University of Anbar
Engineering College
Civil Engineering Department

CHAPTER FIVE

GEOMETRIC DESIGN OF SHALLOW FOUNDATIONS

LECTURE
DR. AHMED H. ABDULKAREEM
DR. MAHER ZUHAIR AL- RAWI
2023 - 2024

5.1. Introduction

The foundations are considered to be shallow if [Df ≤ (3→4)B].

Shallow foundations have several advantages:

- Minimum cost of materials and construction,
- Easy in construction "labor don't need high experience to construct shallow foundations".

On the other hand, the main disadvantage of shallow foundations that if the bearing capacity of the soil supporting the foundation is small, the amount of settlement will be large.

The types of sallow foundations is the following:

1. Isolated Footings (spread footings).
2. Combined Footings.
3. Strap Footings.
4. Mat "Raft" Foundations.

SPREAD FOOTING DESIGN

5.2 Geometric Design of Isolated Footings

- A footing carrying a single column is called a *spread footing*, since its function is to "spread" the column load laterally to the soil so that the stress intensity is reduced to a value that the soil can safely carry.
- These members are sometimes called single or isolated footings.
- Wall footings serve a similar purpose of spreading the wall load to the soil. Often, however, wall footing widths are controlled by factors other than the allowable soil pressure since wall loads (including wall weight) are usually rather low.
- Spread footings with tension reinforcing may be called two-way or one-way depending on whether the steel used for bending runs both

ways (usual case) or in one direction (as is common for wall footings).

- Single footings may be of constant thickness or either stepped or sloped. Stepped or sloped footings are most commonly used to reduce the quantity of concrete away from the column where the bending moments are small and when the footing is not reinforced. When labor costs are high relative to material, it is usually more economical to use constant-thickness reinforced footings.
- Figure 5-1 illustrates several spread footings.
- A pedestal (Fig. 5-Ie) may be used to interface metal columns with spread or wall footings that are located at the depth in the ground.

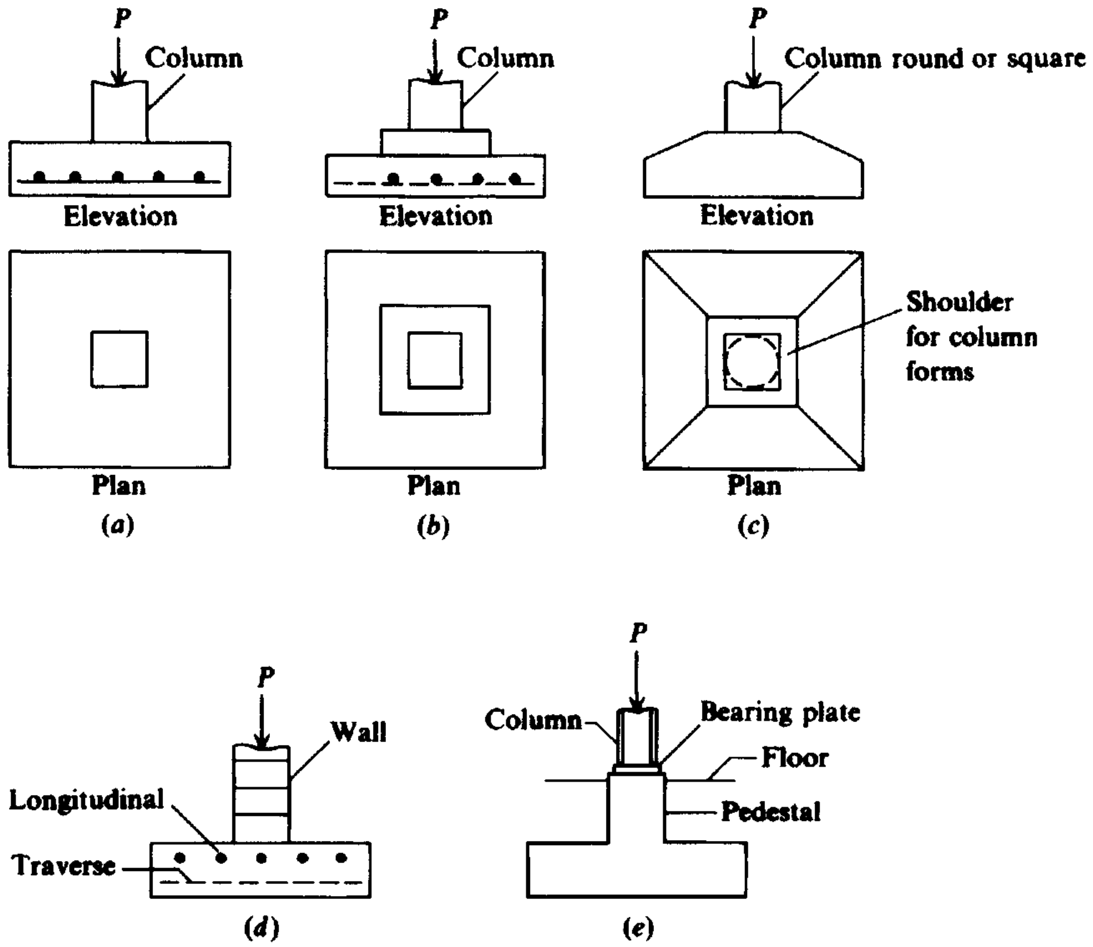


Figure 5-1 Typical footings, *(a)* Single or spread footings; *(b)* stepped footing; *(c)* sloped footing; *(d)* wall footing; *(e)* footing with pedestal.

5.2.1 Design Procedures:

1. Calculate the net allowable bearing capacity:

The first step for geometric design of foundations is to calculate the allowable bearing capacity of the foundations as we discussed in previous chapters as shown in Fig. 5.2.

$$q_{all,net} = \frac{q_{u,net}}{FS}$$
$$q_{u,net} = q_{u,gross} - \gamma_c h_c - \gamma_s h_s$$

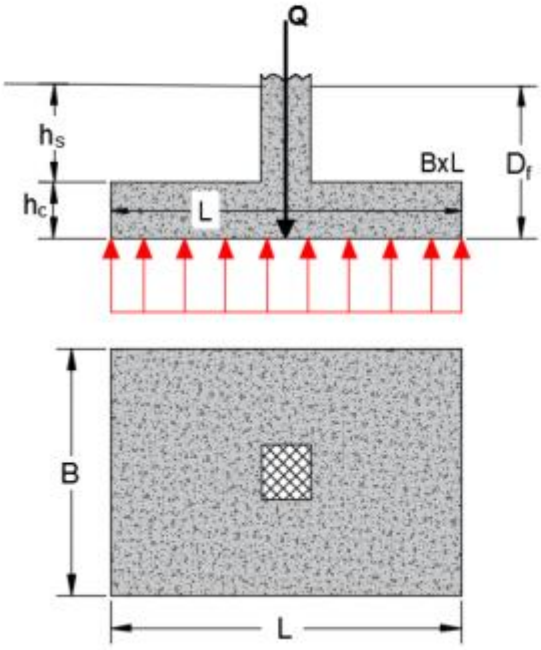


Fig. 5.2 spread foundation

2. Calculate the required area of the footing:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L$$

Assume B or L then find the other dimension.

If the footing is square:

$$A_{req} = B^2 \rightarrow B = \sqrt{A_{req}}$$
$$Q_{service} = P_D + P_L$$

Why we use $Q_{service}$:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = A_{req} = \frac{Q_{service}}{\frac{q_{u,net}}{FS}} = \frac{FS \times Q_{service}}{q_{u,net}}$$

Note:

- The equation of calculating the required a $\left(A_{req} = \frac{Q_{service}}{q_{all,net}}\right)$ is valid only if the pressure under the base of the foundation is uniform.

5.3 STRUCTURAL DESIGN OF SPREAD FOOTINGS

- The allowable soil pressure controls the plan *(B* X L) dimensions of a spread footing.
- Structural (such as a basement) and environmental factors locate the footing vertically in the soil.
- Shear stresses usually control the footing thickness *D.*
- Two-way action shear always controls the depth for centrally loaded square footings.
- Wide-beam shear may control the depth for rectangular footings when the *L/B* ratio is greater than about 1.2 and may control for other *L/B* ratios when there are overturning or eccentric loadings as Fig. 5-3a.

The depth of footing for two-way action produces a quadratic equation that is developed from Fig. 5-3b, c using

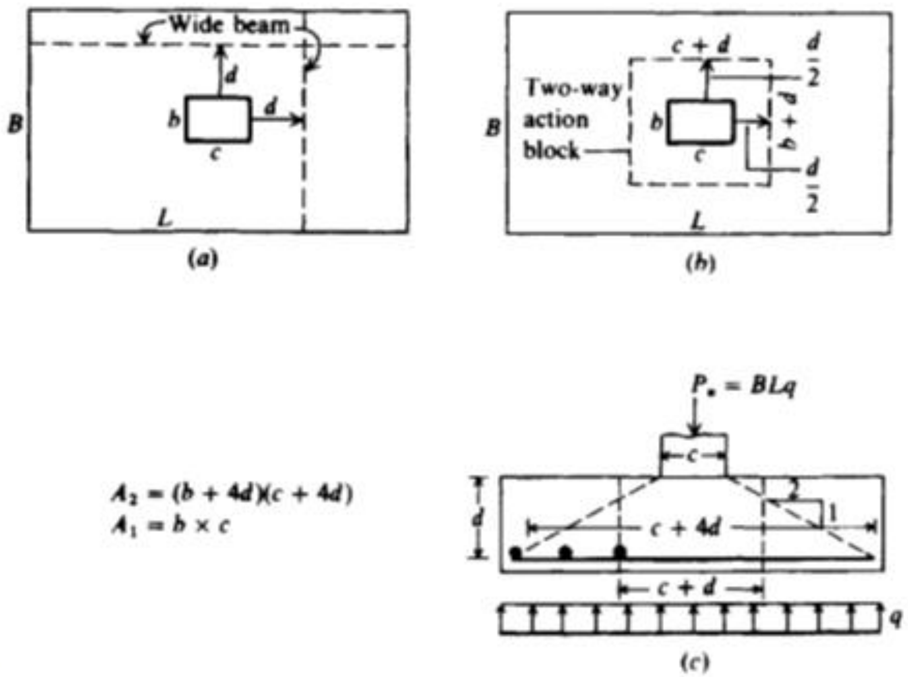


Fig.5.3 (*a*) Section for wide-beam shear; (*b*) section for diagonal-tension shear; (*c*) method of computing area A_2 for allowable column bearing stress.

