.6238
	37	0.2698	0.3175	0.3726	0.4373	0.5146	0.6089
	38	0.2578	0.3050	0.3595	0.4237	0.5006	0.5945
	39	0.2463	0.2929	0.3470	0.4106	0.4871	0.5805
	40	0.2353	0.2813	0.3348	0.3980	0.4740	0.5671
	41	0.2247	0.2702	0.3231	0.3858	0.4613	0.5541
	42	0.2146	0.2594	0.3118	0.3740	0.4491	0.5415
20	28	0.4602	0.5205	0.5900	0.6714	0.7689	0.8880
	29	0.4364	0.4958	0.5642	0.6445	0.7406	0.8581
	30	0.4142	0.4728	0.5403	0.6195	0.7144	0.8303
	31	0.3935	0.4513	0.5179	0.5961	0.6898	0.8043
	32	0.3742	0.4311	0.4968	0.5741	0.6666	0.7799
	33	0.3559	0.4121	0.4769	0.5532	0.6448	0.7569
	34	0.3388	0.3941	0.4581	0.5335	0.6241	0.7351
	35	0.3225	0.3771	0.4402	0.5148	0.6044	0.7144
	36	0.3071	0.3609	0.4233	0.4969	0.5856	0.6947
	37	0.2925	0.3455	0.4071	0.4799	0.5677	0.6759
	38	0.2787	0.3308	0.3916	0.4636	0.5506	0.6579
	39	0.2654	0.3168	0.3768	0.4480	0.5342	0.6407
	40	0.2529	0.3034	0.3626	0.4331	0.5185	0.6242
	41	0.2408	0.2906	0.3490	0.4187	0.5033	0.6083
	42	0.2294	0.2784	0.3360	0.4049	0.4888	0.5930

Table 2.4 Values of K_a Eq(2.17) for $\delta' = 1/2$

α (deg)	φ′ (deg)	90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
	36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702

Table 2.4 Values of K_a Eq(2.17) for $\delta' = 1/2$

		β (deg)					
α (deg)	φ′ (deg)	90	85	80	75	70	65
	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4177
5	28	0.3477	0.3879	0.4327	0.4837	0.5425	0.6115
	29	0.3337	0.3737	0.4185	0.4694	0.5282	0.5972
	30	0.3202	0.3601	0.4048	0.4556	0.5144	0.5833
	31	0.3072	0.3470	0.3915	0.4422	0.5009	0.5698
	32	0.2946	0.3342	0.3787	0.4292	0.4878	0.5566
	33	0.2825	0.3219	0.3662	0.4166	0.4750	0.5437
	34	0.2709	0.3101	0.3541	0.4043	0.4626	0.5312
	35	0.2596	0.2986	0.3424	0.3924	0.4505	0.5190
	36	0.2488	0.2874	0.3310	0.3808	0.4387	0.5070
	37	0.2383	0.2767	0.3199	0.3695	0.4272	0.4954
	38	0.2282	0.2662	0.3092	0.3585	0.4160	0.4840
	39	0.2185	0.2561	0.2988	0.3478	0.4050	0.4729
	40	0.2090	0.2463	0.2887	0.3374	0.3944	0.4620
	41	0.1999	0.2368	0.2788	0.3273	0.3840	0.4514
	42	0.1911	0.2276	0.2693	0.3174	0.3738	0.4410
10	28	0.3743	0.4187	0.4688	0.5261	0.5928	0.6719
	29	0.3584	0.4026	0.4525	0.5096	0.5761	0.6549
	30	0.3432	0.3872	0.4368	0.4936	0.5599	0.6385
	31	0.3286	0.3723	0.4217	0.4782	0.5442	0.6225
	32	0.3145	0.3580	0.4071	0.4633	0.5290	0.6071
	33	0.3011	0.3442	0.3930	0.4489	0.5143	0.5920
	34	0.2881	0.3309	0.3793	0.4350	0.5000	0.5775
	35	0.2757	0.3181	0.3662	0.4215	0.4862	0.5633
	36	0.2637	0.3058	0.3534	0.4084	0.4727	0.5495
	37	0.2522	0.2938	0.3411	0.3957	0.4597	0.5361
	38	0.2412	0.2823	0.3292	0.3833	0.4470	0.5230
	39	0.2305	0.2712	0.3176	0.3714	0.4346	0.5103
	40	0.2202	0.2604	0.3064	0.3597	0.4226	0.4979
	41	0.2103	0.2500	0.2956	0.3484	0.4109	0.4858
	42	0.2007	0.2400	0.2850	0.3375	0.3995	0.4740
15	28	0.4095	0.4594	0.5159	0.5812	0.6579	0.7498
	29	0.3908	0.4402	0.4964	0.5611	0.6373	0.7284
	30	0.3730	0.4220	0.4777	0.5419	0.6175	0.7080
	31	0.3560	0.4046	0.4598	0.5235	0.5985	0.6884
	32	0.3398	0.3880	0.4427	0.5059	0.5803	0.6695
	33	0.3244	0.3721	0.4262	0.4889	0.5627	0.6513
	34	0.3097	0.3568	0.4105	0.4726	0.5458	0.6338
	35	0.2956	0.3422	0.3953	0.4569	0.5295	0.6168

