University of Anbar
Engineering College
Civil Engineering Department

CHAPTER FIVE

GEOMETRIC DESIGN OF SHALLOW FOUNDATIONS

LECTURE
DR. AHMED H. ABDULKAREEM
DR. MAHER ZUHAIR AL-RAWI
2023 - 2024

Mat Foundations

5.5 Common Types of Mat Foundations

The mat foundation, which is sometimes referred to as a *raft foundation*, is a combined footing that may cover the entire area under a structure supporting several columns and walls.

Mat foundations are sometimes preferred for soils that have low load-bearing capacities, but that will have to support high column or wall loads. Under some conditions, spread footings would have to cover more than half the building area, and mat foundations might be more economical.

Several types of mat foundations are used currently. Some of the common ones are shown schematically in Figure 8.4 and include the following:

1. Flat plate (Figure 5.4a). The mat is of uniform thickness.
2. Flat plate thickened under columns (Figure 5.4b).
3. Beams and slab (Figure 5.4c). The beams run both ways, and the columns are located at the intersection of the beams.
4. Flat plates with pedestals (Figure 5.4d).
5. Slab with basement walls as a part of the mat (Figure 5.4e). The walls act as stiffeners for the mat.

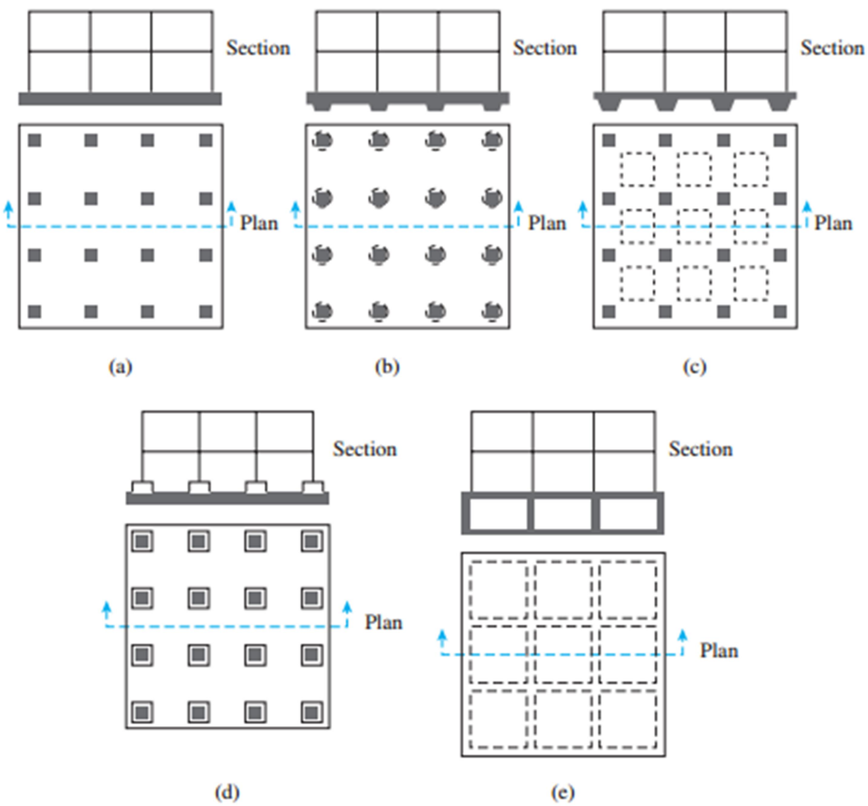


Figure **5.4** Common types of mat foundations

Mats may be supported by piles, which help reduce the settlement of structure built over highly compressible soil. Where the water table is high, mats are often placed over piles to control buoyancy. Figure 5.5 shows the difference between the depth D_f and the width B of isolated foundations and mat foundations. Figure 5.6 shows a flat-plate mat foundation under construction.

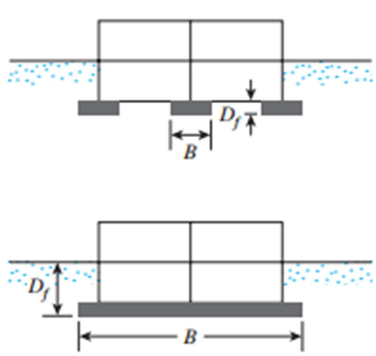


Figure **5.5** Comparison of isolated foundation and mat foundation (B = width, D_f = depth)

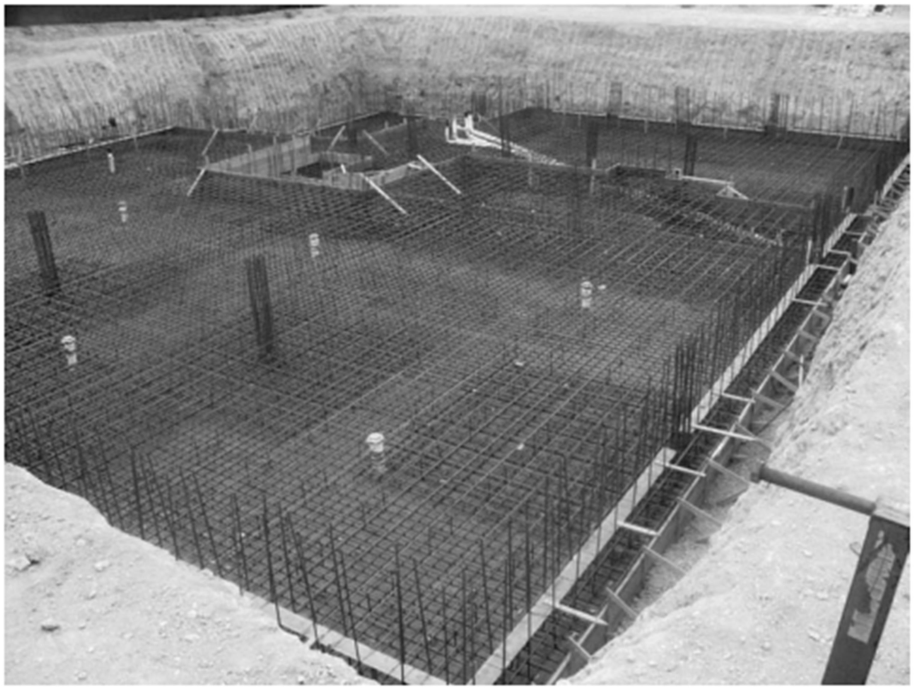


Figure **5.6** A flat plate mat foundation under construction (*Courtesy of Dharma Shakya, Geotechnical Solutions, Inc., Irvine, California*)

5.6 Bearing Capacity of Mat Foundations

The *gross ultimate bearing capacity* of a mat foundation can be determined by the same equation used for shallow foundations, or

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

The term B in Eq. above is the smallest dimension of the mat. The *net ultimate capacity* of a mat foundation is

$$q_{net(u)} = q_u - q$$

A suitable factor of safety should be used to calculate the net *allowable* bearing capacity. For mats on clay, the factor of safety should not be less than 3 under dead load or maximum live load. However, under the most extreme conditions, the factor of safety should be at least 1.75 to 2. For mats constructed over sand, a factor of safety of 3 should normally be used. Under most working conditions, the factor of safety against bearing capacity failure of mats on sand is very large.