$\sum F_y = 0$

on the two-way action zone shown. Noting the footing block weight cancels, we have:

$$P_u = 2dv_c(b + d) + 2dv_c(c + d) + (c + d)(b + d)q$$

Substitution of P_u or $P_d = BLq$ and using shear stress v_c gives

$$d^2(4v_c + q) + d(2v_c + q)(b + c) = (BL - cb)q$$

For a square column $c = b = w$ we obtain

$$d^2\left(v_c + \frac{q}{4}\right) + d\left(v_c + \frac{q}{2}\right)w = (BL - w^2)\frac{q}{4}$$

For a round column, a = diameter, the expression is

$$d^2\left(v_c + \frac{q}{4}\right) + d\left(v_c + \frac{q}{2}\right)a = (BL - A_{col})\frac{q}{\pi}$$

- If we neglect the upward soil pressure on the diagonal tension block, an *approximate* effective concrete depth d can be obtained for rectangular and round columns as

Rectangular: $$4d^2 + 2(b + c)d = \frac{BLq}{v_c} = \frac{P_v}{v_c}$$

Round: $$d^2 + ad = \frac{P_u}{\pi v_c}$$

- Steps in square or rectangular spread footing design with a centrally loaded column and no moments are as follows:

1. Compute the footing plan dimensions $B \times L$ or B using the allowable soil pressure:

Square: $$B = \sqrt{\frac{\text{Critical load combination}}{q_a}} = \sqrt{\frac{P}{q_a}}$$

Rectangular: $$BL = \frac{P}{q_a}$$

A rectangular footing may have a number of satisfactory solutions unless either B or L is fixed.

2. Convert the allowable soil pressure q_a to an ultimate value $q_{ult} = q$ for footing depth

$$\frac{P_u}{BL} = q = \frac{P_{\text{ult}}}{P_{\text{design}}} q_a$$

Obtain P_u by applying appropriate load factors to the given design loading.

3. Obtain the allowable two-way action shear stress v_c and using the Equations above to compute the effective footing depth d.

4. If the footing is rectangular, immediately check wide-beam shear. Use the larger d from two-way action (step 3) or wide-beam.

5. Compute the required steel for bending, and use the same amount each way for square footings. Use the effective d to the intersection of the two bar layers for square footings and if d > 305 mm or 12 in. For d less than this and for rectangular footings use the actual d for the two directions. The bending moment is computed at the critical section shown in Fig. 5-4. For the length l shown the ultimate bending moment/unit width is

$$M_u = \frac{ql^2}{2}$$

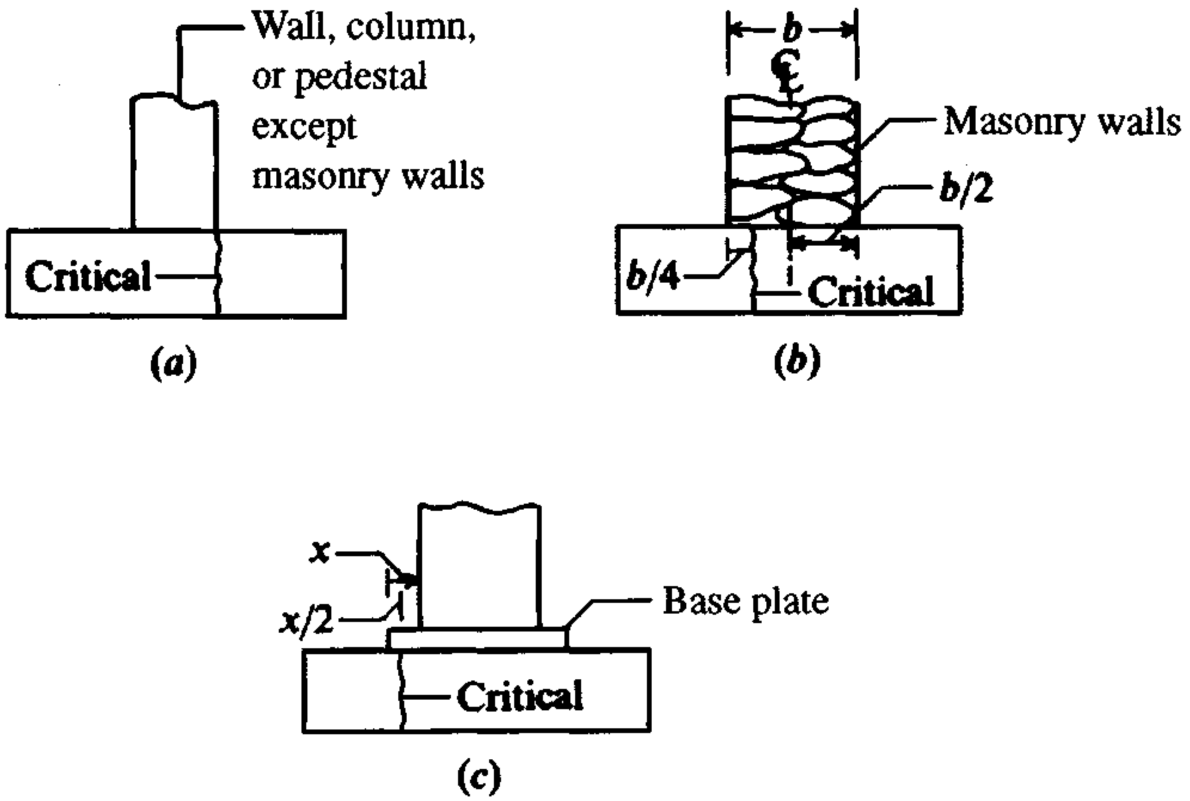


Fig. 5.4 Sections for computing bending moment.

6. Compute column bearing and use dowels for bearing if the allowable bearing stress is exceeded. In that case, compute the required dowels based on the difference between actual and allowable stresses X column area. This force, divided by *fy*, is the required area of dowels for bearing.

5.4 Reinforced Concrete Design of Shallow Foundations

5.4.1 Fundamentals of Reinforced Concrete Design

At the present time, most reinforced concrete designs are based on the recommendations of the building code prepared by the American Concrete Institute—that is, ACI 318-11. The basis for this code is the *ultimate strength design* or *strength design*. Some of the fundamental recommendations of the code are briefly summarized in the following sections.

Load Factors

According to ACI Code Section 9.2, depending on the type, the ultimate load-carrying capacity of a structural member should be one of the following:

$$U = 1.4D \quad \text{(A.1a)}$$
$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad \text{(A.1b)}$$
$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \quad \text{(A.1c)}$$
$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \quad \text{(A.1d)}$$
$$U = 1.2D + 1.0E + 1.0L + 0.2S \quad \text{(A.1e)}$$
$$U = 0.9D + 1.0W \quad \text{(A.1f)}$$

or

$$U = 0.9D + 1.0E \quad \text{(A.1g)}$$

where

U = ultimate load-carrying capacity of a member
D = dead loads
E = effects of earthquake
L = live loads
L_r = roof live loads
R = rain load
S = snow load
W = wind load

Strength Reduction Factor

The design strength provided by a structural member is equal to the nominal strength times a strength reduction factor, ϕ, or

$$\text{Design strength} = \phi\,(\text{nominal strength})$$

The reduction factor, ϕ, takes into account the inaccuracies in the design assumptions, changes in property or strength of the construction materials, and so on. Following are some of the recommended values of ϕ (ACI Code Section 9.3):

Condition	Value of ϕ
a. Axial tension; flexure with or without axial tension	0.9
b. Shear or torsion	0.75
c. Axial compression with spiral reinforcement	0.75
d. Axial compression without spiral reinforcement	0.65
e. Bearing on concrete	0.65
f. Flexure in plain concrete	0.65

Design Concepts for a Rectangular Section in Bending

Figure A.1a shows a section of a concrete beam having a width *b* and a depth *h*. The assumed stress distribution across the section at ultimate load is shown in Figure A.1b. The following notations have been used in this figure:

f'_c = compressive strength of concrete at 28 days
A_s = area of steel tension reinforcement
f_y = yield stress of reinforcement in tension
d = effective depth
l = location of the neutral axis measured from the top of the compression face
$a = \beta$

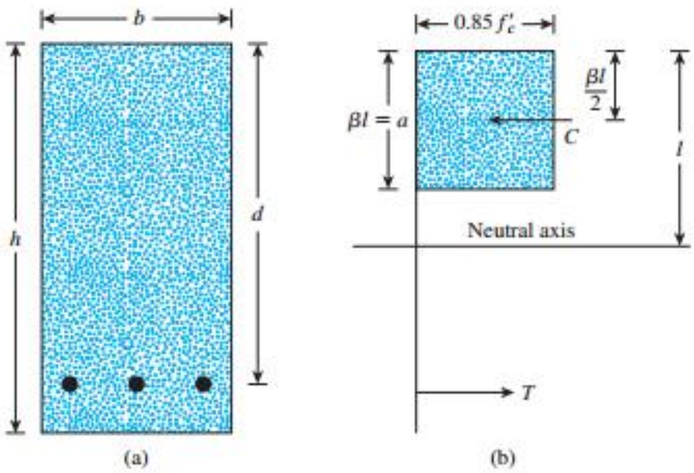


Figure A.1 Rectangular section in heading

$\beta = 0.85$ for f'_c of 28 MN/m^2 (4000 lb/in.^{2}) of less and decreases at the rate of 0.05 for every 7 MN/m^2 (1000 lb/in.^{2}) increase of f'_c. However, it cannot be less than 0.65 in any case (ACI Code Section 10.2.7).