(continued)

Example 2.4

Consider the retaining wall shown in Figure 16.14a. Given: $H = 5$ m; unit weight of soil = 17.6 kN/m^3; angle of friction of soil = 35°; wall friction angle, $\delta' = \frac{2}{3}\phi'$; soil cohesion, $c' = 0$; $\alpha = 0$; and $\beta = 90°$. Calculate the Coulomb's active force per unit length of the wall.

SOLUTION

From Eq. (16.35),

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

From Table 2.3,

, for $\alpha = 0°$, $\beta = 90°$, $\phi' = 35°$, and $\delta' = \frac{2}{3}\phi' = 23.33°$, $K_a =$ 0.2444. Hence,

$$P_a = \frac{1}{2}(17.6)(5)^2(0.2444) = \mathbf{53.77\ kN/m}$$

2.4 Passive Pressure

2.4.1 Rankine Passive Earth Pressure

Figure 2.8a shows a vertical frictionless retaining wall with a horizontal backfill. At depth *z*, the effective vertical pressure on a soil element is $\sigma'_o=\gamma z$ Initially, if the wall does not yield at all, the lateral stress at that depth will be $\sigma'_h=K_o\sigma'_o$. This state of stress is illustrated by the Mohr's circle *a* in Figure 2.8b. Now, if the wall is pushed into the soil mass by an amount Δx as shown in Figure 2.8a, the vertical stress at depth *z* will stay the same; however, the horizontal stress will increase. Thus, σ'_h will be greater than $K_o\sigma'_o$. The state of stress can now be represented by the Mohr's circle *b* in Figure 2.8b. If the wall moves farther inward (i.e., is increased still more), the stresses at depth *z* will ultimately reach the state represented by Mohr's circle *c*. Note that this Mohr's circle touches the Mohr–Coulomb failure envelope, which implies that the soil behind the wall will fail by being pushed upward. The horizontal stress, σ'_h , at this point is referred to as the ***Rankine passive pressure***, or $\sigma'_h = \sigma'_p$.

For Mohr's circle *c* in Figure 2.8b, the major principal stress is σ'_p and the minor principal stress is σ'_o .Substituting these quantities into Eq. (2.8) yields

$$\sigma'_p = \sigma'_o tan^2\left(45+\frac{\phi'}{2}\right) + 2c'\tan\left(45+\frac{\phi'}{2}\right) \qquad (2.18)$$

K_p= Rankine passive earth-pressure coefficient

$$K_p = tan^2\left(45+\frac{\phi'}{2}\right) \qquad (2.19)$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p} \qquad (2.20)$$

Equation (2.20) produces (Figure 2.18c), the passive pressure diagram for the wall shown in Figure 2.18a. Note that at z=0

$$\sigma'_o = 0 \;\; and \;\; \sigma'_p = 2c'\sqrt{K_p}$$

and at z= H

$$\sigma'_o = \gamma H \;\; and \;\; \sigma'_p = \gamma H K_p + 2c'\sqrt{K_p}$$

The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c'H\sqrt{K_p} \qquad (2.21)$$

The approximate magnitudes of the wall movements, Δx , required to develop failure under passive conditions are as follows:

Soil type	Wall movement for passive condition, Δx
Dense sand	$0.005H$
Loose sand	$0.01H$
Stiff clay	$0.01H$
Soft clay	$0.05H$