For saturated clays with $\phi = 0$ and a vertical loading condition, Eq. above gives

$$q_u = c_u N_c F_{cs} F_{cd} + q \qquad \textbf{5.10}$$

where c_u = undrained cohesion. (*Note:* $N_c = 5.14$, $N_q = 1$, and $N_\gamma = 0$.)
From Table **3.3** for $\phi = 0$,

$$F_{cs} = 1 + \frac{B}{L}\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{B}{L}\right)\left(\frac{1}{5.14}\right) = 1 + \frac{0.195B}{L}$$

and

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

Substitution of the preceding shape and depth factors into Eq. **above** yields

$$q_u = 5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) + q \qquad \textbf{(5.11)}$$

Hence, the net ultimate bearing capacity is

$$q_{\text{net}(u)} = q_u - q = 5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) \qquad \textbf{(5.12)}$$

For FS = 3, the net allowable soil bearing capacity becomes

$$q_{\text{net(all)}} = \frac{q_{u(\text{net})}}{\text{FS}} = 1.713c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) \qquad \textbf{(5.13)}$$

- The net allowable bearing capacity for mats constructed over granular soil deposits can be adequately determined from the standard penetration resistance numbers. For shallow foundations,

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right)$$

where

N_{60} = standard penetration resistance
B = width (m)
$F_d = 1 + 0.33(D_f/B) \leq 1.33$
S_e = settlement, (mm)

When the width B is large, the preceding equation can be approximated as

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08} F_d\left(\frac{S_e}{25}\right) = \frac{N_{60}}{0.08}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right]\left[\frac{S_e(\text{mm})}{25}\right] \leq 16.63 N_{60}\left[\frac{S_e(\text{mm})}{25}\right] \qquad (5.14)$$

- Generally, shallow foundations are designed for a maximum settlement of 25 mm (1 in.) and a differential settlement of about 19 mm (0.75 in.).
- However, the width of the raft foundations are larger than those of the isolated spread footings. The depth of significant stress increase in the soil below a foundation depends on the width of the foundation. Hence, for a raft foundation, the depth of the zone of influence is likely to be much larger than that of a spread footing. Thus, the loose soil pockets under a raft may be more evenly distributed, resulting in a smaller differential settlement. Accordingly, the customary assumption is that, *for a maximum raft settlement of* 50 mm (2 in.), *the differential settlement would be* 19 mm (0.75 in.). Using this logic and conservatively assuming that $F_d = 1$, we can respectively approximate Eqs. above as

$$q_{net(all)} = q_{net}(\text{kN/m}^2) \approx 25N_{60} \tag{5.16a}$$

and

$$q_{net(all)} = q_{net}(\text{kip/ft}^2) = 0.5N_{60} \tag{5.16b}$$

The net pressure applied on a foundation (see Figure 8.7) may be expressed as

$$q = \frac{Q}{A} - \gamma D_f \tag{5.17}$$

where

Q = dead weight of the structure and the live load
A = area of the raft

In all cases, q should be less than or equal to allowable $q_{net(all)}$.

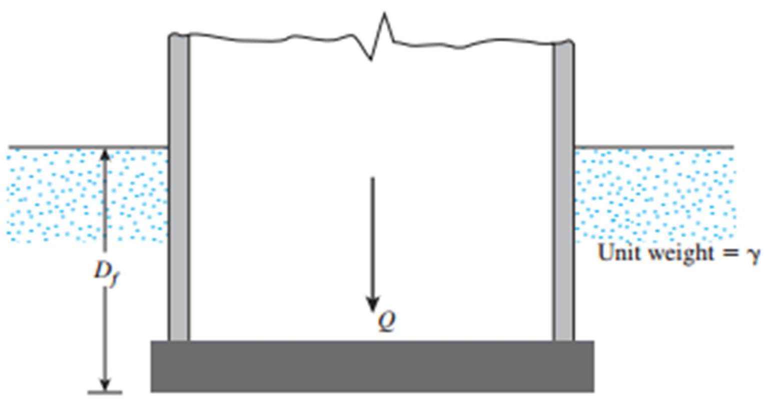


Figure 8.7 Definition of net pressure on soil caused by a mat foundation

Example 5.1

Determine the net ultimate bearing capacity of a mat foundation measuring 20 m × 8 m on a saturated clay with c_u = 85 kN/m², ϕ = 0, and D_f = 1.5 m.

Solution

From Eq. (8.12),

$$q_{net(u)} = 5.14c_u\left[1 + \left(\frac{0.195B}{L}\right)\right]\left[1 + 0.4\frac{D_f}{B}\right]$$

$$= (5.14)(85)\left[1 + \left(\frac{0.195 \times 8}{20}\right)\right]\left[1 + \left(\frac{0.4 \times 1.5}{8}\right)\right]$$

$$= \mathbf{506.3\ kN/m^2}$$

Example 5.2

What will be the net allowable bearing capacity of a mat foundation with dimensions of 15 m × 10 m constructed over a sand deposit? Here, $D_f = 2$ m, the allowable settlement is 25 mm, and the corrected average penetration number $N_{60} = 10$.

Solution

$$q_{net(all)} = 11.98N_{60}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right]\left(\frac{S_e}{25}\right) \leq 15.93N_{60}\left(\frac{S_e}{25}\right)$$

or

$$q_{net(all)} = (11.98)(10)\left[1 + \frac{0.33 \times 2}{10}\right]\left(\frac{25}{25}\right) = \mathbf{127.7\ kN/m^2}$$ ■

5.6 Compensated Foundation

- Figure 5.7 and Eq. (5.17) indicate that the net pressure increase in the soil under a mat foundation can be reduced by increasing the depth D_f of the mat. This approach is generally referred to as the *compensated foundation design* and is extremely useful when structures are to be built on very soft clays.
- In this design, a deeper basement is made below the higher portion of the superstructure, so that the net pressure increase in soil at any depth is relatively uniform. (See Figure 5.8.) From Eq. (5.17) and Figure 5.7, the net average applied pressure on soil is

$$q = \frac{Q}{A} - \gamma D_f$$

For no increase in the net pressure on soil below a mat foundation, q should be zero. Thus,

$$D_f = \frac{Q}{A\gamma} \tag{5.21}$$

This relation for D_f is usually referred to as the depth of a *fully compensated foundation*.