From the principles of statics, for the section

$$\Sigma \text{ compressive force, } C = \Sigma \text{ tensile force, } T$$

Thus,

$$0.85 f'_c ab = A_s f_y$$

or

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{A.2}$$

Also, for the beam section, the nominal ultimate moment can be given as

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{A.3}$$

where M_n = theoretical ultimate moment.

The design ultimate moment, M_u, can be given as

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{A.4}$$

Combining Eqs. (A.2) and (A.4)

$$M_u = \phi A_s f_y \left[d - \left(\frac{1}{2} \right) \frac{A_s f_y}{0.85 f'_c b} \right] = \phi A_s f_y \left(d - \frac{0.59\, A_s f_y}{f'_c b} \right) \tag{A.5}$$

The steel percentage is defined by the equation

$$s = \frac{A_s}{bd} \tag{A.6}$$

In a balanced beam, failure would occur by sudden simultaneous yielding of tensile steel and crushing of concrete. The balanced percentage of steel (for Young's modulus) of steel, E_s = 200 MN/m^2) can be given as

$$s_b = \frac{0.85 f'_c}{f_y} (\beta) \left(\frac{600}{600 + f_y} \right) \tag{A.7a}$$

where f'_c and f_y are in MN/m^2.

In conventional English units (with $E_s = 29 \times 10^6$ lb/in.^{2})

$$s_b = \frac{0.85 f'_c}{f_y} (\beta) \left(\frac{87{,}000}{87{,}000 + f_y} \right) \tag{A.7b}$$

where f'_c and f_y and in lb/in.^{2}

To avoid sudden failure without warning, ACI Code Section 10.3.5 recommends that the maximum steel percentage (s_{max}) should be limited to a net tensile strain (ϵ_t) of 0.004. For all practical purposes,

$$s_{max} \approx 0.75\, s_b \tag{A.8}$$

The nominal or theoretical shear strength of a section, V_n, can be given as

$$V_n = V_c + V_s \tag{A.9}$$

where V_c = nominal shear strength of concrete
V_s = nominal shear strength of reinforcement

The permissible shear strength, V_u, can be given by

$$V_u = \phi V_n = \phi(V_c + V_s) \tag{A.10}$$

The values of V_c can be given by the following equations (ACI Code Sections 11.2 and 11.11).

$$V_c = 0.17\lambda\sqrt{f_c'}\,bd \quad \text{(for member subjected to shear and flexure)} \tag{A.11a}$$

and

$$V_c = 0.33\lambda\sqrt{f_c'}\,bd \quad \text{(for member subjected to diagonal tension)} \tag{A.11b}$$

where f_c' is in MN/m^2, V_c is in MN, b and d are in m, and $\lambda = 1$ for normal weight concrete.

In conventional English units, Eqs. (A.11a) and (A.11b) take the following form:

$$V_c = 2\lambda\sqrt{f_c'}\,bd \tag{A.12a}$$

$$V_c = 4\lambda\sqrt{f_c'}\,bd \tag{A.12b}$$

where V_c is in lb, f_c' is in lb/in.^{2}, and b and d are in inches.

Note that

$$v_c = \frac{V_c}{bd} \tag{A.13}$$

where v_c is the shear stress.

Now, combining Eqs. (A.11a), and (A.13), one obtains

$$\text{Permissible shear stress} = v_u = \frac{V_u}{bd} = 0.17\,\phi\lambda\sqrt{f_c'} \tag{A.14a}$$

Similarly, from Eqs. (A.11b), and (A.13),

$$v_u = 0.33\lambda\phi\sqrt{f_c'} \tag{A.14b}$$

Reinforcing Bars

The nominal sizes of reinforcing bars commonly used in the United States are given in Table A.1.

Table A.1 Nominal Sizes of Reinforcing Bars Used in the United States

	Diameter		Area of cross section	
Bar No.	(mm)	(in.)	(mm^2)	($in.^2$)
3	9.52	0.375	71	0.11
4	12.70	0.500	129	0.20
5	15.88	0.625	200	0.31
6	19.05	0.750	284	0.44
7	22.22	0.875	387	0.60
8	25.40	1.000	510	0.79
9	28.65	1.128	645	1.00
10	32.26	1.270	819	1.27
11	35.81	1.410	1006	1.56
14	43.00	1.693	1452	2.25
18	57.33	2.257	2580	4.00

Reinforcing-bar sizes in the metric system have been recommended by UNESCO (1971). (Bars in Europe will be specified to comply with the standard EN 100080).

Bar diameter, mm	Area, mm^2
6	28
8	50
10	79
12	113
14	154
16	201
18	254
20	314
22	380
25	491
30	707
32	804
40	1256
50	1963
60	2827

This appendix uses the standard bar diameters recommended by UNESCO.

Development Length

The development length, L_d , is the length of embedment required to develop the yield stress in the tension reinforcement for a section in flexure. ACI Code Section 12.2 lists the basic development lengths for tension reinforcement.

Example 5.1: Design Example of a Continuous Wall Foundation

Let it be required to design a load-bearing wall with the following data:

Dead load $= D = 43.8$ kN/m
Live load $= L = 17.5$ kN/m
Gross allowable bearing capacity of soil $= 94.9$ kN/m^2
Depth of the top of foundation from the ground surface $= 1.2$ m
$f_y = 413.7$ MN/m^2
$f'_c = 20.68$ MN/m^2
Unit weight of soil $= \gamma = 17.27$ kN/m^3
Unit weight of concrete $= \gamma_c = 22.97$ kN/m^3

General Considerations

For this design, assume the foundation thickness to be 0.3 m. Refer to ACI Code Section 7.7.1, which recommends a minimum cover of 76 mm over steel reinforcement, and assume that the steel bars to be used are 12 mm in diameter (Figure A.2a). Thus,

$$d = 300 - 76 - \frac{12}{2} = 218 \text{ mm}$$

Also,

$$\text{Weight of the foundation} = (0.3)\gamma_c = (0.3)(22.97) = 6.89 \text{ kN/m}^2$$

$$\text{Weight of soil above the foundation} = (1.2)\gamma = (1.2)(17.27) = 20.72 \text{ kN/m}^2$$

So, the net allowable soil bearing capacity is

$$q_{net(all)} = 94.9 - 6.89 - 20.72 = 67.29 \text{ kN/m}^2$$

Hence, the required width of foundation is

$$B = \frac{D + L}{q_{net(all)}} = \frac{43.8 + 17.5}{67.29} = 0.91 \text{ m}$$

So, assume $B = 1$ m.
According to ACI Code Section 9.2,

$$U = 1.2D + 1.6L = (1.2)(43.8) + (1.6)(17.5) = 80.56 \text{ kN/m}$$

Converting the net allowable soil pressure to an ultimate (factored) value,

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{(1)(1)} = 80.56 \text{ kN/m}^2$$