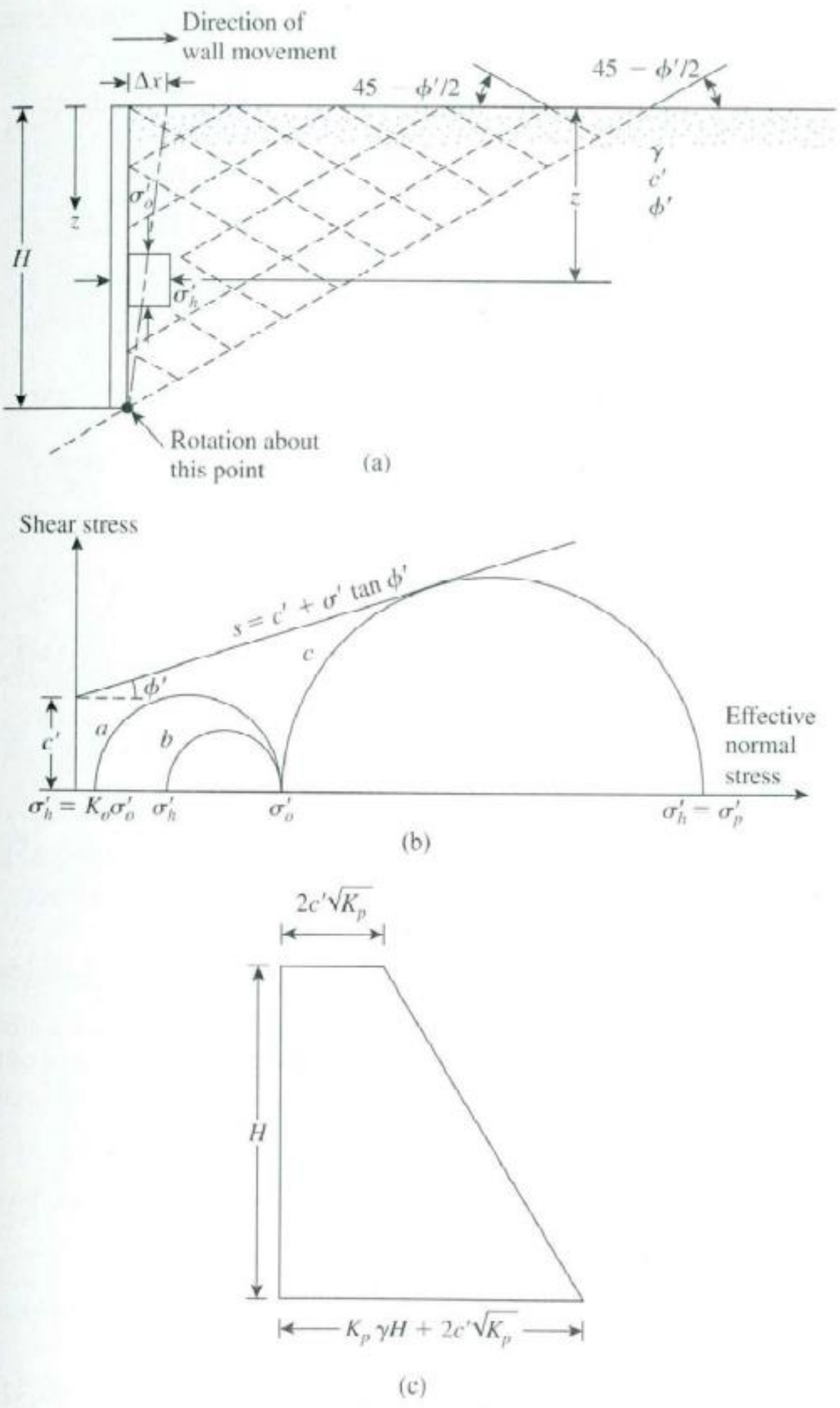


Figure 2.8 Rankine passive pressure

If the backfill behind the wall is a granular soil (i.e., $c' = 0$), then, from Eq. (2.21), the passive force per unit length of the wall will be

$$P_p = \frac{1}{2}\gamma H^2 K_p \quad (2.22)$$

2.4.2 Rankine Passive Earth Pressure for Inclined Backfill

For a frictionless vertical retaining wall (Figure 2.5) with a *granular backfill* ($c' = 0$), the Rankine passive pressure at any depth can be determined in a manner similar to that done in the case of active pressure in Section 2.3.2. The pressure is

$$\sigma'_p = \gamma z K_p \quad (2.23)$$

And the passive force is

$$P_p = 1/2\, \gamma H^2 K_p \quad (2.24)$$

where

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}} \quad (2.25)$$

As in the case of the active force, the resultant force, P_p, is inclined at an angle α with the horizontal and intersects the wall at a distance H/3 from the bottom of the wall. The values of K_p (the passive earth pressure coefficient) for various values of α and ϕ' are given in **Table 2.6**.