The factor of safety against bearing capacity failure for partially compensated foundations (i.e., $D_f < Q/A\gamma$) may be given as

$$FS = \frac{q_{net(u)}}{q} = \frac{q_{net(u)}}{\frac{Q}{A} - \gamma D_f} \tag{5.22}$$

where $q_{net(u)}$ = net ultimate bearing capacity.

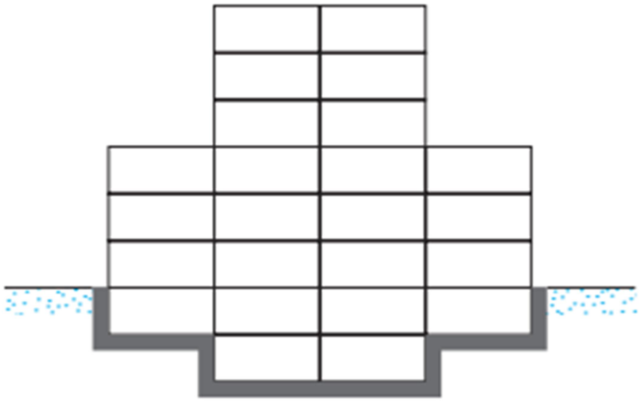


Figure 5.8 Compensated foundation

- For saturated clays, the factor of safety against bearing capacity failure can thus be obtained by substituting Eq. (5.12) into Eq. (5.22):

$$FS = \frac{5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right)}{\frac{Q}{A} - \gamma D_f} \quad (5.23)$$

Example 5.3

The mat shown in Figure 8.7 has dimensions of 20 m × 30 m. The total dead and live load on the mat is 110 MN. The mat is placed over a saturated clay having a unit weight of 18 kN/m³ and c_u = 140 kN/m². Given that D_f = 1.5 m, determine the factor of safety against bearing capacity failure.

Solution

From Eq. (8.23), the factor of safety

$$FS = \frac{5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right)}{\frac{Q}{A} - \gamma D_f}$$

We are given that $c_u = 140 \text{ kN/m}^2$, $D_f = 1.5$ m, $B = 20$ m, $L = 30$ m, and $\gamma = 18 \text{ kN/m}^3$. Hence,

$$FS = \frac{(5.14)(140)\left[1 + \frac{(0.195)(20)}{30}\right]\left[1 + 0.4\left(\frac{1.5}{20}\right)\right]}{\left(\frac{110{,}000 \text{ kN}}{20 \times 30}\right) - (18)(1.5)} = \mathbf{5.36}$$

■

Example 5.4

Consider a mat foundation 30 m × 40 m in plan, as shown in Figure 8.9. The total dead load and live load on the raft is 200 × 10^3 kN. Estimate the consolidation settlement at the center of the foundation.

Solution

From Eq. (2.65)

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}\right)$$

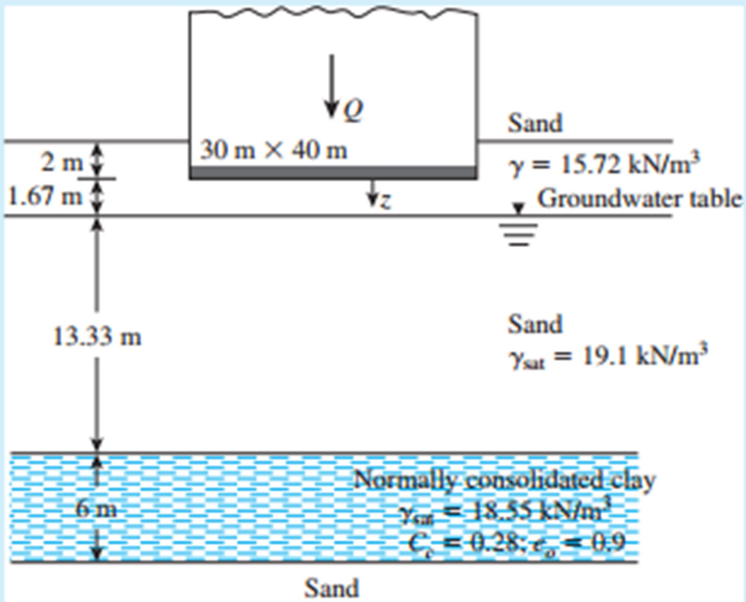


Figure 8.9 Consolidation settlement under a mat foundation

$$\sigma'_o = (3.67)(15.72) + (13.33)(19.1 - 9.81) + \frac{6}{2}(18.55 - 9.81) \approx 208 \text{ kN/m}^2$$

$H_c = 6$ m

$C_c = 0.28$

$e_o = 0.9$

For $Q = 200 \times 10^3$ kN, the net load per unit area is

$$q = \frac{Q}{A} - \gamma D_f = \frac{200 \times 10^3}{30 \times 40} - (15.72)(2) \approx 135.2 \text{ kN/m}^2$$

In order to calculate $\Delta\sigma'_{av}$ we refer to Section 6.8. The loaded area can be divided into four areas, each measuring 15 m × 20 m. Now using Eq. (6.23), we can calculate the average stress increase in the clay layer below the corner of each rectangular area, or

$$\Delta\sigma'_{av(H_2/H_1)} = q_o\left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right]$$

$$= 135.2\left[\frac{(1.67 + 13.33 + 6)I_{a(H_2)} - (1.67 + 13.33)I_{a(H_1)}}{6}\right]$$

For $I_{a(H_2)}$,

$$m_2 = \frac{B}{H_2} = \frac{15}{1.67 + 13.33 + 6} = 0.71$$

$$n_2 = \frac{L}{H_2} = \frac{20}{21} = 0.95$$

From Fig. 6.11, for $m_2 = 0.71$ and $n_2 = 0.95$, the value of $I_{a(H_2)}$ is 0.21. Again, for $I_{a(H_1)}$,

$$m_2 = \frac{B}{H_1} = \frac{15}{15} = 1$$

$$n_2 = \frac{L}{H_1} = \frac{20}{15} = 1.33$$

From Figure 6.11, $I_{a(H_1)} = 0.225$, so

$$\Delta\sigma'_{av(H_2/H_1)} = 135.2\left[\frac{(21)(0.21) - (15)(0.225)}{6}\right] = 23.32 \text{ kN/m}^2$$

So, the stress increase below the center of the 30 m × 40 m area is (4)(23.32) = 93.28 kN/m². Thus

$$S_{c(p)} = \frac{(0.28)(6))}{1 + 0.9}\log\left(\frac{208 + 93.28}{208}\right) = 0.142 \text{ m}$$

$$= \mathbf{142\ mm}$$

Example 5.5

Refer to Figure 4.6. The mat has dimensions of 30 m × 40 m. The live load and dead load on the mat are 200 MN. The mat is placed over a layer of soft clay. The unit weight of the clay is 18.75 kN/m³. Find D_f for a fully compensated foundation.

