University of Anbar
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CHAPTER FOUR

SETTLEMENT OF SHALLOW FOUNDATIONS

LECTURE
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VERTICAL STRESSES INCREASE IN SOIL

4.1. Introduction

It was mentioned in Chapter 3 that, in many foundation may control the allowable bear may be controlled by local building codes. the smaller of the following two conditions:

$$q_{all} = \begin{cases} \dfrac{q_u}{FS} \\ \text{or} \\ q_{\text{allowable settlement}} \end{cases}$$

For the calculation of foundation settlement, it is required that we estimate the vertical stress increase in the soil mass due to the net load applied on the foundation. Hence, in this chapter, we will discuss the general principles for estimating the increase of vertical stress at various depths in soil due to the application of (on the ground surface).

- A point load
- Circularly loaded area
- Vertical line load
- Strip load
- Rectangular loaded area

4.2 Stress Due to a Concentrated Load

In 1885, Boussinesq developed the mathematical relationships for determining the normal and shear stresses at any point inside homogeneous, elastic, and isotropic mediums due to a concentrated point load located at the surface, as shown in Figure 4.1.

According to his analysis, the vertical stress increase at point A caused by a point load of magnitude P is given by

$$\Delta\sigma = \frac{3P}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} \qquad (4.1)$$

where

$r = \sqrt{x^2 + y^2}$

x, y, z = coordinates of the point A

.Note that Eq. (4.1) is not a function of Poisson's ratio of the soil

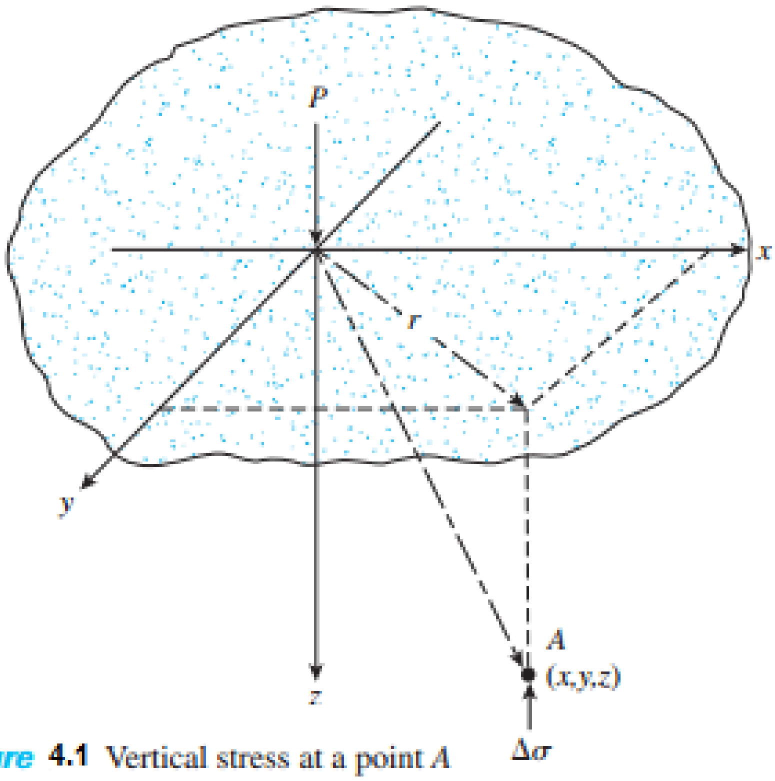

Figure 4.1 Vertical stress at a point A caused by a point load on the surface

4.3 Stress Due to a Circularly Loaded Area

The Boussinesq equation (4.1) can also be used to determine the vertical stress below the center of a flexible circularly loaded area, as shown in Figure 4.2. Let the radius of the loaded area be B/2, and let q_o be the uniformly distributed load per unit area. To determine the stress increase at a point A, located at a depth z below the center of the circular area, consider an elemental area on the circle. The load on this elemental area may be taken to be a point load and expressed as $q_o\ r\ d\theta\ dr$. The stress increase at A caused by this load can be determined from Eq. (4.1) as

$$d\sigma = \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \qquad (4.2)$$

The total increase in stress caused by the entire loaded area may be obtained by integrating Eq. (4.2), or

$$\Delta\sigma = \int d\sigma = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=B/2} \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= q_o\left\{1 - \frac{1}{\left[1 + \left(\frac{B}{2z}\right)^2\right]^{3/2}}\right\} \qquad (4.3)$$

Similar integrations could be performed to obtain the vertical stress increase at A', located a distance r from the center of the loaded area at a depth z (Ahlvin and Ulery, 1962). Table 4.1 gives the variation of $\Delta\sigma/q_o$ with r/(B/2) and z/(B/2) [for $0 \le r/(B/2) \le 1$]. Note that the variation of $\Delta\sigma/q_o$ with depth at r/(B/2) =0 can be obtained from Eq. (4.3).

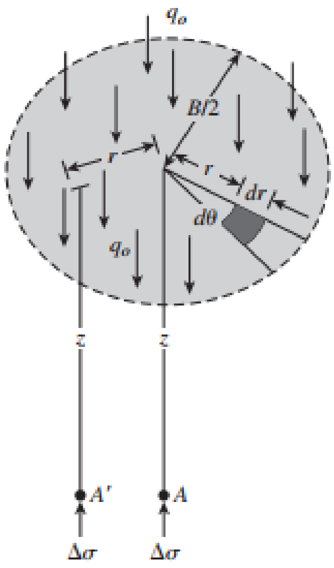

Table 4.1 Variation of $\Delta\sigma/q_o$ for a Uniformly Loaded Flexible Circular Area

	r/(B/2)					
z/(B/2)	0	0.2	0.4	0.6	0.8	1.0
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348
1.0	0.646	0.633	0.591	0.525	0.434	0.332
1.2	0.546	0.535	0.501	0.447	0.377	0.300
1.5	0.424	0.416	0.392	0.355	0.308	0.256
2.0	0.286	0.286	0.268	0.248	0.224	0.196
2.5	0.200	0.197	0.191	0.180	0.167	0.151
3.0	0.146	0.145	0.141	0.135	0.127	0.118
4.0	0.087	0.086	0.085	0.082	0.080	0.075

4.4 Stress Due to a Line Load

Figure 4.3 shows a vertical flexible line load of infinite length that has an intensity q/unit length on the surface of a semi-infinite soil mass. The vertical stress increase, Δσ, inside the soil mass can be determined by using the principles of the theory of elasticity, or

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2} \quad (4.4)$$

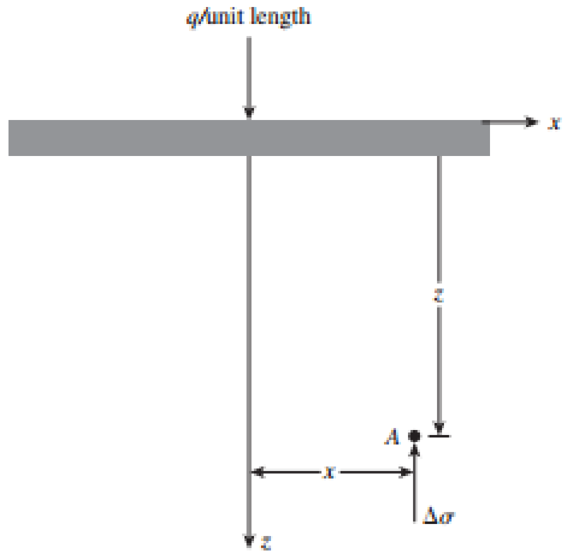

Figure 4.3 Line load over the surface of a semi-infinite soil mass

This equation can be rewritten as

$$\Delta\sigma = \frac{2q}{\pi z[(x/z)^2 + 1]^2}$$

$$\frac{\Delta\sigma}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2} \quad (4.5)$$

Note that Eq. (4.5) is in a nondimensional form. Using this variation of Δσ/(q/z) with x/z. This is given in Table 4.2.