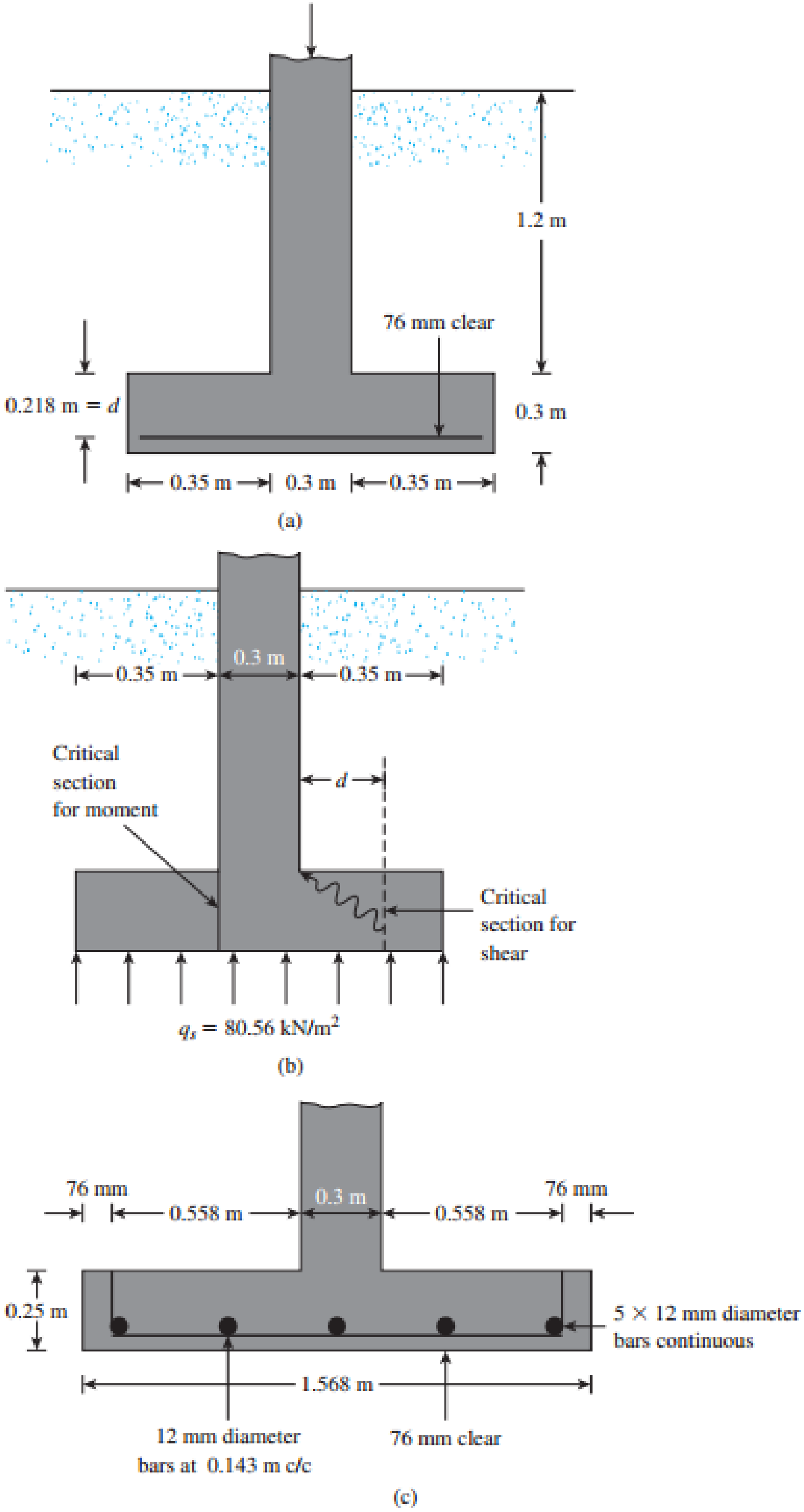


Figure A.2 Continuous wall foundation

Investigation of Shear Strength of the Foundation

The critical section for shear occurs at a distance *d* from the face of the wall (ACI Code Section 11.11.3), as shown in Figure A.2b. So, shear at critical section

$$V_u = (0.35 - d)q_s = (0.35 - 0.218)(80.56) = 10.63 \text{ kN/m}$$

From Eq. (A.11a) with $\lambda = 1$,

$$V_c = 0.17\sqrt{f_c'}\,bd = 0.17\sqrt{20.68}(1)(0.218) = 0.1685 \text{ MN/m} \approx 168 \text{ kN/m}$$

Also,

$$\phi V_c = (0.75)(168) = 126 \text{ kN/m} > V_u = 10.63 \text{ kN/m—O.K.}$$

(*Note:* $\phi = 0.75$ for shear—ACI Code Section 9.3.2.3.)

Because $V_u < \phi V_c$, the total thickness of the foundations could be reduced to 250 mm. So, the modified

$$d = 250 - 76 - \frac{12}{2} = 168 \text{ mm} > 152 \text{ mm} = d_{\min} \text{ (ACI Code Section 15.7)}$$

Neglecting the small difference in footing weight, if $d = 168$ mm,

$$\phi V_c = (0.75)(0.17)\sqrt{20.68}(1)(0.168) = 0.0974 \text{ MN}$$

$$= 97.4 \text{ kN} > V_u\text{—O.K.}$$

Flexural Reinforcement

For steel reinforcement, factored moment at the face of the wall has to be determined (ACI Code Section 15.4.2). The bending of the foundation will be in one direction only. So, according to Figure A.2b, the design ultimate moment

$$M_u = \frac{q_s l^2}{2}$$

$$l = 0.35 \text{ m}$$

So,

$$M_u = \frac{(80.56)(0.35)^2}{2} = 4.93 \text{ kN-m/m}$$

From Eqs. (A.2) and (A.3),

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1)} = 23.5351 A_s$$

Thus,

$$M_n = (A_s)(413.7)\left(0.168 - \frac{23.5351}{2} A_s\right)$$

or

$$M_n(\text{MN-m/m}) = 69.5 A_s - 4868.24 A_s^2$$

Again, from Eq. (A.4)

$$M_u \leqslant \phi M_n$$

where $\phi = 0.9$.

Thus,

$$4.93 \times 10^{-3}(\text{MN-m/m}) = 0.9\,(69.5 A_s - 4868.24 A_s^2)$$

Solving for A_s, one gets

$$A_{s(1)} = 0.0128 \text{ m}^2;\ A_{s(2)} = 0.0001 \text{ m}^2$$

Hence, steel percentage with $A_{s(1)}$ is

$$s_1 = \frac{A_{s(1)}}{bd} = \frac{0.0128}{(1)(0.168)} = 0.0762$$

Similarly, steel percentage with $A_{s(2)}$ is

$$s_2 = \frac{A_{s(2)}}{bd} = \frac{0.0001}{(1)(0.168)} = 0.0006 < s_{\min} = 0.0018 \text{ (ACI Code Section 7.12.2.1)}$$

The maximum steel percentage that can be provided is given in Eqs. (A.7a) and (A.8). Thus,

$$s_{\max} = (0.75)(0.85)\frac{f'_c}{f_y}\beta\left(\frac{600}{600 + f_y}\right)$$

Note that $\beta = 0.85$. Substituting the proper values of β, f'_c, and f_y in the preceding equation, one obtains

$$s_{\max} = 0.016$$

Note that $s_1 = 0.0762 > s_{\max} = 0.016$. So use $s = s_{\min} = 0.0018$. So,

$$A_s = (s_{\min})(b)(d) = (0.0018)(1)(0.168) = 0.000302 \text{ m}^2 = 302 \text{ mm}^2$$

Use 12-mm diameter bars @ 350 mm c/c. Hence,

$$A_s(\text{provided}) = \frac{1000}{350}\left(\frac{\pi}{4}\right)(12)^2 = 323 \text{ mm}^2$$

Development Length of Reinforcement Bars (*Ld*)

According to ACI Code Section 12.2, the minimum development length *Ld* for 12 mm diameter bars is about 558 mm (approximately equivalent to No. 4 U.S. bar). Assuming a 76-mm cover to be on both sides of the footing, the minimum footing width should be [2(558 + 76) + 300] mm = 1568 mm = 1.568 m. Hence, the revised calculations are

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{1.568} = 51.38\,\text{kN/m}^2$$

$$M_u = \frac{q_s l^2}{2} = \frac{1}{2}(51.38)(0.558 + 0.076)^2$$

$$= 10.326\ \text{kN}\cdot\text{m/m} = 10.326 \times 10^{-3}\,\text{MN}\cdot\text{m/m}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{A_s(413.7)}{(0.85)(20.68)(1.568)} = 15.01 A_s$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = A_s(413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

$$\phi M_n \geq M_u$$

$$10.326 \times 10^{-3} = 0.9 A_s(413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

and

$$A_s = 0.00016\ \text{m}^2$$

The steel percentage is $s = \frac{A_s}{bd} = \frac{0.00016}{(1.568)(0.25)} < 0.0018.$

(*Note:* Use gross area when $s_{min} = 0.0018$ is used.)

Use $A_s = (0.0018)(1.568)(0.25) = 0.000706\ \text{m}^2 = 706\ \text{mm}^2$. Provide 7 × 12 mm bars ($A_s = 565\ \text{mm}^2$).

Minimum reinforcement should be furnished in the long direction to offset shrinkage and temperature effects (ACI Code Section 7.12.). So,

$$A_s = (0.0018)(b)(d) = (0.0018)\,[(0.558 + 0.076)(2) + 0.3](0.168)$$
$$= 0.000474\ \text{m}^2 = 474\ \text{mm}^2$$

Provide 5 × 12 mm bars ($A_s = 565\ \text{mm}^2$).

The final design sketch is shown in Figure A.2c.

Example 5.2: Design Example of a Square Foundation for a Column

Figure A.3a shows a square column foundation with the following conditions:

Live load= L = 675 kN

Dead load = D = 1125 kN

Allowable gross soil-bearing capacity = q_{all} = 145 kN/m^2

Column size = 0.5 m × 0.5 m

f'_c = 20.68 MN/m^2

f_y = 413.7 MN/m^2

Let it be required to design the column foundation.