Solution

From Eq. (4.15),

$$D_f = \frac{Q}{A\gamma} = \frac{200 \times 10^3 \text{ kN}}{(30 \times 40)(18.75)} = \mathbf{8.89\ m}$$

5.7 Structural Design of Mat Foundations

The structural design of mat foundations can be carried out by two conventional methods:

- the conventional rigid method and the approximate flexible method. Finite-difference and finite-element methods can also be used, but this section covers only the basic concepts of the first design method.

5.7.1 Conventional Rigid Method

The *conventional rigid method* of mat foundation design can be explained step by step with reference to Figure 8.10:

Step 1. Figure 8.10a shows mat dimensions of $L \times B$ and column loads of Q_1, Q_2, Q_3, ... Calculate the total column load as

$$Q = Q_1 + Q_2 + Q_3 + \cdots \tag{8.24}$$

Step 2. Determine the pressure on the soil, q, below the mat at points A, B, C, D, ..., by using the equation

$$q = \frac{Q}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} \tag{8.25}$$

where

$A = BL$
$I_x = (1/12)BL^3$ = moment of inertia about the x-axis
$I_y = (1/12)LB^3$ = moment of inertia about the y-axis
M_x = moment of the column loads about the x-axis = Qe_y
M_y = moment of the column loads about the y-axis = Qe_x

The load eccentricities, e_x and e_y, in the x and y directions can be determined by using (x', y') coordinates:

$$x' = \frac{Q_1 x'_1 + Q_2 x'_2 + Q_3 x'_3 + \cdots}{Q} \tag{8.26}$$

and

$$e_x = x' - \frac{B}{2} \tag{8.27}$$

Similarly,

$$y' = \frac{Q_1 y'_1 + Q_2 y'_2 + Q_3 y'_3 + \cdots}{Q} \tag{8.28}$$

and

$$e_y = y' - \frac{L}{2} \tag{8.29}$$

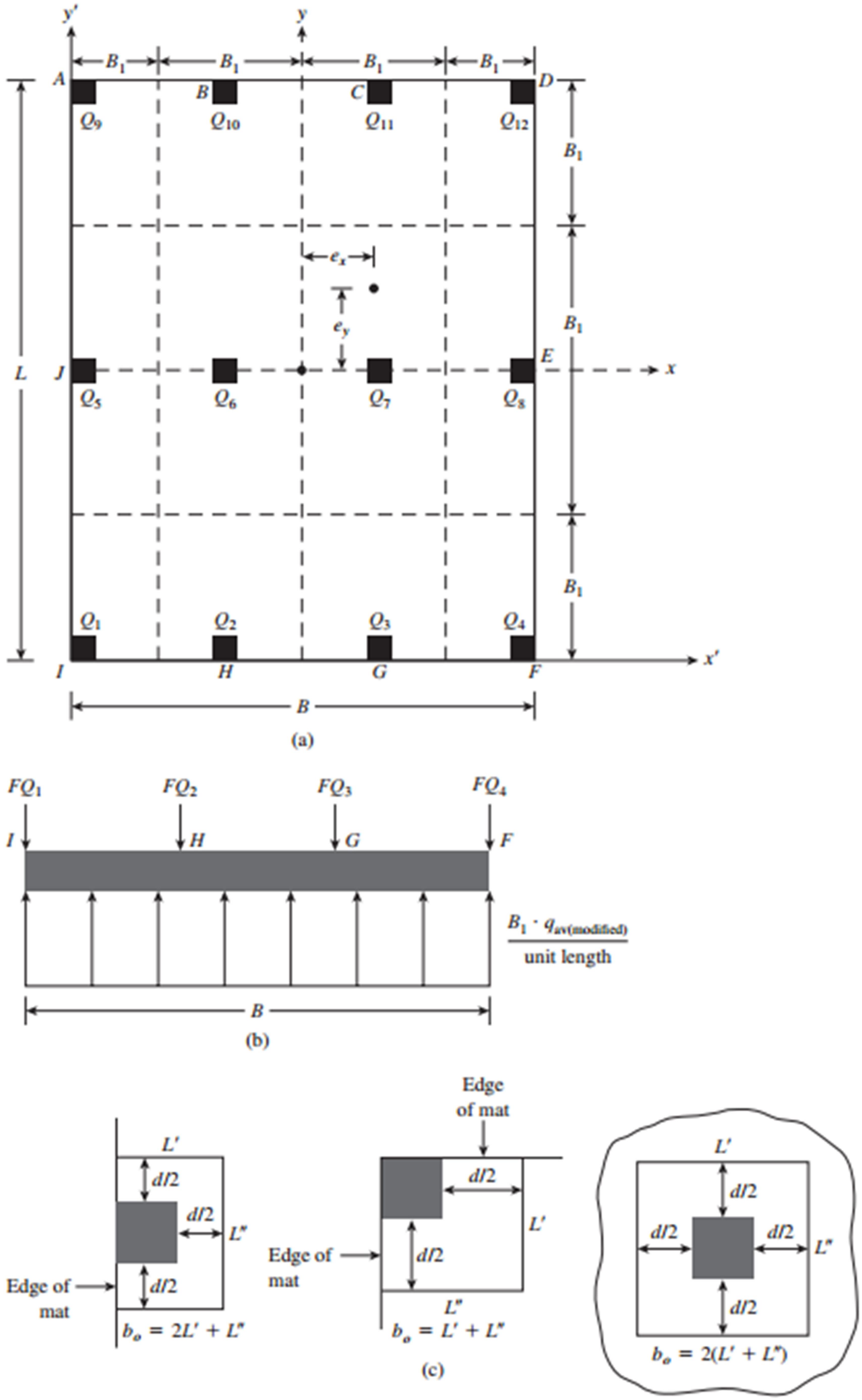


Figure 8.10 Conventional rigid mat foundation design

Step 3. Compare the values of the soil pressures determined in Step 2 with the net allowable soil pressure to determine whether $q \leq q_{all(net)}$.

Step 4. Divide the mat into several strips in the x and y directions. (See Figure 8.10). Let the width of any strip be B_1.