Table 4.2 Variation of $\Delta\sigma/(q/z)$ with x/z [Eq. (4.5)]

x/z	$\Delta\sigma/(q/z)$	x/z	$\Delta\sigma/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006

4.5 Stresses below a Rectangular Area

The integration technique of Boussinesq's equation also allows the vertical stress at any point A below the corner of a flexible rectangular loaded area to be evaluated. (See Figure 4.5.) To do so, consider an elementary area dA= dx dy on the flexible loaded area. If the load per unit area is q_o, the total load on the elemental area is

$$dP = q_o \, dx \, dy \tag{4.8}$$

The total stress increase Δσ caused by the entire loaded area at point A may now be obtained by integrating the preceding equation:

$$\Delta\sigma = \int_{y=0}^{L}\int_{x=0}^{B} \frac{3q_o\,(dx\,dy)z^3}{2\pi(x^2+y^2+z^2)^{5/2}} = q_o I \tag{4.9}$$

Here,

$$I = \text{influence factor} = \frac{1}{4\pi}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right) \quad (4.10)$$

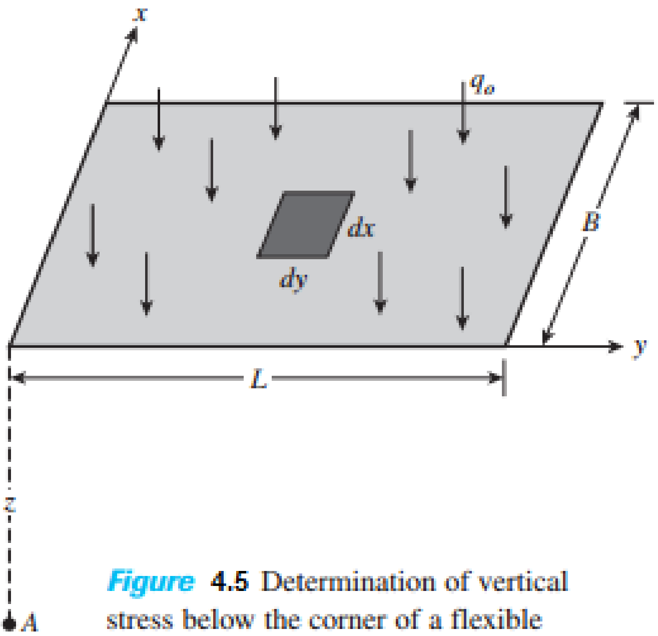

Figure 4.5 Determination of vertical stress below the corner of a flexible rectangular loaded area

where

$$m = \frac{B}{z} \quad (4.11)$$

and

$$n = \frac{L}{z} \quad (4.12)$$

The arctangent term in Eq. **(4.10)** must be a positive angle in radians. When $m^2 + n^2 + 1 < m^2n^2$, it becomes a negative angle. So a term π should be added to that angle. The variations of the influence values with m and n are given in Table **4.4**.

The stress increase at any point below a rectangular loaded area can also be found by using Eq. (4.9) in conjunction with Figure 4.6. To determine the stress at a depth z below point O, divide the loaded area into four rectangles, with O the corner common to each. Then use Eq. (6.9) to calculate the increase in stress at a depth z below O caused by each rectangular area. The total stress increase caused by the entire loaded area may now be expressed as

$$\Delta\sigma = q_o (I_1 + I_2 + I_3 + I_4) \qquad (4.13)$$

where I_1, I_2, I_3, and I_4 = the influence values of rectangles 1, 2, 3, and 4, respectively.

In most cases, the vertical stress below the center of a rectangular area is of importance. This can be given by the relationship

$$\Delta\sigma = q_o I_c \qquad (4.14)$$

where

$$I_c = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}} \frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right]$$

$$m_1 = \frac{L}{B}$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)}$$

The variation of I_c with m_1 and n_1 is given in Table 4.5.

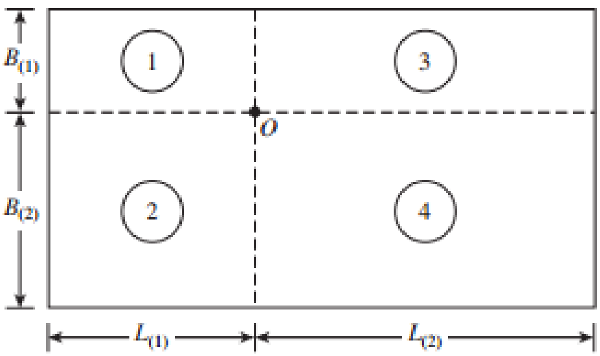

Figure 4.6 Stress below any point of a loaded flexible rectangular area

Table 4.4 Variation of Influence Value I [Eq. (6.10)][a]

	n											
m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00470	0.00917	0.01323	0.01678	0.01978	0.02223	0.02420	0.02576	0.02698	0.02794	0.02926	0.03007
0.2	0.00917	0.01790	0.02585	0.03280	0.03866	0.04348	0.04735	0.05042	0.05283	0.05471	0.05733	0.05894
0.3	0.01323	0.02585	0.03735	0.04742	0.05593	0.06294	0.06858	0.07308	0.07661	0.07938	0.08323	0.08561
0.4	0.01678	0.03280	0.04742	0.06024	0.07111	0.08009	0.08734	0.09314	0.09770	0.10129	0.10631	0.10941
0.5	0.01978	0.03866	0.05593	0.07111	0.08403	0.09473	0.10340	0.11035	0.11584	0.12018	0.12626	0.13003
0.6	0.02223	0.04348	0.06294	0.08009	0.09473	0.10688	0.11679	0.12474	0.13105	0.13605	0.14309	0.14749
0.7	0.02420	0.04735	0.06858	0.08734	0.10340	0.11679	0.12772	0.13653	0.14356	0.14914	0.15703	0.16199
0.8	0.02576	0.05042	0.07308	0.09314	0.11035	0.12474	0.13653	0.14607	0.15371	0.15978	0.16843	0.17389
0.9	0.02698	0.05283	0.07661	0.09770	0.11584	0.13105	0.14356	0.15371	0.16185	0.16835	0.17766	0.18357
1.0	0.02794	0.05471	0.07938	0.10129	0.12018	0.13605	0.14914	0.15978	0.16835	0.17522	0.18508	0.19139
1.2	0.02926	0.05733	0.08323	0.10631	0.12626	0.14309	0.15703	0.16843	0.17766	0.18508	0.19584	0.20278
1.4	0.03007	0.05894	0.08561	0.10941	0.13003	0.14749	0.16199	0.17389	0.18357	0.19139	0.20278	0.21020
1.6	0.03058	0.05994	0.08709	0.11135	0.13241	0.15028	0.16515	0.17739	0.18737	0.19546	0.20731	0.21510
1.8	0.03090	0.06058	0.08804	0.11260	0.13395	0.15207	0.16720	0.17967	0.18986	0.19814	0.21032	0.21836
2.0	0.03111	0.06100	0.08867	0.11342	0.13496	0.15326	0.16856	0.18119	0.19152	0.19994	0.21235	0.22058
2.5	0.03138	0.06155	0.08948	0.11450	0.13628	0.15483	0.17036	0.18321	0.19375	0.20236	0.21512	0.22364
3.0	0.03150	0.06178	0.08982	0.11495	0.13684	0.15550	0.17113	0.18407	0.19470	0.20341	0.21633	0.22499
4.0	0.03158	0.06194	0.09007	0.11527	0.13724	0.15598	0.17168	0.18469	0.19540	0.20417	0.21722	0.22600
5.0	0.03160	0.06199	0.09014	0.11537	0.13737	0.15612	0.17185	0.18488	0.19561	0.20440	0.21749	0.22632
6.0	0.03161	0.06201	0.09017	0.11541	0.13741	0.15617	0.17191	0.18496	0.19569	0.20449	0.21760	0.22644
8.0	0.03162	0.06202	0.09018	0.11543	0.13744	0.15621	0.17195	0.18500	0.19574	0.20455	0.21767	0.22652
10.0	0.03162	0.06202	0.09019	0.11544	0.13745	0.15622	0.17196	0.18502	0.19576	0.20457	0.21769	0.22654
∞	0.03162	0.06202	0.09019	0.11544	0.13745	0.15623	0.17197	0.18502	0.19577	0.20458	0.21770	0.22656