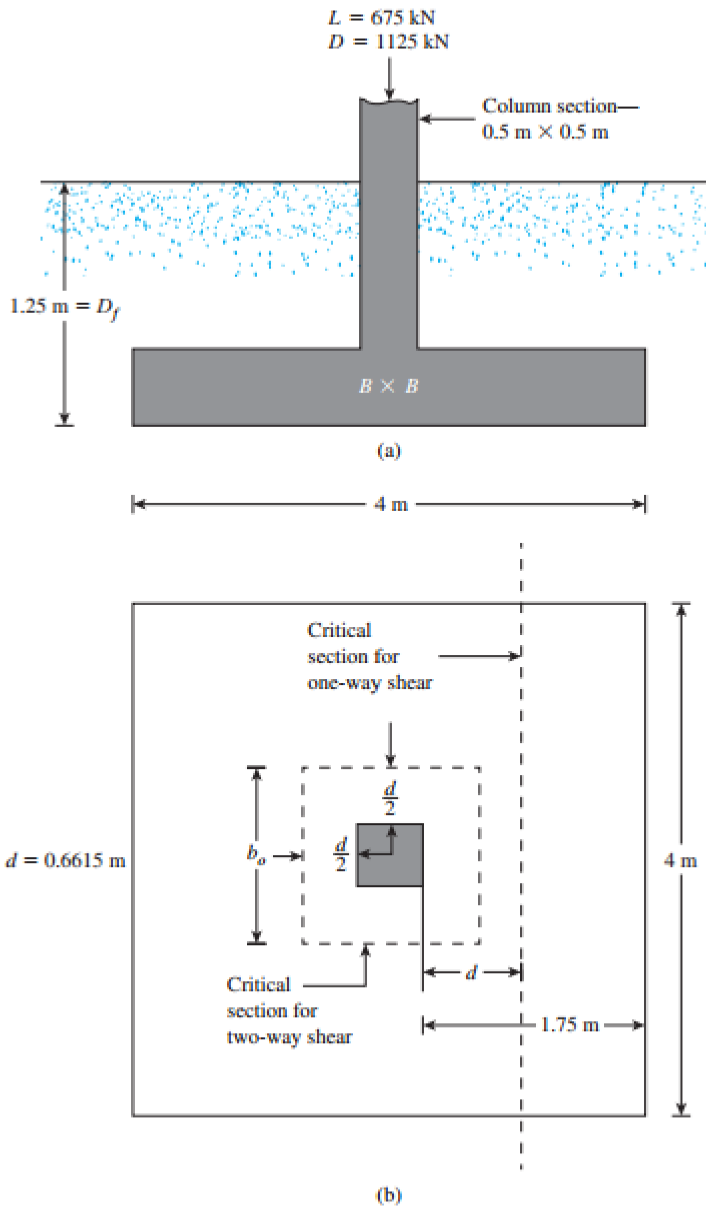


Figure A.3 Square foundation for a column

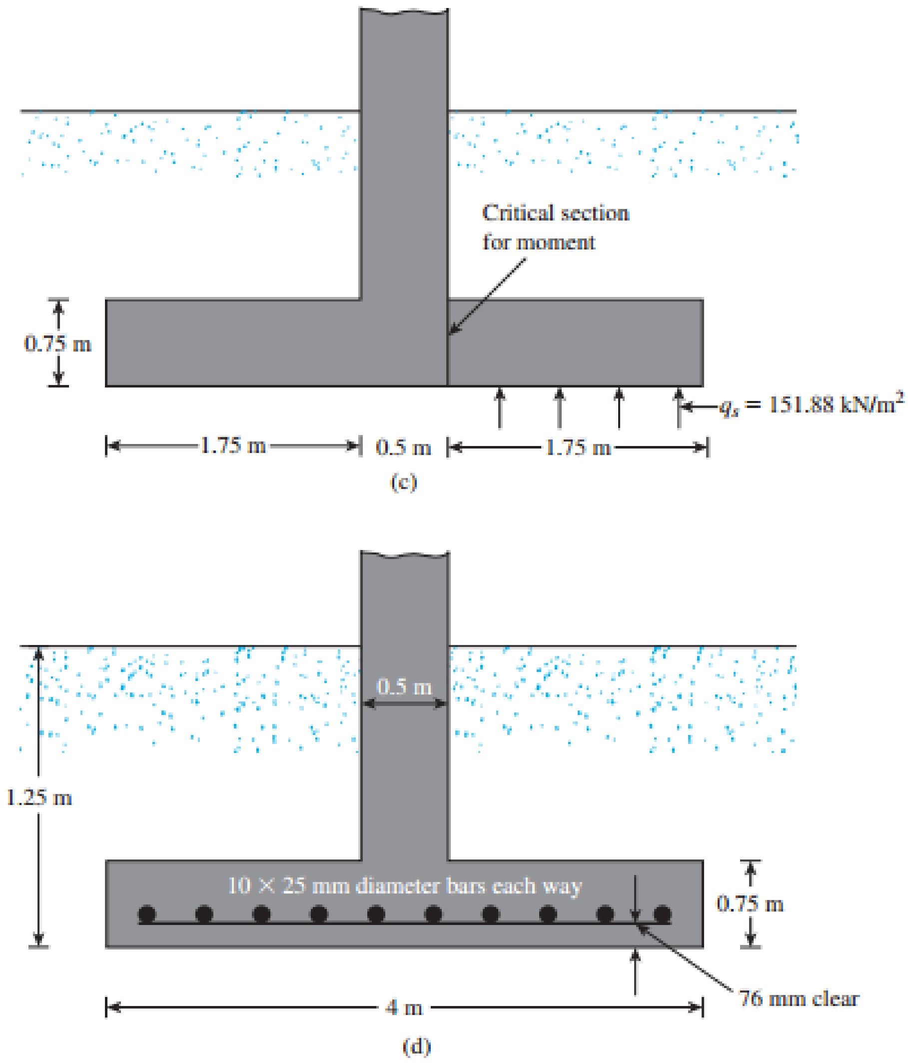


Figure A.3 (*continued*)

General Considerations

Let the average unit weight of concrete and soil above the base of the foundation be 21.97 kN/m^3. So, the net allowable soil-bearing capacity

$$q_{all(net)} = 145 - (D_f)(21.97) = 145 - (1.25)(21.97) = 117.54 \text{ kN/m}^2$$

Hence, the required foundation area is

$$A = B^2 = \frac{D + L}{q_{all(net)}} = \frac{675 + 1125}{117.54} = 15.31 \text{ m}^2$$

Use a foundation with dimensions (B) of 4 m × 4 m.

The factored load for the foundation is

$$U = 1.2D + 1.6L = (1.2)(1125) + (1.6)(675) = 2430 \text{ kN}$$

Hence, the factored soil pressure is

$$q_s = \frac{U}{B^2} = \frac{2430}{16} = 151.88\,\text{kN/m}^2$$

Assume the thickness of the foundation to be equal to 0.75 m. With a clear cover of 76 m over the steel bars and an assumed bar diameter of 25 mm, we have

$$d = 0.75 - 0.076 - \frac{0.025}{2} = 0.6615\,\text{m}$$

Check for Shear

As we have seen in Section A.4, V_u should be equal to or less than ϕV_c. For one-way shear [with $\lambda = 1$ in Eq. (A.11a)],

$$V_u \leq \phi(0.17)\sqrt{f'_c}\,bd$$

The critical section for one-way shear is located at a distance d from the edge of the column (ACI Code Section 11.1.3) as shown in Figure A.3b. So

$$V_u = q_s \times \text{critical area} = (151.88)(4)(1.75 - 0.6615) = 661.3\text{ kN}$$

Also (with $\lambda = 1$),

$$\phi V_c = (0.75)(0.17)(\sqrt{20.68})(4)(0.6615)(1000) = 1534.2\text{ kN}$$

So,

$$V_u = 661.3\text{ kN} \leq \phi V_c = 1534.2\text{ kN—O.K.}$$

For two-way shear, the critical section is located at a distance of $d/2$ from the edge of the column (ACI Code Section 11.11.1.2). This is shown in Figure A.3b. For this case, [with $\lambda = 1$ in Eq. (A.11b)]

$$\phi V_c = \phi(0.33)\sqrt{f'_c}\,b_o d$$

The term b_o is the perimeter of the critical section for two-way shear. Or for this design,

$$b_o = 4[0.5 + 2(d/2)] = 4[0.5 + 2(0.3308)] = 4.65\text{ m}$$

Hence,

$$\phi V_c = (0.75)(0.33)(\sqrt{20.68})(4.65)(0.6615) = 3.462\,\text{MN} = 3462\text{ kN}$$

Also,

$$V_u = (q_s)(\text{critical area})$$
$$\text{Critical area} = (4 \times 4) - (0.5 + 0.6615)^2 = 14.65\text{ m}^2$$

So,

$$V_u = (151.88)\,(14.65) = 2225.18\text{ kN}$$
$$V_u = 2225.18\text{ kN} < \phi V_c = 3462\text{ kN—O.K.}$$

The assumed depth of foundation is more than adequate.

Flexural Reinforcement

According to Figure A.3c, the moment at critical section (ACI Code Section 15.4.2) is

$$M_u = (q_s B)\left(\frac{1.75}{2}\right)^2 = \frac{[(151.88)(4)](1.75)^2}{2} = 930.27 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_v}{0.85 f_o' b} \qquad (Note\text{: } b = B)$$

or

$$A_s = \frac{0.85 f_c' Ba}{f_y} = \frac{(0.85)(20.68)(4)a}{413.7} = 0.17a$$

From Eq. (A.4),

$$M_u \le \phi A_s f_v \left(d - \frac{a}{2}\right)$$

With $\phi = 0.9$ and $A_s = 0.17a$,

$$M_u = 930.27 = (0.9)(0.17a)(413700)\left(0.6615 - \frac{a}{2}\right)$$

Solution of the preceding equation given $a = 0.0226$ m. Hence,

$$A_s = 0.17a = (0.17)(0.0226) = 0.0038 \text{ m}^2$$

The percentage of steel is

$$s = \frac{A_s}{bd} = \frac{A_s}{Bd} = \frac{0.0038}{(4)(0.6615)} = 0.0015 < s_{\min}$$
$$= 0.0018 \text{ (ACI Code Section 7.12)}$$

So,

$$A_{s(\min)} = (0.0018)(B)(d) = (0.0018)(4)(0.6615)$$
$$= 0.004762 \text{ m}^2 = 47.62 \text{ cm}^2$$

Provide 10 × 25-mm diameter bars each way [$A_s = (4.91)(10) = 49.1 \text{ cm}^2$].