Table 2.6 Passive Earth Pressure Coefficient [from Eq. (2.25)]

↓α (deg)	ϕ' (deg)→ 28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

2.4.3 Coulomb's Passive Earth Pressure

Coulomb (1776) also presented an analysis for determining the passive earth pressure (i.e., when the wall moves *into* the soil mass) for walls possessing friction (δ'=angle of wall friction) and retaining a granular backfill material similar to that discussed in Section 2.3.3.

To understand the determination of Coulomb's passive force, P_p , consider the wall shown in Figure 2.9a. As in the case of active pressure, Coulomb assumed that the potential failure surface in soil is a plane. For a trial failure wedge of soil, such as ABC_1, the forces per unit length of the wall acting on the wedge are

1. The weight of the wedge, W
2. The resultant, R, of the normal and shear forces on the plane and
3. The passive force, P_p

Figure 2.9b shows the force triangle at equilibrium for the trial wedge ABC_1. From this force triangle, the value of P_p can be determined, because the direction of all three forces and the magnitude of one force are known.

Similar force triangles for several trial wedges, such as ABC_1, ABC_2, ABC_3, … can be constructed, and the corresponding values of P_p can be determined. The top part of Figure 2.9a shows the nature of variation of P_p the values for different wedges. The *minimum value of* P_p in this diagram is *Coulomb's passive force,* mathematically expressed as

$$P_a = 1/2\,\gamma H^2 K_p \qquad (2.26)$$

Where

K_a= Coulomb's passive earth-pressure coefficient

$$= \frac{sin^2(\beta-\emptyset')}{sin^2\beta\,\sin(\beta+\delta')\left[1-\sqrt{\frac{\sin(\emptyset'+\delta')\sin(\emptyset'+\alpha)}{\sin(\beta+\delta')\sin(\alpha+\beta)}}\right]^2} \qquad (2.27)$$

and H= height of the wall.

The values of the passive pressure coefficient, K_p, for various values of $\emptyset'$ and δ' are given in Table 2.7 ($\beta = 90^o$, $\alpha = 0^o$).

Note that the resultant passive force, P_p , will act at a distance H/3 from the bottom of the wall and will be inclined at an angle δ' to the normal drawn to the back face of the wall.