Table 4.4 (*Continued*)

	n										
m	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	∞
0.1	0.03058	0.03090	0.03111	0.03138	0.03150	0.03158	0.03160	0.03161	0.03162	0.03162	0.03162
0.2	0.05994	0.06058	0.06100	0.06155	0.06178	0.06194	0.06199	0.06201	0.06202	0.06202	0.06202
0.3	0.08709	0.08804	0.08867	0.08948	0.08982	0.09007	0.09014	0.09017	0.09018	0.09019	0.09019
0.4	0.11135	0.11260	0.11342	0.11450	0.11495	0.11527	0.11537	0.11541	0.11543	0.11544	0.11544
0.5	0.13241	0.13395	0.13496	0.13628	0.13684	0.13724	0.13737	0.13741	0.13744	0.13745	0.13745
0.6	0.15028	0.15207	0.15326	0.15483	0.15550	0.15598	0.15612	0.15617	0.15621	0.15622	0.15623
0.7	0.16515	0.16720	0.16856	0.17036	0.17113	0.17168	0.17185	0.17191	0.17195	0.17196	0.17197
0.8	0.17739	0.17967	0.18119	0.18321	0.18407	0.18469	0.18488	0.18496	0.18500	0.18502	0.18502
0.9	0.18737	0.18986	0.19152	0.19375	0.19470	0.19540	0.19561	0.19569	0.19574	0.19576	0.19577
1.0	0.19546	0.19814	0.19994	0.20236	0.20341	0.20417	0.20440	0.20449	0.20455	0.20457	0.20458
1.2	0.20731	0.21032	0.21235	0.21512	0.21633	0.21722	0.21749	0.21760	0.21767	0.21769	0.21770
1.4	0.21510	0.21836	0.22058	0.22364	0.22499	0.22600	0.22632	0.22644	0.22652	0.22654	0.22656
1.6	0.22025	0.22372	0.22610	0.22940	0.23088	0.23200	0.23236	0.23249	0.23258	0.23261	0.23263
1.8	0.22372	0.22736	0.22986	0.23334	0.23495	0.23617	0.23656	0.23671	0.23681	0.23684	0.23686
2.0	0.22610	0.22986	0.23247	0.23614	0.23782	0.23912	0.23954	0.23970	0.23981	0.23985	0.23987
2.5	0.22940	0.23334	0.23614	0.24010	0.24196	0.24344	0.24392	0.24412	0.24425	0.24429	0.24432
3.0	0.23088	0.23495	0.23782	0.24196	0.24394	0.24554	0.24608	0.24630	0.24646	0.24650	0.24654
4.0	0.23200	0.23617	0.23912	0.24344	0.24554	0.24729	0.24791	0.24817	0.24836	0.24842	0.24846
5.0	0.23236	0.23656	0.23954	0.24392	0.24608	0.24791	0.24857	0.24885	0.24907	0.24914	0.24919
6.0	0.23249	0.23671	0.23970	0.24412	0.24630	0.24817	0.24885	0.24916	0.24939	0.24946	0.24952
8.0	0.23258	0.23681	0.23981	0.24425	0.24646	0.24836	0.24907	0.24939	0.24964	0.24973	0.24980
10.0	0.23261	0.23684	0.23985	0.24429	0.24650	0.24842	0.24914	0.24946	0.24973	0.24981	0.24989
∞	0.23263	0.23686	0.23987	0.24432	0.24654	0.24846	0.24919	0.24952	0.24980	0.24989	0.25000

[a]Based on Saika, 2012

Table 4.5 Variation of I_c with m_1 and n_1

n_1	m_1 1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

Stress Increase under a Rectangular Foundation- 2:1 Method

Foundation engineers often use an approximate method to determine the increase in stress with depth caused by the construction of a foundation. The method is referred to as the 2:1 method. (See Figure 4.7). According to this method, the increase in stress at depth z is

$$\Delta\sigma = \frac{q_o \times B \times L}{(B+z)(L+z)}$$

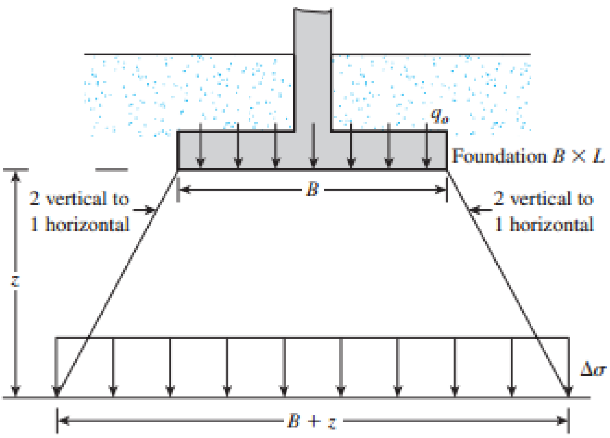

Figure 4.7 2:1 method of finding stress increase under a foundation

Note that Equation above is based on the assumption that the stress from the foundation spreads out along lines with a vertical-to-horizontal slope of 2:1.

Settlement of Shallow Foundations

4.6 Type of Shallow Foundations

The settlement of a shallow foundation can be divided into two major categories:

(a) Elastic, or immediate, settlement and

(b) Consolidation settlement.

- Immediate, or elastic, settlement of a foundation takes place during or immediately after the construction of the structure.
- Consolidation settlement occurs over time. Pore water is extruded from the void spaces of saturated clayey soils submerged in water.
- The total settlement of a foundation is the sum of the elastic settlement and the consolidation settlement. Consolidation settlement comprises two phases: ***primary* and *secondary***. Primary consolidation settlement is more significant than secondary settlement in inorganic clays and silty soils. However, in organic soils, secondary consolidation settlement is more significant.

4.7 Elastic Settlement of Shallow Foundation on Saturated Clay ($\mu_s = 0.5$)

- Janbu et al. (1956) proposed an equation for evaluating the average settlement of flexible foundations on saturated clay soils (Poisson's ratio, **(μ_s = 0.5)**. Referring to Figure 4.8, this relationship can be expressed as

$$S_e = A_1 A_2 \frac{q_o B}{E_s} \qquad (4.1)$$

where

$A_1 = f(H/B,\ L/B)$
$A_2 = f(D_f/B)$
L = length of the foundation
B = width of the foundation
D_f = depth of the foundation
H = depth of the bottom of the foundation to a rigid layer
q_o = net load per unit area of the foundation