Check for Development Length (L_d)

From ACI Code Section 12.2.2, for 25 mm diameter bars, $L_d \approx 1338$ mm. Actual L_d provided is $(4 - 0.5/2) - 0.076$ (cover) $= 1.674$ m > 1338 mm—O.K.

Check for Bearing Strength

ACI Code Section 10.14 indicates that the bearing strength should be at least $0.85\phi f_c' A_1 \sqrt{A_2/A_1}$ with a limit of $\sqrt{A_2/A_1} \le 2$. For this problem, $\sqrt{A_2/A_1} = \sqrt{(4 \times 4)/(0.5 \times 0.5)} = 8$. So, use $\sqrt{A_2/A_1} = 2$. Also $\phi = 0.7$. Hence, the design bearing strength = (0.85)(0.65)(20.68)(0.5 × 0.5)(2) = 5.713 MN = 5713 kN. However, the factored column load U = 2430 kN < 5713 kN—O.K.

The final design section is shown in Figure A.3d.

Example 5.3: Design Example of a Rectangular Foundation for a Column

This section describes the design of a rectangular foundation to support a column having dimensions of 0.4 m × 0.4 m in cross section. Other details are as follows:

Dead load = D = 290 kN
Live load = L = 110 kN
Depth from the ground surface to the top of the foundation = 1.2 m
Allowable gross soil-bearing capacity = 120 kN/m^2
Maximum width of foundation = B = 1.5 m
f_y = 413.7 MN/m^2
f'_c = 20.68 MN/m^2
Unit weight of soil = γ = 17.27 kN/m^3
Unit weight of concrete = γ_c = 22.97 kN/m^3

General Considerations

For this design, let us assume a foundation thickness of 0.45 m (Figure A.4a). The weight of foundation/m^2 = 0.45 γ_c = (0.45) (22.97) = 10.34 kN/m^2, and the weight of soil above the foundation/m^2 = (1.2)γ = (1.2) (17.27) = 20.72 kN/m^2. Hence, the net allowable soil-bearing capacity [$q_{net(all)}$] = 120 − 10.34 − 20.72 = 88.94 kN/m^2.

The required area of the foundation = $(D + L)/q_{net(all)}$ = (290 + 110)/88.94 = 4.5 m^2. Hence, the length of the foundation is 4.5 m^2/B = 4.5/1.5 = 3 m.

The factored column load = 1.2D + 1.6L = 1.2(290) + 1.6(110) = 524 kN.

The factored soil-bearing capacity, q_s = factored load/foundation area = 524/4.5 = 116.44 kN/m^2.

Shear Strength of Foundation

Assume that the steel bars to be used have a diameter of 16 mm. So, the effective depth d = 450 − 76 − 16/2 = 366 mm. (Note that the assumed clear cover is 76 mm.)

Figure A.4a shows the critical section for one-way shear (ACI Code Section 11.11.1.1). According to this figure

$$V_u = \left(1.5 - \frac{0.4}{2} - 0.366\right)Bq_s = (0.934)(1.5)(116.44) = 163.13 \text{ kN}$$

The nominal shear capacity of concrete for one-way beam action [with λ = 1 in Eq. (11.a)]

$$V_c = 0.17\sqrt{f'_c}Bd = 0.17(\sqrt{20.68})(1.5)(0.366) = 0.4244 \text{ MN} = 424.4 \text{ kN}$$

The critical section for two-way shear is also shown in Figure A.4a. This is based on the recommendations given by ACI Code Section 11.11.1.2. For this section

$$V_u = q_s[(1.5)(3) - 0.766^2] = 455.66 \text{ kN}$$

The nominal shear capacity of the foundation can be given as (ACI Code Section 11.11.2)

$$V_c = v_c b_o d = 0.33\lambda\sqrt{f_c'}\, b_o d$$

where b_o = perimeter of the critical section

or

$$V_c = (0.33)(1)(\sqrt{20.68})(4 \times 0.766)(0.366) = 1.683 \text{ MN}$$

So, for two-way shear condition

$$V_u = 455.66 \text{ kN} < \phi V_c = (0.75)(1683) = 1262.25 \text{ kN}$$

Therefore, the section is adequate.

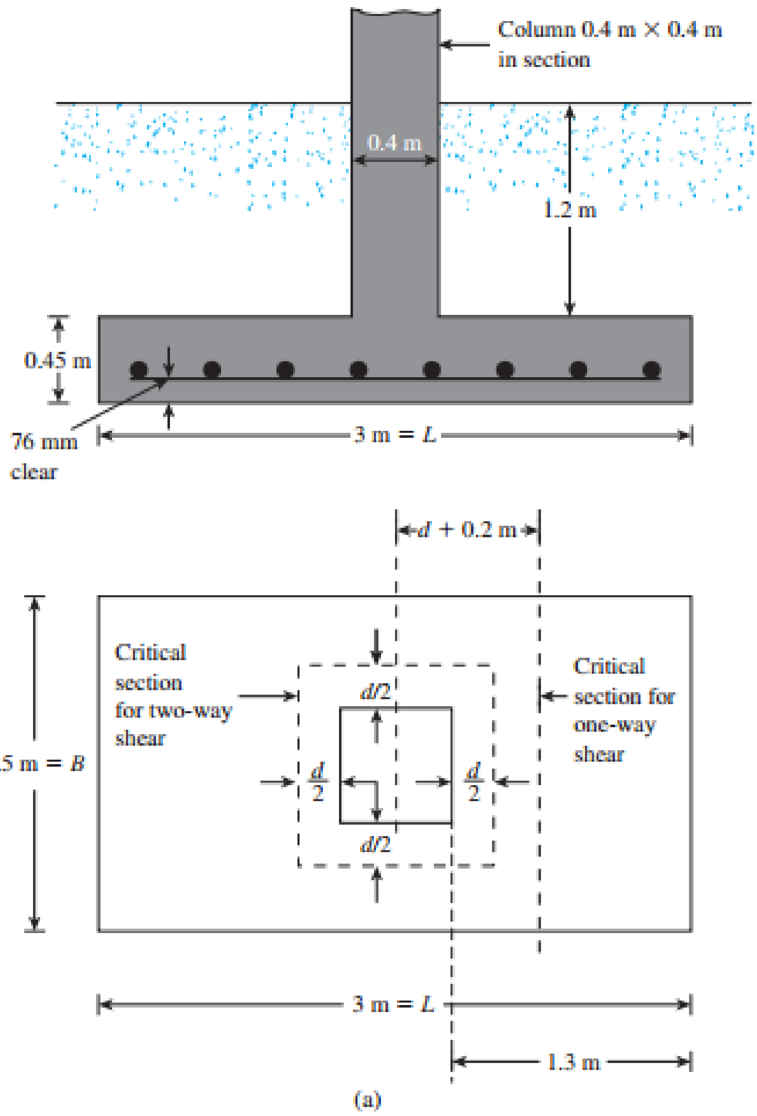


Figure A.4 Rectangular foundation for a column

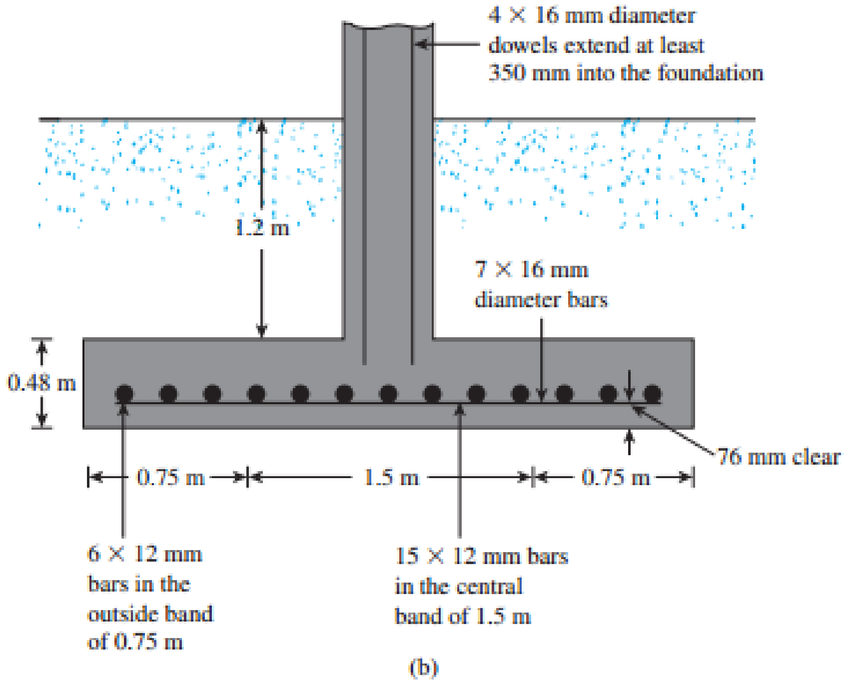


Figure A.4 (*continued*)

Check for Bearing Capacity of Concrete Column at the interface with Foundation

According to ACI Code Section 10.14.1, the bearing strength is equal to $0.85\ \phi f'_c A_1$ ($\phi = 0.65$). For this problem, $U = 524$ kN < bearing strength $= (0.85)(0.65)(20.68)(0.4)^2 = 1.828$ MN.