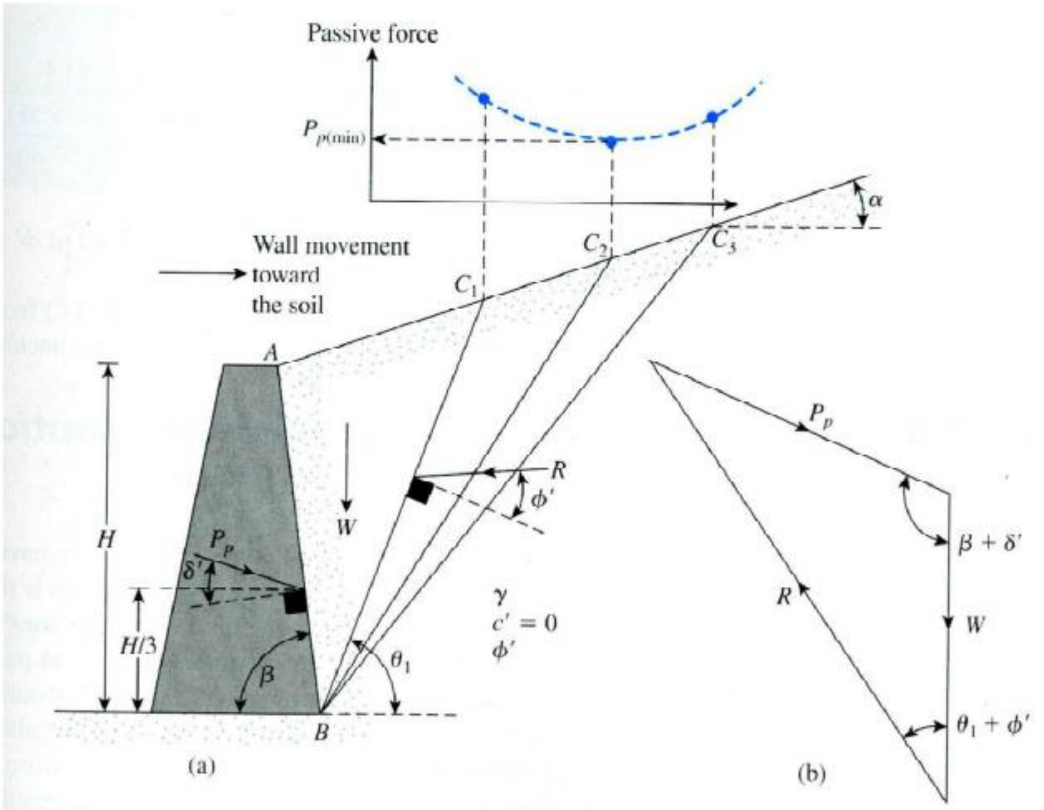


Figure 2.9 Coulomb's passive pressure

Table 7.10 Values of [from Eq. (2.27)] for β=90° and α=0°

	δ′ (deg)				
φ′ (deg)	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

Example 2.5

A 3 m high wall is shown in Figure 16.22a. Determine the Rankine passive force per unit length of the wall.

SOLUTION

For the top layer,

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi'_1}{2}\right) = \tan^2(45 + 15) = 3$$

From the bottom soil layer,

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

where

σ'_o = effective vertical stress

at $z = 0$, $\sigma'_o = 0$, $c'_1 = 0$, $\sigma'_p = 0$

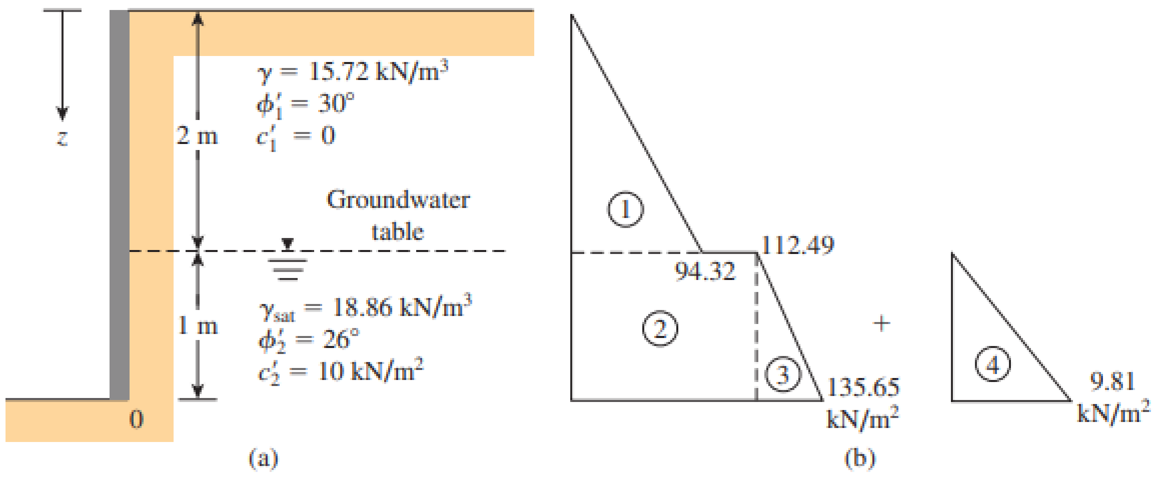
at $z = 2$ m, $\sigma'_o = (15.72)(2) = 31.44$ kN/m^2, $c'_1 = 0$

So, for the top soil layer,

$$\sigma'_p = 31.44K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth (that is, $z = 2$ m), for the bottom soil layer,

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56}$$
$$= 80.49 + 32 = 112.49 \text{ kN/m}^2$$



Again, at $z = 3$ m,

$$\sigma'_o = (15.72)(2) + (\gamma_{sat} - \gamma_w)(1)$$
$$= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2$$

Hence,

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6)$$
$$= \mathbf{135.65 \text{ kN/m}^2}$$

Note that, because a water table is present, the hydrostatic stress, u, also has to be taken into consideration. For $z = 0$ to 2 m, $u = 0$; at $z = 3$ m, $u = (1)(\gamma_w) = 9.81$ kN/m^2.

The passive pressure diagram is plotted in Figure 16.22b. The passive force per unit length of the wall can be determined from the area of the pressure diagram as follows:

Area no.	Area	
1	$(\frac{1}{2})(2)(94.32)$	= 94.32
2	$(112.49)(1)$	= 112.49
3	$(\frac{1}{2})(1)(135.65 - 112.49)$	= 11.58
4	$(\frac{1}{2})(9.81)(1)$	= 4.905
		$P_p \approx$ **223.3 kN/m**