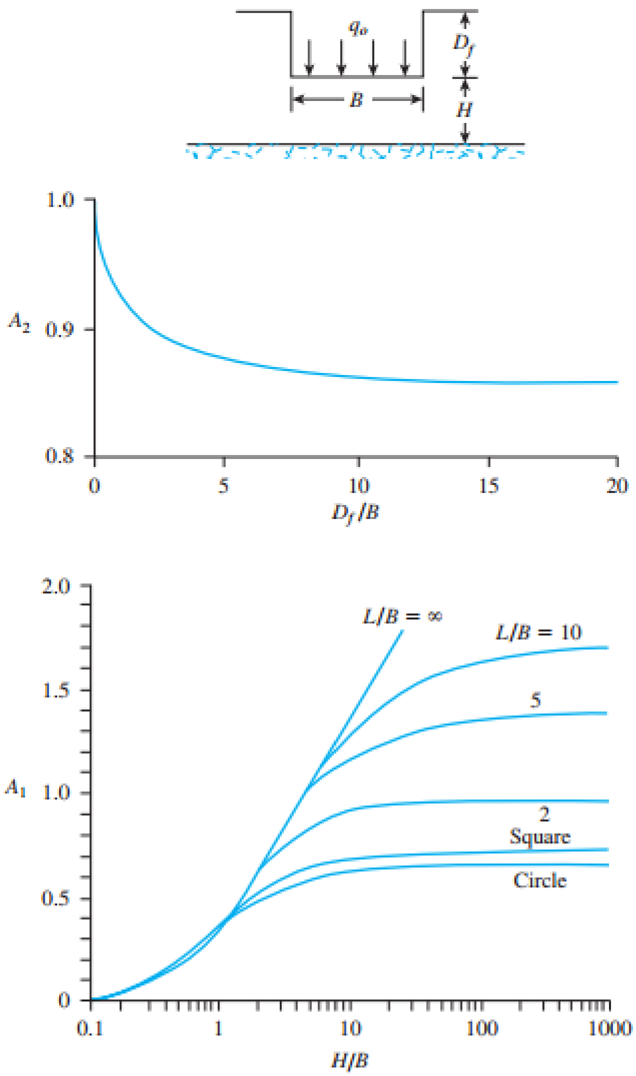

Figure 4.8 Values of A_1 and A_2 for elastic settlement calculation–

- Christian and Carrier (1978) modified the values of A_1 and A_2 to some extent and is presented in Figure 4.8.
- The modulus of elasticity (E_s) for saturated clays can, in general, be given as

$$E_s = \beta c_u \qquad (4.2)$$

- The parameter β is primarily a function of the plasticity index and overconsolidation ratio (OCR). Table 4.1 provides a general range for β based on that proposed by Duncan and Buchignani (1976). In any case, proper judgment should be used in selecting the magnitude of β.

Table 4.1 Range of β for Saturated Clay [Eq. (4.2)][a]

Plasticity Index	β				
	OCR = 1	OCR = 2	OCR = 3	OCR = 4	OCR = 5
<30	1500–600	1380–500	1200–580	950–380	730–300
30 to 50	600–300	550–270	580–220	380–180	300–150
>50	300–150	270–120	220–100	180–90	150–75

[a]Based on Duncan and Buchignani (1976)

Example 4.1

Consider a shallow foundation 2 m × 1 m in plan in a saturated clay layer. A rigid rock layer is located 8 m below the bottom of the foundation. Given:

Foundation: $D_f = 1$ m, $q_o = 120$ kN/m^2

Clay: $c_u = 150$ kN/m^2, OCR = 2, and Plasticity index, PI = 35

Estimate the elastic settlement of the foundation.

Solution

From Eq. (7.1),

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$

Given:

$$\frac{L}{B} = \frac{2}{1} = 2$$

$$\frac{D_f}{B} = \frac{1}{1} = 1$$

$$\frac{H}{B} = \frac{8}{1} = 8$$

$$E_s = \beta c_u$$

For OCR = 2 and PI = 35, the value of $\beta \approx 480$ (Table 7.1). Hence,

$$E_s = (480)(150) = 72{,}000 \text{ kN/m}^2$$

Also, from Figure 7.1, $A_1 = 0.9$ and $A_2 = 0.92$. Hence,

$$S_e = A_1 A_2 \frac{q_o B}{E_s} = (0.9)(0.92)\frac{(120)(1)}{72{,}000} = 0.00138 \text{ m} = \mathbf{1.38\ mm}$$

4.8 Settlement Based on the Theory of Elasticity

- The elastic settlement of a shallow foundation can be estimated by using the theory of elasticity. From Hooke's law, as applied to Figure 4.9, we obtain

$$S_e = \int_0^H \varepsilon_z dz = \frac{1}{E_s}\int_0^H (\Delta\sigma_z - \mu_s\Delta\sigma_x - \mu_s\Delta\sigma_y)dz \qquad 4.3$$

where

S_e = elastic settlement
E_s = modulus of elasticity of soil
H = thickness of the soil layer
μ_s = Poisson's ratio of the soil
$\Delta\sigma_x, \Delta\sigma_y, \Delta\sigma_z$ = stress increase due to the net applied foundation load in the x, y, and z directions, respectively

Theoretically, if the foundation is perfectly flexible (see Figure 4.10 and Bowles, 1987), the settlement may be expressed as

$$S_e = q_o(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f \qquad (4.4)$$

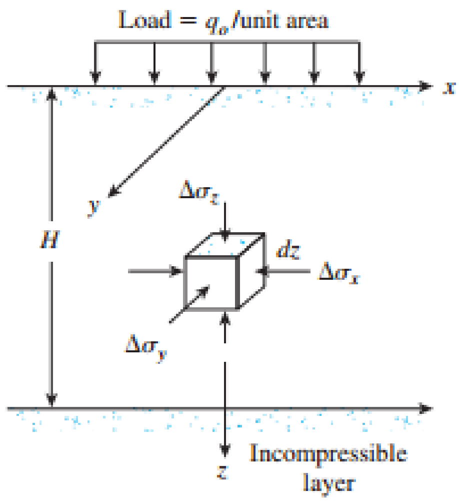

Figure 4.9 Elastic settlement of shallow foundation

where

q_o = net applied pressure on the foundation
μ_s = Poisson's ratio of soil
E_s = average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 5B$
$B' = B/2$ for center of foundation
$= B$ for corner of foundation
I_s = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2 \quad 4.5$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1) \quad 4.6$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}A_2 \quad 4.7$$

$$A_0 = m'\ln\frac{\left(1 + \sqrt{m'^2 + 1}\right)\sqrt{m'^2 + n'^2}}{m'\left(1 + \sqrt{m'^2 + n'^2 + 1}\right)} \quad 4.8$$

$$A_1 = \ln\frac{\left(m' + \sqrt{m'^2 + 1}\right)\sqrt{1 + n'^2}}{m' + \sqrt{m'^2 + n'^2 + 1}} \quad 4.9$$

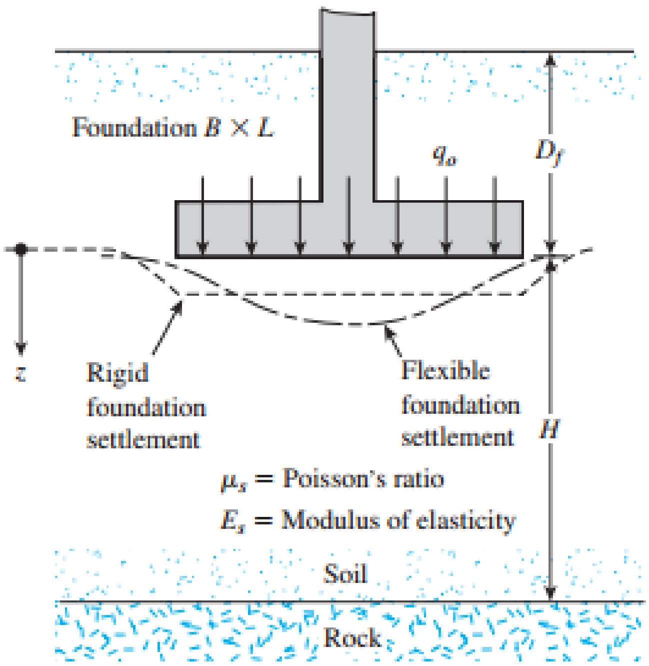

Figure 4.10 Elastic settlement of flexible and rigid foundations

$$A_2 = \frac{m'}{n'\sqrt{m'^2 + n'^2 + 1}} \quad 4.10$$

$$I_f = \text{depth factor (Fox, 1948)} = f\left(\frac{D_f}{B}, \mu_s, \text{ and } \frac{L}{B}\right) \quad 4.11$$

α = a factor that depends on the location on the foundation where settlement is being calculated

To calculate settlement at the *center* of the foundation, we use

$$\alpha = 4$$

$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

To calculate settlement at a *corner* of the foundation,

$$\alpha = 1$$

$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{B}$$

The variations of F_1 and F_2 [see Eqs. (4.6) and (4.7)] with m' and n' are given in Tables 4.2 and 4.3. Also, the variation of I_f with D_f/B (for μ_s = 0.3, 0.4, and 0.5) is given in Table 4.4. These values are also given in more detailed form by Bowles (1987).