Step 5. Draw the shear, V, and the moment, M, diagrams for each individual strip (in the x and y directions). For example, the average soil pressure of the bottom strip in the x direction of Figure 8.10a is

$$q_{av} \approx \frac{q_I + q_F}{2} \tag{8.30}$$

where q_I and q_F = soil pressures at points I and F, as determined from Step 2.

The total soil reaction is equal to $q_{av}B_1B$. Now obtain the total column load on the strip as $Q_1 + Q_2 + Q_3 + Q_4$. The sum of the column loads on the strip will not equal $q_{av}B_1B$, because the shear between the adjacent strips has not been taken into account. For this reason, the soil reaction and the column loads need to be adjusted, or

$$\text{Average load} = \frac{q_{av}B_1B + (Q_1 + Q_2 + Q_3 + Q_4)}{2} \tag{8.31}$$

Now, the modified average soil reaction becomes

$$q_{av(modified)} = q_{av}\left(\frac{\text{average load}}{q_{av}B_1B}\right) \tag{8.32}$$

and the column load modification factor is

$$F = \frac{\text{average load}}{Q_1 + Q_2 + Q_3 + Q_4} \tag{8.33}$$

So the modified column loads are FQ_1, FQ_2, FQ_3, and FQ_4. This modified loading on the strip under consideration is shown in Figure 8.10b. The shear and the moment diagram for this strip can now be drawn, and the procedure is repeated in the x and y directions for all strips.

Step 6. Determine the effective depth d of the mat by checking for diagonal tension shear near various columns. For the critical section,

$$V_c \geq U \tag{8.34}$$

where

U = factored column load according to ACI Code 318-11 (2011)
V_c = shear capacity at the column location

According to ACI Code 318-11 (Section 11.11.2.1) for non-prestressed slabs and footings, V_c shall be the smallest of (8.35a), (8.35b), and (8.35c). In US customary units, the equations are

$$V_c = \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c}\,b_0d \tag{8.35a}$$

$$V_c = \left(2 + \frac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c}\, b_0 d \tag{8.35b}$$

$$V_c = 4\lambda\sqrt{f'_c}\, b_0 d \tag{8.35c}$$

where

β = ratio of long side to short side of the column
α_s = 40 for interior columns
= 30 for edge columns
= 20 for corner columns
b_0 = perimeter of the critical section for shear
f'_c = compressive strength of concrete at 28 days (psi)
λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative to normal weight concrete of the same compressive strength
d = effective depth of the mat

The expression for b_0 in terms of d, which depends on the location of the column with respect to the plan of the mat, can be obtained from Figure 8.10c.

In SI units, the equations for V_c are

$$V_c = \frac{1}{6}\left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f'_c}\, b_0 d \tag{8.35d}$$

$$V_c = \frac{1}{12}\left(2 + \frac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c}\, b_0 d \tag{8.35e}$$

$$V_c = \frac{1}{3}\lambda\sqrt{f'_c}\, b_0 d \tag{8.35f}$$

Step 7. From the moment diagrams of all strips *in one direction* (x or y), obtain the maximum positive and negative moments per unit width (i.e., $M_u = M/B_1$). Since factored column loads are used in accordance with ACI Code 318-11 (see Step 6), M_u is the factored moment.

Step 8. Determine the area of steel per unit width for positive and negative reinforcement in the x and y directions. We have

$$M_u = \phi A_s f_y\left(d - \frac{a}{2}\right) \tag{8.36}$$

and

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{8.37}$$

where

A_s = area of steel per unit width
f_y = yield stress of reinforcement in tension
M_u = factored moment
ϕ = 0.9 = reduction factor

Example 5.6

The plan of a mat foundation is shown in Figure 8.14. Calculate the soil pressure at points *A*, *B*, *C*, *D*, *E*, and *F*. (*Note*: All column sections are planned to be 0.5 m × 0.5 m.) All loads shown are factored loads according to ACI 381-11 (2011).

Solution

Eq. (8.25): $q = \dfrac{Q}{A} \pm \dfrac{M_y x}{I_y} \pm \dfrac{M_x y}{I_x}$

$$A = (20.5)(27.5) = 563.75 \text{ m}^2$$

$$I_x = \frac{1}{12}BL^3 = \frac{1}{12}(20.5)(27.5)^3 = 35{,}528 \text{ m}^4$$

$$I_y = \frac{1}{12}LB^3 = \frac{1}{12}(27.5)(20.5)^3 = 19{,}743 \text{ m}^4$$

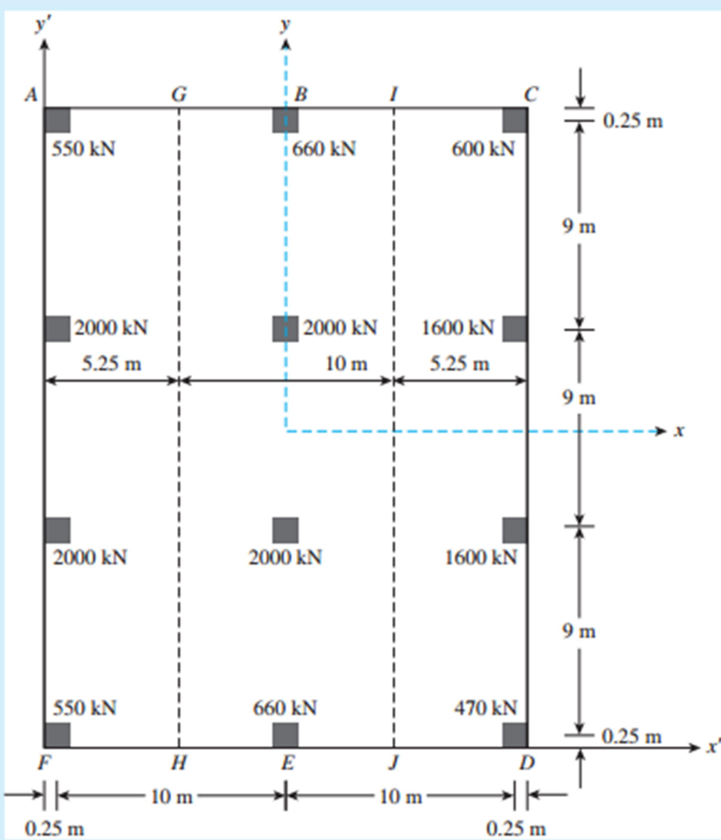
$$Q = 470 + (2)(550) + 600 + (2)(660) + (2)(1600) + (4)(2000) = 14{,}690 \text{ kN}$$

$$M_y = Qe_x; \quad e_x = x' - \frac{B}{2}$$

$$x' = \frac{Q_1 x_1' + Q_2 x_2' + Q_3 x_3' + \cdots}{Q}$$

$$= \frac{1}{14{,}690}\left[\begin{aligned} &(10.25)(660 + 2000 + 2000 + 660) \\ &+ (20.25)(470 + 1600 + 1600 + 600) \\ &+ (0.25)(550 + 2000 + 2000 + 550) \end{aligned}\right] = 9.686 \text{ m}$$

$$e_x = x' - \frac{B}{2} = 9.686 - 10.25 = -0.565 \text{ m} \approx -0.57 \text{ m}$$