So, a minimum area of dowels should be provided across the interface of the column and the foundation (ACI Code Section 15.8.2). Based on ACI Code Section 15.8.2.1

$$\text{Minimum area of steel} = (0.005)\ (\text{area of column}) = (0.005)\ (400^2) = 800\ \text{mm}^2$$

So use 4 × 16-mm diameter bars as dowels.

The minimum required length of development (L_d) of dowels in the foundation is $(0.24 f_y d_b)/\lambda\sqrt{f'_c}$, but not less than $0.043\, f_y d_b$ (ACI Code Section 12.3.2). So,

$$L_d = \frac{0.24 f_y d_b}{\lambda\sqrt{f'_c}} = \frac{(0.24)(413.7)(16)}{(1)(\sqrt{20.68})} = 349.33\ \text{mm}$$

Also,

$$L_d = 0.043\, f_y d_b = (0.043)(413.7)(16) = 284.6\ \text{mm}$$

Hence, $L_d = 349.33$-mm controls.

Available depth for the dowels (Figure A.4a) is $450 - 76 - 16 - 16 = 342$ mm. Since hooks cannot be used, the foundation depth must be increased. Let the new depth be equal to 480 mm to accommodate the required $L_d = 349.33$ mm. Hence, the new value of d is equal to $480 - 76 - 16 - 16 = 372$ mm.

Flexural Reinforcement in the Long Direction

According to Figure A.4a, the design moment about the column face is

$$M_u = \frac{(q_s B)1.3^2}{2} = \frac{(116.44)(1.5)(1.3)^2}{2} = 147.59 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1.5)} = 15.69 A_s$$

Again, from Eq. (A.4),

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

or

$$147.59 = (0.9)(A_s)(413.7 \times 10^3)\left[0.396 - \frac{15.69}{2}(A_s)\right]$$

$$147.59 = 147{,}444.7\, A_s - 2{,}920{,}928 A_s^2$$

(*Note:* $d = 0.396$ m, assuming that these bars are placed as the bottom layer.)

The solution of the preceding equation gives

$$A_s = 0.00102 \text{ m}^2 \left[\text{that is, steel percentage} = \frac{A_s}{Bd} = \frac{0.00102}{(1.5)(0.396)} = 0.0017\right]$$

Also, from ACI Code Section 7.12.2, $s_{min} = 0.0018$. Hence, provide 7×16-mm diameter bars (A_s provided is 0.001407 m^2).

Flexural Reinforcement in the Short Direction

According to Figure A.4a, the moment at the face of the column is

$$M_u = \frac{(q_s L)(0.55)^2}{2} = \frac{(116.44)(3)(0.55)^2}{2} = 52.83 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(3)} = 7.845 A_s$$

From Eq. (A.4),

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

or

$$52.83 = (0.9)(A_s)\,(413.7 \times 10^3)\left[0.380 - \frac{7.845}{2}(A_s)\right]$$

(*Note:* $d = 480 - 76 - 16 - \frac{16}{2} = 380$ mm for short bars in the upper layer.)

The solution of the preceding equation gives

$$A_s = 0.0004 \text{ m}^2 \qquad (\text{thus } s < s_{\min})$$

So, use $s = s_{\min}$, or

$$A_s = s_{\min}\, bd = (0.0018)(3)(0.48) \approx 0.0026 \text{ m}^2$$

(*Note* : Use gross area when $s_{\min} = 0.0018$ is used.)

Use 13 × 16-mm diameter bars.

According to ACI Code Section 12.2, the development length L_d for 16 mm diameter bars is about 693 mm. For such a case, the footing width needs to be [2(0.693 + 0.076) + 0.4] = 1.938 m. Since the footing width is limited to 1.5 m, we should use 12-mm diameter bars.

So, use 23 × 12 mm diameter bars.

Final Design Sketch

According to ACI Code Section 15.4.4, a portion of the reinforcement in the short direction shall be distributed uniformly over a bandwidth equal to the smallest dimension of the foundation. The remainder of the reinforcement should be distributed uniformly outside the central band of the foundation. The reinforcement in the central band can be given to the equal to $2/(\beta_c + 1)$ (where $\beta_c = L/B$). For this problem, $\beta_c = 2$. Hence, 2/3 of the reinforcing bars (that is, 15 bars) should be placed in the center band of the foundation. The remaining bars should be placed outside the central band. However, one needs to check the steel percentage in the outside band, or

$$s = \frac{A_s}{bd} = \frac{(2)(113\text{ mm}^2)}{\left(\dfrac{3000 - 1500}{2}\right)(380)} = 0.00079 < s_{\min} = 0.0018$$

So, use $A_s = (s_{\min})(b)(d) = (0.0018)(750)(480) = 648 \text{ mm}^2$. Hence, 6 × 12-mm diameter bars on each side of the central band will be sufficient.

The final design sketch is shown in Figure A.4b.

5.5 Geometric Design of Combined Footings

5.5.1 Combined Footings

Types:

1. Rectangular Combined Footing (two columns).

2. Trapezoidal Combined Footing (two columns).

3. Strip Footing (more than two columns and may be rectangular or trapezoidal).

Usage:

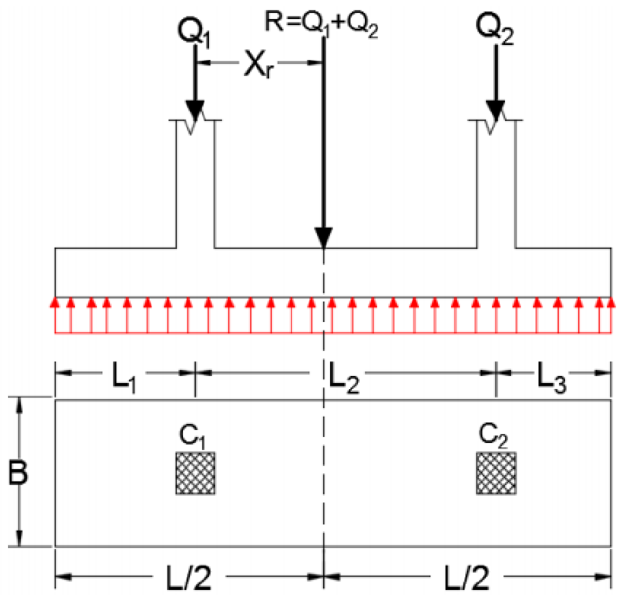
1. Used when the loads on the columns are heavy and the distance between these columns is relatively small (i.e. when the distance between isolated footings is less than 30 cm).

2. Used as an alternative to neighbor footing which is an eccentrically loaded footing and it's danger if used when the load on the column is heavy.

5.4.1 Design of Rectangular Combined Footings:

There are three cases:

1. Extension is permitted from both side of the footing:



- The resultant force R is more closed to the column which have largest load.

- To keep the pressure under the foundation uniform, the resultant force of all columns loads (R) must be at the center of the footing, and since the footing is rectangular, R must be at the middle of the footing (at distance L/2) from each edge to keep uniform pressure.

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1+Q_2}{q_{all,net}} = B \times L$$

How we can find L:

$L_2 =$ distance between centers of the two c

$X_r =$ distance between the resultant forc

OR column (2) as u like ☺.

L_1 and $L_3 =$ extensions from left and right

Now take summation moments at C_1 equals

$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_2 + (W_{footing} + W_{soil}$

$(W_{footing} + W_{soil})$ are located at the cente

If we are not given any information about (

$Q_2 L_2 = R \times X_r \rightarrow X_r = \checkmark$.

Now, to keep uniform pressure under the fo

$X_r + L_1 = \frac{L}{2}$ (Two unknowns "L_1" and "

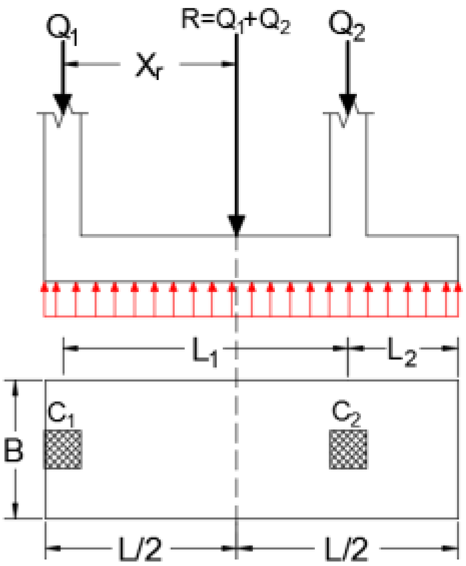
The value of L_1 can be assumed according

$\rightarrow L = \checkmark \rightarrow\rightarrow B = \frac{A_{req}}{L} = \checkmark$.