- The elastic settlement of a *rigid foundation* can be estimated as

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})} \quad (4.12)$$

Due to the nonhomogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average of E_s in Eq. (4.4), or

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}} \quad 4.13$$

where

$E_{s(i)}$ = soil modulus of elasticity within a depth Δz

$\bar{z}$ = H or $5B$, whichever is smaller

Table 4.2 Variation of F_1 with m' and n'

	m'									
n'	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.014	0.013	0.012	0.011	0.011	0.011	0.010	0.010	0.010	0.010
0.50	0.049	0.046	0.044	0.042	0.041	0.040	0.038	0.038	0.037	0.037
0.75	0.095	0.090	0.087	0.084	0.082	0.080	0.077	0.076	0.074	0.074
1.00	0.142	0.138	0.134	0.130	0.127	0.125	0.121	0.118	0.116	0.115
1.25	0.186	0.183	0.179	0.176	0.173	0.170	0.165	0.161	0.158	0.157
1.50	0.224	0.224	0.222	0.219	0.216	0.213	0.207	0.203	0.199	0.197
1.75	0.257	0.259	0.259	0.258	0.255	0.253	0.247	0.242	0.238	0.235
2.00	0.285	0.290	0.292	0.292	0.291	0.289	0.284	0.279	0.275	0.271
2.25	0.309	0.317	0.321	0.323	0.323	0.322	0.317	0.313	0.308	0.305
2.50	0.330	0.341	0.347	0.350	0.351	0.351	0.348	0.344	0.340	0.336
2.75	0.348	0.361	0.369	0.374	0.377	0.378	0.377	0.373	0.369	0.365
3.00	0.363	0.379	0.389	0.396	0.400	0.402	0.402	0.400	0.396	0.392
3.25	0.376	0.394	0.406	0.415	0.420	0.423	0.426	0.424	0.421	0.418
3.50	0.388	0.408	0.422	0.431	0.438	0.442	0.447	0.447	0.444	0.441
3.75	0.399	0.420	0.436	0.447	0.454	0.460	0.467	0.458	0.466	0.464
4.00	0.408	0.431	0.448	0.460	0.469	0.476	0.484	0.487	0.486	0.484
4.25	0.417	0.440	0.458	0.472	0.481	0.484	0.495	0.514	0.515	0.515
4.50	0.424	0.450	0.469	0.484	0.495	0.503	0.516	0.521	0.522	0.522
4.75	0.431	0.458	0.478	0.494	0.506	0.515	0.530	0.536	0.539	0.539
5.00	0.437	0.465	0.487	0.503	0.516	0.526	0.543	0.551	0.554	0.554
5.25	0.443	0.472	0.494	0.512	0.526	0.537	0.555	0.564	0.568	0.569
5.50	0.448	0.478	0.501	0.520	0.534	0.546	0.566	0.576	0.581	0.584
5.75	0.453	0.483	0.508	0.527	0.542	0.555	0.576	0.588	0.594	0.597
6.00	0.457	0.489	0.514	0.534	0.550	0.563	0.585	0.598	0.606	0.609
6.25	0.461	0.493	0.519	0.540	0.557	0.570	0.594	0.609	0.617	0.621
6.50	0.465	0.498	0.524	0.546	0.563	0.577	0.603	0.618	0.627	0.632
6.75	0.468	0.502	0.529	0.551	0.569	0.584	0.610	0.627	0.637	0.643
7.00	0.471	0.506	0.533	0.556	0.575	0.590	0.618	0.635	0.646	0.653
7.25	0.474	0.509	0.538	0.561	0.580	0.596	0.625	0.643	0.655	0.662
7.50	0.477	0.513	0.541	0.565	0.585	0.601	0.631	0.650	0.663	0.671
7.75	0.480	0.516	0.545	0.569	0.589	0.606	0.637	0.658	0.671	0.680
8.00	0.482	0.519	0.549	0.573	0.594	0.611	0.643	0.664	0.678	0.688
8.25	0.485	0.522	0.552	0.577	0.598	0.615	0.648	0.670	0.685	0.695
8.50	0.487	0.524	0.555	0.580	0.601	0.619	0.653	0.676	0.692	0.703
8.75	0.489	0.527	0.558	0.583	0.605	0.623	0.658	0.682	0.698	0.710
9.00	0.491	0.529	0.560	0.587	0.609	0.627	0.663	0.687	0.705	0.716
9.25	0.493	0.531	0.563	0.589	0.612	0.631	0.667	0.693	0.710	0.723
9.50	0.495	0.533	0.565	0.592	0.615	0.634	0.671	0.697	0.716	0.719
9.75	0.496	0.536	0.568	0.595	0.618	0.638	0.675	0.702	0.721	0.735
10.00	0.498	0.537	0.570	0.597	0.621	0.641	0.679	0.707	0.726	0.740
20.00	0.529	0.575	0.614	0.647	0.677	0.702	0.756	0.797	0.830	0.858
50.00	0.548	0.598	0.640	0.678	0.711	0.740	0.803	0.853	0.895	0.931
100.00	0.555	0.605	0.649	0.688	0.722	0.753	0.819	0.872	0.918	0.956

(Continued)

Table 4.2 *(Continued)*

	m'									
n'	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
0.75	0.073	0.073	0.072	0.072	0.072	0.072	0.071	0.071	0.071	0.071
1.00	0.114	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.25	0.155	0.154	0.153	0.152	0.152	0.151	0.151	0.150	0.150	0.150
1.50	0.195	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
1.75	0.233	0.232	0.229	0.228	0.227	0.226	0.225	0.223	0.223	0.223
2.00	0.269	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
2.25	0.302	0.300	0.296	0.294	0.293	0.291	0.291	0.287	0.287	0.287
2.50	0.333	0.331	0.327	0.324	0.322	0.321	0.320	0.316	0.315	0.315
2.75	0.362	0.359	0.355	0.352	0.350	0.348	0.347	0.343	0.342	0.342
3.00	0.389	0.386	0.382	0.378	0.376	0.374	0.373	0.368	0.367	0.367
3.25	0.415	0.412	0.407	0.403	0.401	0.399	0.397	0.391	0.390	0.390
3.50	0.438	0.435	0.430	0.427	0.424	0.421	0.420	0.413	0.412	0.411
3.75	0.461	0.458	0.453	0.449	0.446	0.443	0.441	0.433	0.432	0.432
4.00	0.482	0.479	0.474	0.470	0.466	0.464	0.462	0.453	0.451	0.451
4.25	0.516	0.496	0.484	0.473	0.471	0.471	0.470	0.468	0.462	0.460
4.50	0.520	0.517	0.513	0.508	0.505	0.502	0.499	0.489	0.487	0.487
4.75	0.537	0.535	0.530	0.526	0.523	0.519	0.517	0.506	0.504	0.503
5.00	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.519	0.519
5.25	0.569	0.568	0.564	0.560	0.556	0.553	0.550	0.537	0.534	0.534
5.50	0.584	0.583	0.579	0.575	0.571	0.568	0.585	0.551	0.549	0.548
5.75	0.597	0.597	0.594	0.590	0.586	0.583	0.580	0.565	0.583	0.562
6.00	0.611	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
6.25	0.623	0.623	0.621	0.618	0.615	0.611	0.608	0.592	0.589	0.588
6.50	0.635	0.635	0.634	0.631	0.628	0.625	0.622	0.605	0.601	0.600
6.75	0.646	0.647	0.646	0.644	0.641	0.637	0.634	0.617	0.613	0.612
7.00	0.656	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
7.25	0.666	0.669	0.669	0.668	0.665	0.662	0.659	0.640	0.635	0.634
7.50	0.676	0.679	0.680	0.679	0.676	0.673	0.670	0.651	0.646	0.645
7.75	0.685	0.688	0.690	0.689	0.687	0.684	0.681	0.661	0.656	0.655
8.00	0.694	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
8.25	0.702	0.706	0.710	0.710	0.708	0.705	0.703	0.682	0.676	0.675
8.50	0.710	0.714	0.719	0.719	0.718	0.715	0.713	0.692	0.686	0.684
8.75	0.717	0.722	0.727	0.728	0.727	0.725	0.723	0.701	0.695	0.693
9.00	0.725	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
9.25	0.731	0.737	0.744	0.746	0.745	0.744	0.742	0.719	0.713	0.711
9.50	0.738	0.744	0.752	0.754	0.754	0.753	0.751	0.728	0.721	0.719
9.75	0.744	0.751	0.759	0.762	0.762	0.761	0.759	0.737	0.729	0.727
10.00	0.750	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20.00	0.878	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
50.00	0.962	0.989	1.034	1.070	1.100	1.125	1.146	1.265	1.279	1.261
100.00	0.990	1.020	1.072	1.114	1.150	1.182	1.209	1.408	1.489	1.499