Hence, the resultant line of action is located to the left of the center of the mat. So $M_y = (14{,}690)(0.57) = 8373$ kN-m. Similarly

$$M_x = Qe_y; \quad e_y = y' - \frac{L}{2}$$

$$y' = \frac{Q_1y'_1 + Q_2y'_2 + Q_3y'_3 + \cdots}{Q}$$

$$= \frac{1}{14{,}690}\begin{bmatrix}(0.25)(550 + 660 + 470) + (9.25)(2000 + 2000 + 1600) \\ +(18.25)(2000 + 2000 + 1600) + (27.25)(550 + 660 + 600)\end{bmatrix}$$

$$= 13.86 \text{ m}$$

$$e_y = y' - \frac{L}{2} = 13.86 - 13.75 = 0.11 \text{ m}$$

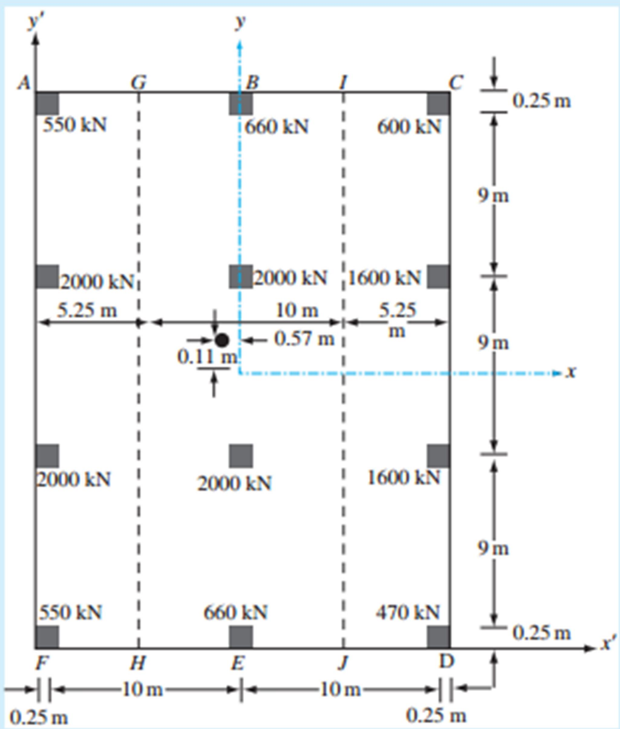


Figure 8.15

The location of the line of action of the resultant column loads is shown in Figure 8.15.

$M_x = (14{,}690)(0.11) = 1616$ kN-m. So

$$q = \frac{14{,}690}{563.75} \pm \frac{8373x}{19743} \pm \frac{1616y}{35{,}528} = 26.0 \pm 0.42x \pm 0.05y \text{ (kN/m}^2\text{)}$$

Therefore,

At *A*: $q = 26 + (0.42)(10.25) + (0.05)(13.75) = \mathbf{31.0\ kN/m^2}$

At *B*: $q = 26 + (0.42)(0) + (0.05)(13.75) = \mathbf{26.68\ kN/m^2}$

At *C*: $q = 26 - (0.42)(10.25) + (0.05)(13.75) = \mathbf{22.38\ kN/m^2}$

At *D*: $q = 26 - (0.42)(10.25) - (0.05)(13.75) = \mathbf{21.0\ kN/m^2}$

At *E*: $q = 26 + (0.42)(0) - (0.05)(13.75) = \mathbf{25.31\ kN/m^2}$

At *F*: $q = 26 + (0.42)(10.25) - (0.05)(13.75) = \mathbf{29.61\ kN/m^2}$

Example 5.7

Divide the mat shown in Figure 8.14 into three strips, such as *AGHF* (B_1 = 5.25 m), *GIJH* (B_1 = 10 m), and *ICDJ* (B_1 = 5.25 m). Use the result of Example 8.7, and determine the reinforcement requirements in the *y* direction. Here, f'_c = 20.7 MN/m², f_y = 413.7 MN/m². *Note:* All column loads are factored loads.

Solution

Determination of Shear and Moment Diagrams for Strips:

Strip *AGHF*:

$$\text{Average soil pressure} = q_{av} = q_{(\text{at } A)} + q_{(\text{at } F)} = \frac{31 + 29.61}{2} = 30.305 \text{ kN/m}^2$$

Total soil reaction $= q_{av}B_1L = (30.305)(5.25)(27.5) = 4375$ kN

$$\text{Average load} = \frac{\text{load due to soil reaction + column loads}}{2}$$

$$= \frac{4375 + 5100}{2} = 4737.5 \text{ kN}$$

So, modified average soil pressure,

$$q_{av(\text{modified})} = q_{av}\left(\frac{4737.5}{4375}\right) = (30.305)\left(\frac{4737.5}{4375}\right) = 32.81 \text{ kN/m}^2$$

The column loads can be modified in a similar manner by multiplying factor

$$F = \frac{4737.5}{5100} = 0.929$$

Figure 8.16 shows the loading on the strip and corresponding shear and moment diagrams. Note that the column loads shown in this figure have been multiplied by

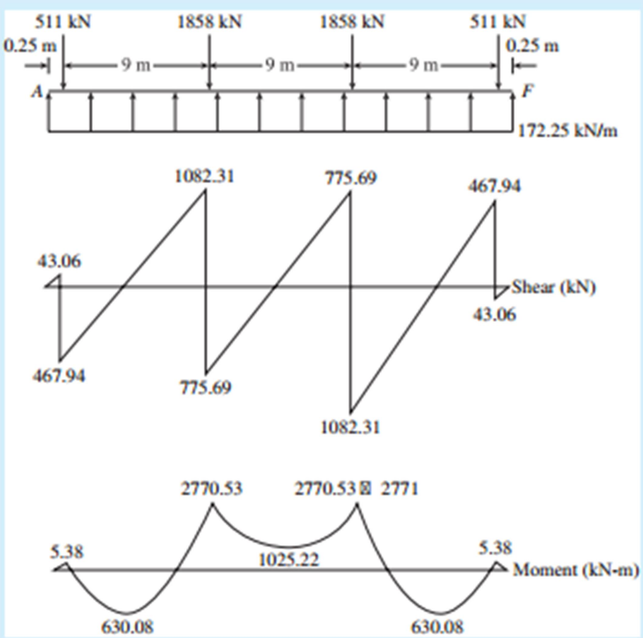


Figure 8.16 Load, shear, and moment diagrams for strip *AGHF*

$F = 0.929$. Also the load per unit length of the beam is equal to $B_1 q_{av(modified)} = (5.25)(32.81) = 172.25$ kN/m.