2. Extension is permitted from one side and prevented from other side:

- The only difference between this case and the previous case that the extension exists from one side and when we find X_r we can easily find L:
  To keep the pressure uniform

$$X_r + \frac{\text{column width}}{2} = \frac{L}{2} \rightarrow L = \checkmark.$$



3. Extension is not permitted from both sides of the footing:

- In this case the resultant force R doesn't in the center of rectangular footing because Q_1 and Q_2 are not equals and no extensions from both sides. So the pressure under the foundation is not uniform and we design the footing in this case as following:

$L = L_1 + W_1 + W_2 = ✓.$

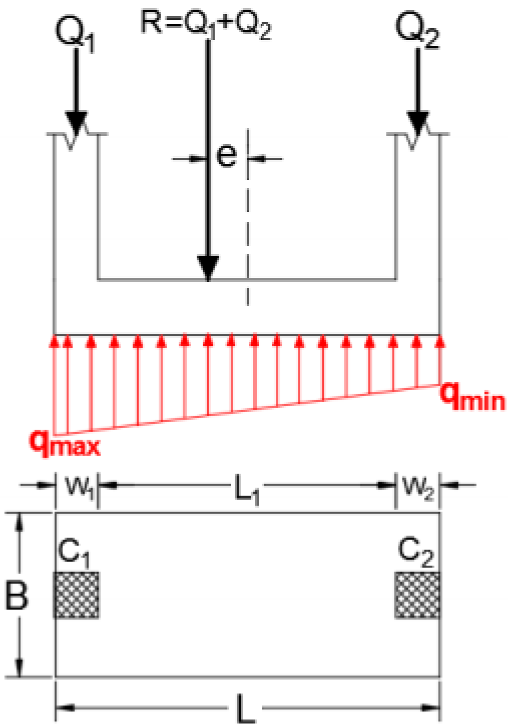
How we can find e:

$$\sum M_{\text{foundation center}} = 0.0$$

$$Q_1 \times \left(\frac{L}{2} - \frac{W_1}{2}\right) - Q_2 \times \left(\frac{L}{2} - \frac{W_2}{2}\right) = R \times e$$

$\rightarrow e = ✓.$

Note: the moment of $W_f + W_s$ is zero because they located at the center of footing.



The eccentricity in the direction of L:

Usually $e < \frac{L}{6}$ (because L is large)

$$q_{max} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$q_{all,gross} \geq q_{max} \rightarrow q_{all,gross} = q_{max}$ (critical case)

$$q_{all,gross} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right) \rightarrow B = ✓.$$

Check for B:

$$q_{min} = \frac{R}{B \times L}\left(1 - \frac{6e}{L}\right) \text{ must be } \geq 0.0$$

If this condition doesn't satisfied, use the modified equation for q_{max} to find B:

$$q_{max,modified} = \frac{4R}{3B(L - 2e)} \rightarrow B = ✓.$$

5.5.2 Design of Trapezoidal Combined Footings:

Advantages:

1. More economical than rectangular combined footing if the extension is not permitted from both sides especially if there is a large difference between columns loads.

2. We can keep uniform contact pressure in case of "extension is not permitted from both sides" if we use trapezoidal footing because the resultant force "R" can be located at the centroid of trapezoidal footing.

Design:

$Q_1 > Q_2 \rightarrow B_1$ at Q_1 and B_2 at Q_2

$L = L_1 + W_1 + W_2 = \checkmark.$

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1 + Q_2}{q_{all,net}}$$

$$\frac{Q_1 + Q_2}{q_{all,net}} = \frac{L}{2}(B_1 + B_2) \rightarrow\rightarrow \text{Eq. (1)}$$

Now take summation moments at C_1 equals zero to find X_r:

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_1 +$$

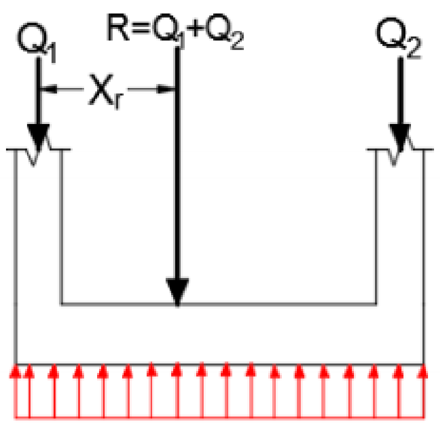
$$(W_f + W_s) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

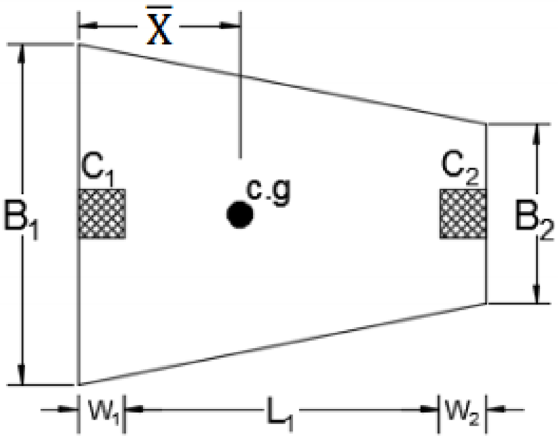
$$X_r + \frac{W_1}{2} = \overline{X} = \checkmark.$$

$$\overline{X} = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq. (2)}$$

Solve Eq. (1) and Eq. (2) $\rightarrow\rightarrow$

$B_1 = \checkmark$ and $B_2 = \checkmark.$

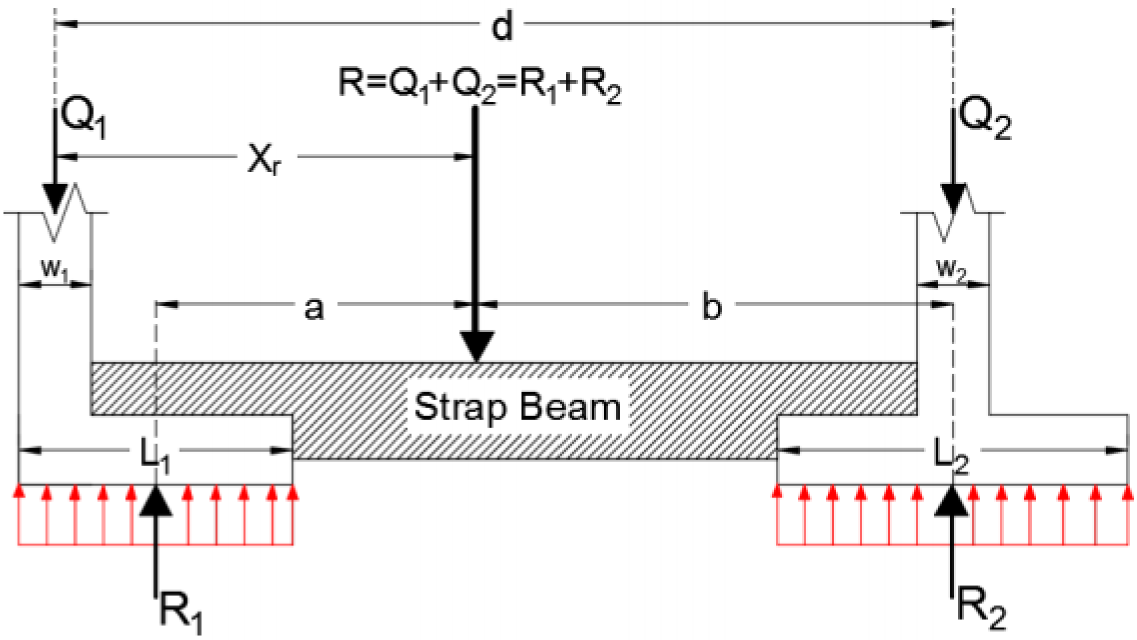




5.5.3 Geometric Design of Strap Footing (Cantilever Footing)

Usage:

1. Used when there is a **property line** which prevents the footing to be extended beyond the face of the edge column. In addition to that the edge column is relatively far from the interior column so that the rectangular and trapezoidal combined footings will be too narrow and long which increases the cost. And may be used to connect between two interior foundations one of them have a large load require a large area but this area not available, and the other foundation have a small load and there is available area to enlarge this footing, so we use strap beam to connect between these two foundations to transfer the load from largest to the smallest foundation.
2. There is a "strap beam" which connects two separated footings. The edge Footing is usually eccentrically loaded and the interior footing is centrically loaded. The **purpose of the beam is to prevent overturning of the eccentrically loaded footing and to keep uniform pressure under this foundation** as shown in figure below**.**



- Note that the strap beam doesn't touch the ground (i.e. there is no contact between the strap beam and soil, so no bearing pressure applied on it).
- This footing also called "cantilever footing" because the overall moment on the strap beam is negative moment.

Design:

$R = Q_1 + Q_2 = R_1 + R_2$ but, $Q_1 \neq R_1$ and $Q_2 \neq R_2$

Q_1 and Q_2 are knowns but R_1 and R_2 are unknowns

Finding X_r:

$\sum M_{Q_1} = 0.0$ (before use of strap beam) $\rightarrow R \times X_r = Q_2 \times d \rightarrow X_r = ✓.$

$a = X_r + \frac{w_1}{2} - \frac{L_1}{2}$ (L_1 should be assumed "if not given")

$b = d - X_r$

Finding R_1:

$\sum M_{R_2} = 0.0$(after use of strap beam) $\rightarrow R_1 \times (a + b) = R \times b \rightarrow R_1 = ✓.$

Finding R_2:

$R_2 = R - R_1$

Design:

$A_1 = \frac{R_1}{q_{all,net}}$, $A_2 = \frac{R_2}{q_{all,net}}$