Table 4.3 Variation of F_2 with m' and n'

n'	m' = 1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.049	0.050	0.051	0.051	0.051	0.052	0.052	0.052	0.052	0.052
0.50	0.074	0.077	0.080	0.081	0.083	0.084	0.086	0.086	0.0878	0.087
0.75	0.083	0.089	0.093	0.097	0.099	0.101	0.104	0.106	0.107	0.108
1.00	0.083	0.091	0.098	0.102	0.106	0.109	0.114	0.117	0.119	0.120
1.25	0.080	0.089	0.096	0.102	0.107	0.111	0.118	0.122	0.125	0.127
1.50	0.075	0.084	0.093	0.099	0.105	0.110	0.118	0.124	0.128	0.130
1.75	0.069	0.079	0.088	0.095	0.101	0.107	0.117	0.123	0.128	0.131
2.00	0.064	0.074	0.083	0.090	0.097	0.102	0.114	0.121	0.127	0.131
2.25	0.059	0.069	0.077	0.085	0.092	0.098	0.110	0.119	0.125	0.130
2.50	0.055	0.064	0.073	0.080	0.087	0.093	0.106	0.115	0.122	0.127
2.75	0.051	0.060	0.068	0.076	0.082	0.089	0.102	0.111	0.119	0.125
3.00	0.048	0.056	0.064	0.071	0.078	0.084	0.097	0.108	0.116	0.122
3.25	0.045	0.053	0.060	0.067	0.074	0.080	0.093	0.104	0.112	0.119
3.50	0.042	0.050	0.057	0.064	0.070	0.076	0.089	0.100	0.109	0.116
3.75	0.040	0.047	0.054	0.060	0.067	0.073	0.086	0.096	0.105	0.113
4.00	0.037	0.044	0.051	0.057	0.063	0.069	0.082	0.093	0.102	0.110
4.25	0.036	0.042	0.049	0.055	0.061	0.066	0.079	0.090	0.099	0.107
4.50	0.034	0.040	0.046	0.052	0.058	0.063	0.076	0.086	0.096	0.104
4.75	0.032	0.038	0.044	0.050	0.055	0.061	0.073	0.083	0.093	0.101
5.00	0.031	0.036	0.042	0.048	0.053	0.058	0.070	0.080	0.090	0.098
5.25	0.029	0.035	0.040	0.046	0.051	0.056	0.067	0.078	0.087	0.095
5.50	0.028	0.033	0.039	0.044	0.049	0.054	0.065	0.075	0.084	0.092
5.75	0.027	0.032	0.037	0.042	0.047	0.052	0.063	0.073	0.082	0.090
6.00	0.026	0.031	0.036	0.040	0.045	0.050	0.060	0.070	0.079	0.087
6.25	0.025	0.030	0.034	0.039	0.044	0.048	0.058	0.068	0.077	0.085
6.50	0.024	0.029	0.033	0.038	0.042	0.046	0.056	0.066	0.075	0.083
6.75	0.023	0.028	0.032	0.036	0.041	0.045	0.055	0.064	0.073	0.080
7.00	0.022	0.027	0.031	0.035	0.039	0.043	0.053	0.062	0.071	0.078
7.25	0.022	0.026	0.030	0.034	0.038	0.042	0.051	0.060	0.069	0.076
7.50	0.021	0.025	0.029	0.033	0.037	0.041	0.050	0.059	0.067	0.074
7.75	0.020	0.024	0.028	0.032	0.036	0.039	0.048	0.057	0.065	0.072
8.00	0.020	0.023	0.027	0.031	0.035	0.038	0.047	0.055	0.063	0.071
8.25	0.019	0.023	0.026	0.030	0.034	0.037	0.046	0.054	0.062	0.069
8.50	0.018	0.022	0.026	0.029	0.033	0.036	0.045	0.053	0.060	0.067
8.75	0.018	0.021	0.025	0.028	0.032	0.035	0.043	0.051	0.059	0.066
9.00	0.017	0.021	0.024	0.028	0.031	0.034	0.042	0.050	0.057	0.064
9.25	0.017	0.020	0.024	0.027	0.030	0.033	0.041	0.049	0.056	0.063
9.50	0.017	0.020	0.023	0.026	0.029	0.033	0.040	0.048	0.055	0.061
9.75	0.016	0.019	0.023	0.026	0.029	0.032	0.039	0.047	0.054	0.060
10.00	0.016	0.019	0.022	0.025	0.028	0.031	0.038	0.046	0.052	0.059
20.00	0.008	0.010	0.011	0.013	0.014	0.016	0.020	0.024	0.027	0.031
50.00	0.003	0.004	0.004	0.005	0.006	0.006	0.008	0.010	0.011	0.013
100.00	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006	0.006

(Continued)

Table 4.3 *(Continued)*

	m'									
n'	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
0.50	0.087	0.087	0.088	0.088	0.088	0.088	0.088	0.088	0.088	0.088
0.75	0.109	0.109	0.109	0.110	0.110	0.110	0.110	0.111	0.111	0.111
1.00	0.121	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.25	0.128	0.130	0.131	0.132	0.132	0.133	0.133	0.134	0.134	0.134
1.50	0.132	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
1.75	0.134	0.136	0.138	0.140	0.141	0.142	0.142	0.144	0.144	0.145
2.00	0.134	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
2.25	0.133	0.136	0.140	0.142	0.144	0.145	0.146	0.149	0.150	0.150
2.50	0.132	0.135	0.139	0.142	0.144	0.146	0.147	0.151	0.151	0.151
2.75	0.130	0.133	0.138	0.142	0.144	0.146	0.147	0.152	0.152	0.153
3.00	0.127	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
3.25	0.125	0.129	0.135	0.140	0.143	0.145	0.147	0.153	0.154	0.154
3.50	0.122	0.126	0.133	0.138	0.142	0.144	0.146	0.153	0.155	0.155
3.75	0.119	0.124	0.131	0.137	0.141	0.143	0.145	0.154	0.155	0.155
4.00	0.116	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
4.25	0.113	0.119	0.127	0.133	0.138	0.141	0.144	0.154	0.156	0.156
4.50	0.110	0.116	0.125	0.131	0.136	0.140	0.143	0.154	0.156	0.156
4.75	0.107	0.113	0.123	0.130	0.135	0.139	0.142	0.154	0.156	0.157
5.00	0.105	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
5.25	0.102	0.108	0.118	0.126	0.131	0.136	0.139	0.154	0.156	0.157
5.50	0.099	0.106	0.116	0.124	0.130	0.134	0.138	0.154	0.156	0.157
5.75	0.097	0.103	0.113	0.122	0.128	0.133	0.136	0.154	0.157	0.157
6.00	0.094	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
6.25	0.092	0.098	0.109	0.118	0.124	0.129	0.134	0.153	0.157	0.158
6.50	0.090	0.096	0.107	0.116	0.122	0.128	0.132	0.153	0.157	0.158
6.75	0.087	0.094	0.105	0.114	0.121	0.126	0.131	0.153	0.157	0.158
7.00	0.085	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
7.25	0.083	0.090	0.101	0.110	0.117	0.123	0.128	0.152	0.157	0.158
7.50	0.081	0.088	0.099	0.108	0.115	0.121	0.126	0.152	0.156	0.158
7.75	0.079	0.086	0.097	0.106	0.114	0.120	0.125	0.151	0.156	0.158
8.00	0.077	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
8.25	0.076	0.082	0.093	0.102	0.110	0.117	0.122	0.150	0.156	0.158
8.50	0.074	0.080	0.091	0.101	0.108	0.115	0.121	0.150	0.156	0.158
8.75	0.072	0.078	0.089	0.099	0.107	0.114	0.119	0.150	0.156	0.158
9.00	0.071	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
9.25	0.069	0.075	0.086	0.096	0.104	0.110	0.116	0.149	0.156	0.158
9.50	0.068	0.074	0.085	0.094	0.102	0.109	0.115	0.148	0.156	0.158
9.75	0.066	0.072	0.083	0.092	0.100	0.107	0.113	0.148	0.156	0.158
10.00	0.065	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20.00	0.035	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
50.00	0.014	0.016	0.019	0.022	0.025	0.028	0.031	0.071	0.113	0.142
100.00	0.007	0.008	0.010	0.011	0.013	0.014	0.016	0.039	0.071	0.113