Strip *GIJH*: In a similar manner,

$$q_{av} = \frac{q_{(at\ B)} + q_{(at\ E)}}{2} = \frac{26.68 + 25.31}{2} = 26.0 \text{ kN/m}^2$$

$$\text{Total soil reaction} = (26)(10)(27.5) = 7150 \text{ kN}$$
$$\text{Total column load} = 5320 \text{ kN}$$

$$\text{Average load} = \frac{7150 + 5320}{2} = 6235 \text{ kN}$$

$$q_{av(modified)} = (26)\left(\frac{6235}{7150}\right) = 22.67 \text{ kN/m}^2$$

$$F = \frac{6235}{5320} = 1.17$$

The load, shear, and moment diagrams are shown in Figure 8.17.

Strip *ICDJ*: Figure 8.18 shows the load, shear, and moment diagrams for this strip.

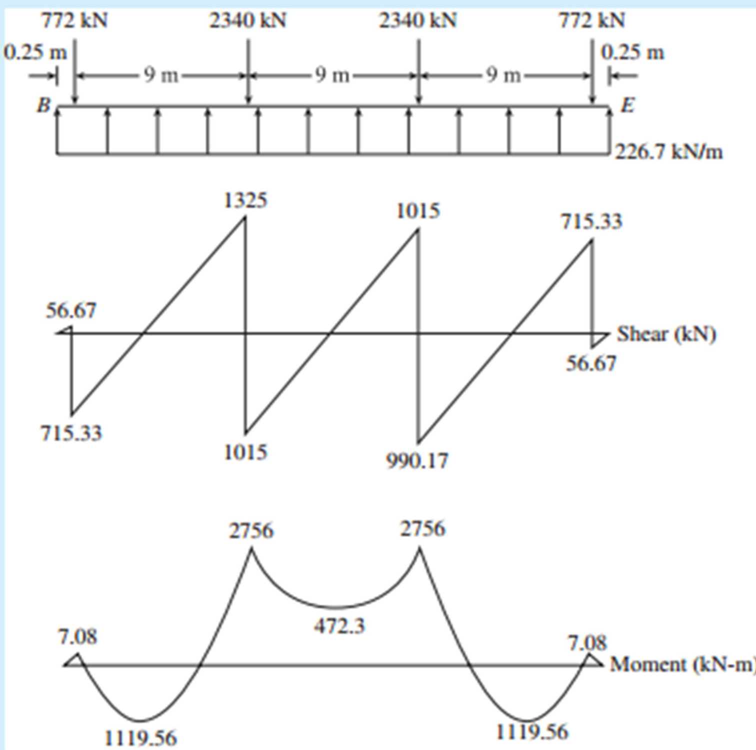


Figure 8.17 Load, shear, and moment diagrams for strip *GIJH*

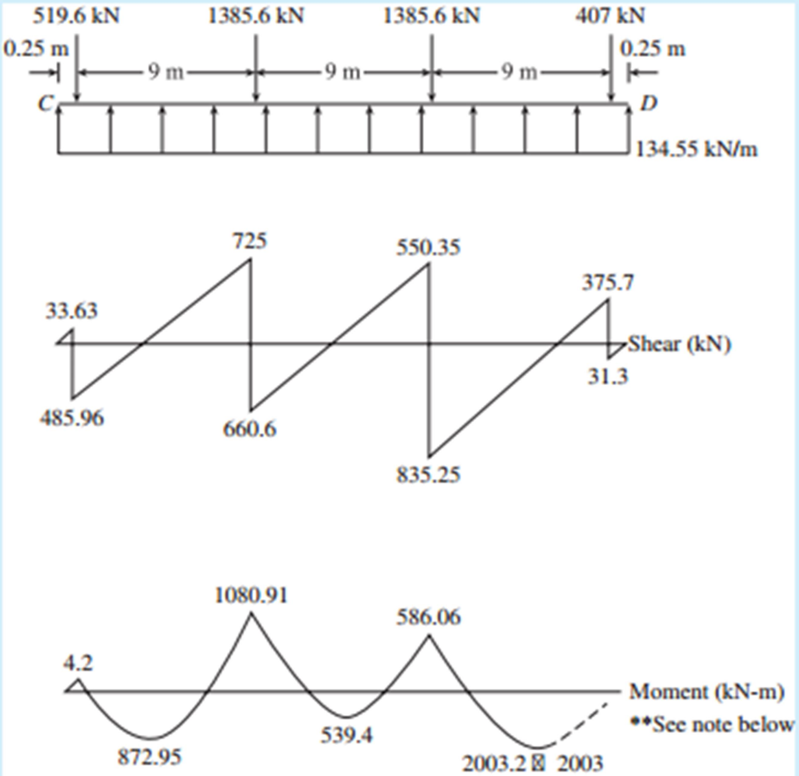


Figure 8.18 Load, shear, and moment diagrams for strip *ICDJ*

**Note: In view of the assumption of uniform soil reaction to non-symmetric loading, there is a discrepancy in the moment values at the right column. As a result, the moment diagram will not "close". This is ignored since it is not the governing design moment

Determination of the Thickness of the Mat

For this problem, the critical section for diagonal tension shear will be at the column carrying 2000 kN of load at the edge of the mat [Figure 8.19]. So

$$U = 2000 \text{ kN} = 2 \text{ MN}$$

$$b_0 = \left(0.5 + \frac{d}{2}\right) + \left(0.5 + \frac{d}{2}\right) + (0.5 + d) = 1.5 + 2d$$

Equations (8.34), (8.35d), (8.35e), and (8.35f) are used to calculate the effective depth, d, given that: $f'_c = 20.7 \text{ MN/m}^2$; $\lambda = 1$ (normal weight concrete); $\beta = 1$ (square columns); and $\alpha_s = 30$ (edge column). Note that the maximum value of d is selected as the design value and it corresponds to the minimum value of V_c obtained from equations (8.35d), (8.35e), and (8.35f).

$$V_c = \frac{1}{6}\left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f'_c}\, b_0 d \qquad (8.35d)$$

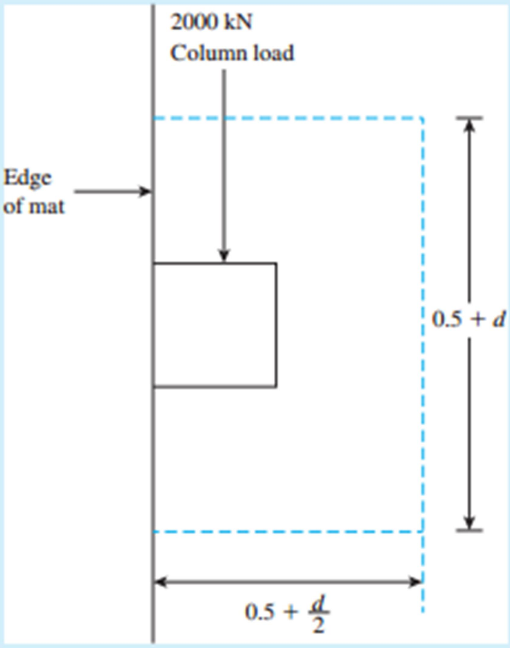


Figure 8.19 Critical perimeter column

$$2 = \frac{1}{6}\left(1 + \frac{2}{1}\right)(1)\sqrt{20.7}(1.5 + 2d)(d)$$

$$2d^2 + 1.5d - 0.8793 = 0$$

So, $d = 0.387$ m.

$$V_c = \frac{1}{12}\left(2 + \frac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c}\, b_0 d \qquad (8.35e)$$

$$2 = \frac{1}{12}\left(2 + \frac{(30)(d)}{1.5 + 2(d)}\right)(1)\sqrt{20.7}(1.5 + 2d)(d)$$

$$34d^2 + 3d - 5.275 = 0$$

So, $d = 0.352$ m.

$$V_c = \frac{1}{3}\lambda\sqrt{f'_c}\, b_0 d \qquad (8.35f)$$

$$2 = \frac{1}{3}(1)\sqrt{20.7}(1.5 + 2d)(d)$$

$$2d^2 + 1.5d - 1.318 = 0$$

So, $d = 0.519$ m.

Therefore, the design mat thickness, $d = 0.519$ m ($\approx$ 20.5 in.)

Assuming a minimum cover of 76 mm over the steel reinforcement and also assuming that the steel bars to be used are 25 mm in diameter, the total thickness of the slab is

$$h = 0.52 + 0.076 + 0.0125 = 0.609 \text{ m} \approx \mathbf{0.61\ m}$$

The thickness of this mat will satisfy the wide beam shear condition across the three strips under consideration.

Determination of Reinforcement

From the moment diagram shown in Figures 8.16, 8.17, and 8.18, it can be seen that the maximum positive moment is located in strip *AGHF*, and its magnitude is

$$M_u = \frac{2771}{B_1} = \frac{2771}{5.25} = 527.8 \text{ kN-m/m}$$

Similarly, the maximum negative moment is located in strip *ICDJ* and its magnitude is

$$M_u = \frac{2003}{B_1} = \frac{2003}{5.25} = 381.52 \text{ kN-m/m}$$

From Eq. (8.36), $M_u = \phi A_s f_y\left(d - \frac{a}{2}\right)$.

For the positive moment,

$$M_u = 527.8 = (\phi)(A_s)(413.7 \times 1000)\left(0.61 - \frac{a}{2}\right)$$

$\phi = 0.9$. Also, from Eq. (8.37),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.7)(1)} = 23.51 A_s; \text{or } A_s = 0.0425a$$

$$527.8 = (0.9)(0.0425a)(413{,}700)\left(0.61 - \frac{a}{2}\right); \text{or } a \approx 0.0573 \text{ m}$$

So, $A_s = (0.0425)(0.0573) = 0.002435 \text{ m}^2/\text{m} = 2435 \text{ mm}^2/\text{m}$.

Use 25-mm diameter bars at 200 mm center-to-center:

$$\left[A_s \text{ provided} = (491)\left(\frac{1000}{200}\right) = 2455 \text{ mm}^2/\text{m}\right]$$

Similarly, for negative reinforcement,

$$M_u = 381.52 = (\phi)(A_s)(413.7 \times 1000)\left(0.61 - \frac{a}{2}\right)$$

$$\phi = 0.9, A_s = 0.0425a$$

So

$$381.52 = (0.9)(0.0425a)(413.7 \times 1000)\left(0.61 - \frac{a}{2}\right); \text{or } a \approx 0.0409 \text{ m}$$

So, $A_s = (0.0409)(0.0425) = 0.001738 \text{ m}^2/\text{m} = 1738 \text{ mm}^2/\text{m}$.

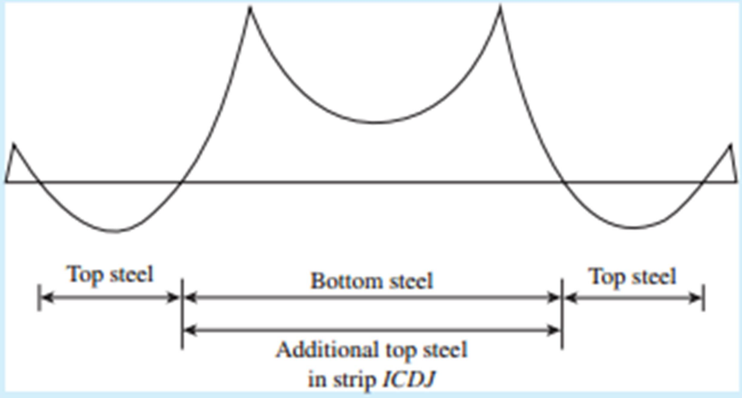


Figure 8.20 General arrangement of reinforcement

Use 25-mm diameter bars at 255 mm center-to-center:

$$[A_s \text{ provided} = 1925 \text{ mm}^2]$$

Because negative moment occurs at midbay of strip *ICDJ*, reinforcement should be provided. This moment is

$$M_u = \frac{539.4}{5.25} = 102.74 \text{ kN-m/m}$$

Hence,

$$M_u = 102.74 = (0.9)(0.0425a)(413.7 \times 1000)\left(0.61 - \frac{a}{2}\right);$$

or $a \approx 0.0107$ m, and

$$A_s = (0.0107)\,(0.0425) = 0.0004547 \text{ m}^2/\text{m} = 455 \text{ mm}^2/\text{m}$$

Provide 16-mm diameter bars at 400 mm center-to-center:

$$[A_s \text{provided} = 502 \text{ mm}^2]$$

For general arrangement of the reinforcement, see Figure 8.20. ■