Table 4.4 Variation of I_f with D_f/B, B/L, and μ_s

		B/L		
μ_s	D_f/B	0.2	0.5	1.0
0.3	0.2	0.95	0.93	0.90
	0.4	0.90	0.86	0.81
	0.6	0.85	0.80	0.74
	1.0	0.78	0.71	0.65
0.4	0.2	0.97	0.96	0.93
	0.4	0.93	0.89	0.85
	0.6	0.89	0.84	0.78
	1.0	0.82	0.75	0.69
0.5	0.2	0.99	0.98	0.96
	0.4	0.95	0.93	0.89
	0.6	0.92	0.87	0.82
	1.0	0.85	0.79	0.72

Example 4.2

A rigid shallow foundation 1 m × 2 m is shown in Figure 7.4. Calculate the elastic settlement at the center of the foundation.

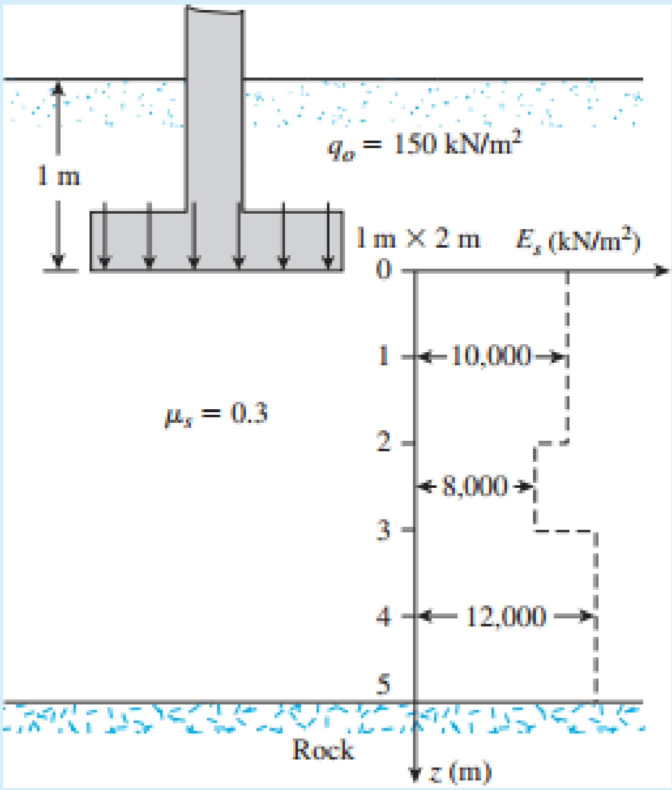

Figure 7.4 Elastic settlement below the center of a foundation

Solution

We are given that $B = 1$ m and $L = 2$ m. Note that $\bar{z} = 5$ m $= 5B$. From Eq. (7.13)

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}}$$

$$= \frac{(10{,}000)(2) + (8{,}000)(1) + (12{,}000)(2)}{5} = 10{,}400 \text{ kN/m}^2$$

For the *center of the foundation,*

$$\alpha = 4$$

$$m' = \frac{L}{B} = \frac{2}{1} = 2$$

and

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{5}{\left(\frac{1}{2}\right)} = 10$$

From Tables 7.2 and 7.3, $F_1 = 0.641$ and $F_2 = 0.031$. From Eq. (7.5),

$$I_s = F_1 + \frac{2 - \mu_s}{1 - \mu_s} F_2$$

$$= 0.641 + \frac{2 - 0.3}{1 - 0.3}(0.031) = 0.716$$

Again, $D_f/B = 1/1 = 1$, $B/L = 0.5$, and $\mu_s = 0.3$. From Table 7.4, $I_f = 0.71$.

Hence,

$$S_{e(\text{flexible})} = q_0(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

$$= (150)\left(4 \times \frac{1}{2}\right)\left(\frac{1 - 0.3^2}{10{,}400}\right)(0.716)(0.71) = 0.0133 \text{ m} = 13.3 \text{ mm}$$

Since the foundation is rigid, from Eq.(7.12) we obtain

$$S_{e(\text{rigid})} = (0.93)(13.3) = \mathbf{12.4\ mm}$$

4.9 Settlement of Sandy Soil: Use of Strain Influence Factor Solution of Schmertmann et al. (1978)

- The settlement of granular soils can also be evaluated by the use of a semiempirical ***strain influence factor*** proposed by Schmertmann et al. (1978). According to this method (Figure 4.9), the settlement is

$$S_e = C_1 C_2 (\bar{q} - q) \sum_0^{z_2} \frac{I_z}{E_s} \Delta z \qquad 4.20$$

where

I_z = strain influence factor
C_1 = a correction factor for the depth of foundation embedment = $1 - 0.5\,[q/(\bar{q} - q)]$
C_2 = a correction factor to account for creep in soil
= $1 + 0.2 \log(\text{time in years}/0.1)$
$\bar{q}$ = stress at the level of the foundation
$q = \gamma D_f$ = effective stress at the base of the foundation
E_s = modulus of elasticity of soil

The variation of the strain influence factor with the depth below the foundation is shown in Figure 4.9. Note that,

For square or circular foundations,

$I_z = 0.1$ at $z = 0$

$I_z = 0.5$ at $z = z_1 = 0.5B$

and

$I_z = 0$ at $z = z_2 = 2B$

Similarly, for foundations with $L/B \geq 10$,

$I_z = 0.2$ at $z = 0$

$I_z = 0.5$ at $z = z_1 = B$

and

$I_z = 0$ at $z = z_2 = 4B$

Where B = width of the foundation and L = length of the foundation. Values of L/B between 1 and 10 can be interpolated.

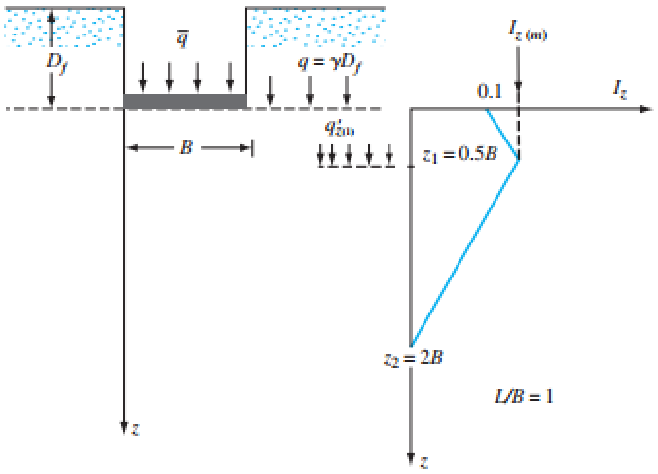

Fig. 4.9 Variation of strain influence factor with depth and L/B

The procedure for calculating elastic settlement using Eq. (4.20) is given here (Figure 4.10).

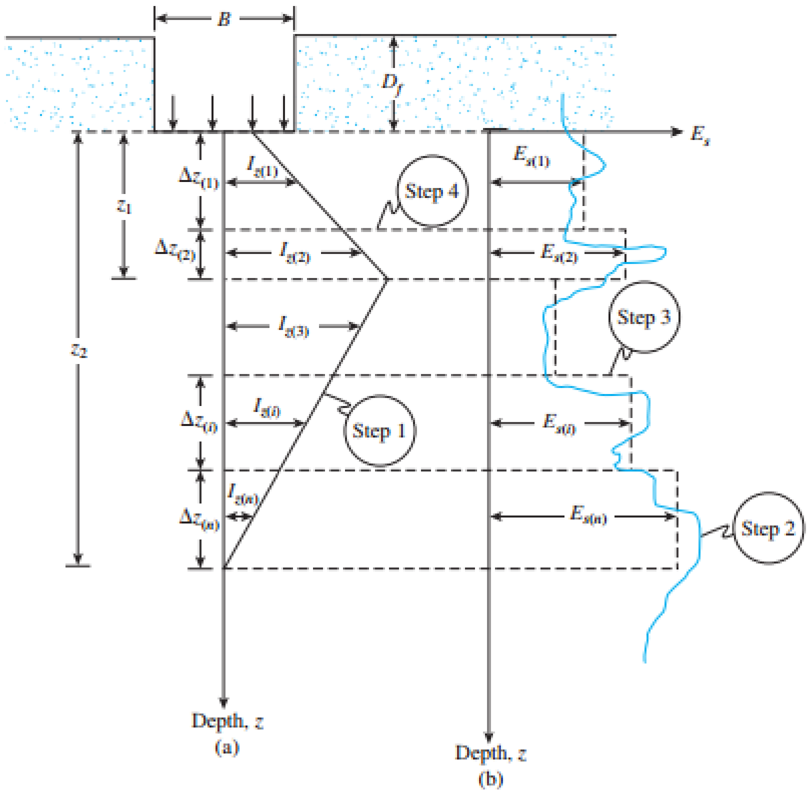

Fig. 4.10 Procedure for calculation of S_e using the strain influence factor

Table 4.5 Calculation of $\Sigma \frac{I_z}{E_s} \Delta z$

Layer no.	Δz	E_s	I_z at the middle of the layer	$\frac{I_z}{E_s} \Delta z$
1	$\Delta z_{(1)}$	$E_{s(1)}$	$I_{z(1)}$	$\frac{I_{z(1)}}{E_{s(1)}} \Delta z_1$
2	$\Delta z_{(2)}$	$E_{s(2)}$	$I_{z(2)}$	
⋮	⋮	⋮	⋮	
i	$\Delta z_{(i)}$	$E_{s(i)}$	$I_{z(i)}$	$\frac{I_{z(i)}}{E_{s(i)}} \Delta z_i$
⋮	⋮	⋮	⋮	⋮
n	$\Delta z_{(n)}$	$E_{s(n)}$	$I_{z(n)}$	$\frac{I_{z(n)}}{E_{s(n)}} \Delta z_n$
				$\Sigma \frac{I_z}{E_s} \Delta z$

Step 1. Plot the foundation and the variation of I_z with depth to scale (Figure 7.10a).

Step 2. Using the correlation from standard penetration resistance (N_{60}) or cone penetration resistance (q_c), plot the actual variation of E_s with depth (Figure 7.10b).

Step 3. Approximate the actual variation of E_s into a number of layers of soil having a constant E_s, such as $E_{s(1)}, E_{s(2)}, \ldots, E_{s(i)}, \ldots E_{s(n)}$ (Figure 7.10b).

Step 4. Divide the soil layer from $z = 0$ to $z = z_2$ into a number of layers by drawing horizontal lines. The number of layers will depend on the break in continuity in the I_z and E_s diagrams.

Step 5. Prepare a table (such as Table 7.5) to obtain $\Sigma \frac{I_z}{E_s} \Delta z$.

Step 6. Calculate C_1 and C_2.

Step 7. Calculate S_e from Eq. (7.20).

Example 4.3

Figure 3.19a shows a shallow foundation on a deposit of sandy soil that is 3 m × 3 m in plan. The actual variation of the values of the modulus of elasticity with depth determined by using the standard penetration numbers are also shown in Figure 3.19a. Using the strain influence factor method, estimate the elastic settlement of the foundation after five years of construction.

Solution

By observing the actual variation of the modulus of elasticity with depth one can plot an estimated idealized form of the variation of E_s, as shown in Figure 3.19a. Figure 3.19b shows the plot of the strain influence factor. The following table can

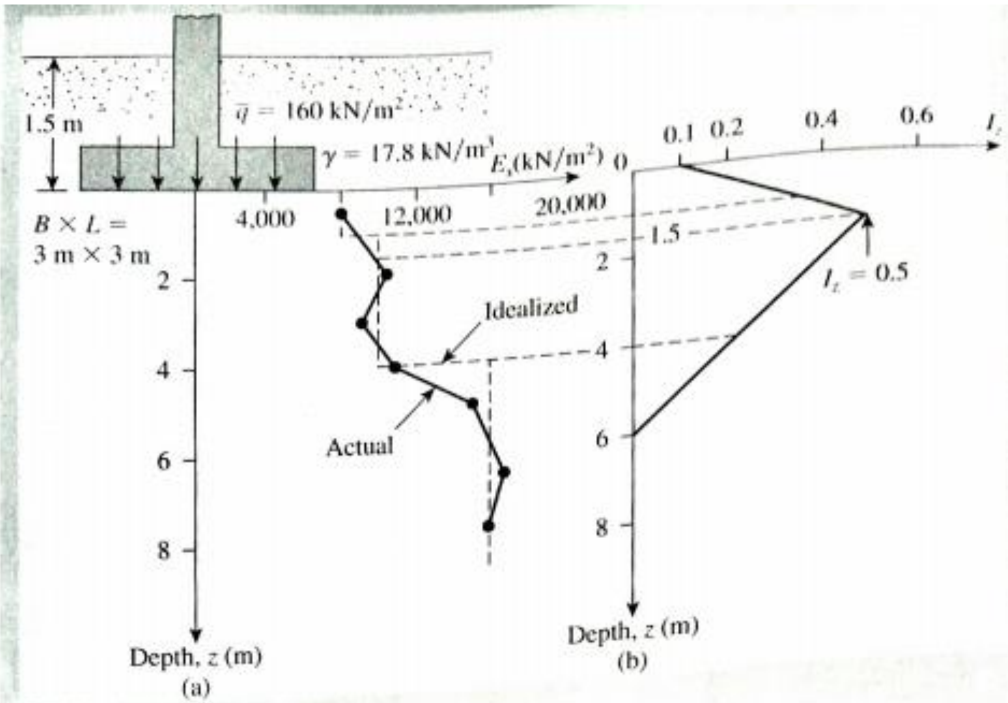

Figure 3.19

Depth (m)	Δz (m)	E_s (kN/m²)	Average I_z	$\frac{I_z}{E_s} \cdot \Delta z$ (m³/kN)
0–1	1	8,000	0.233	0.291×10^{-4}
1.0–1.5	0.5	10,000	0.433	0.217×10^{-4}
1.5–4	2.5	10,000	0.361	0.903×10^{-4}
4.0–6	2	16,000	0.111	0.139×10^{-4}
				$\sum = 1.55 \times 10^{-4}$

$$C_1 = 1 - 0.5\left(\frac{q}{\bar{q} - q}\right) = 1 - 0.5\left[\frac{17.8 \times 1.5}{160 - (17.8 \times 1.5)}\right] = 0.9$$

$$C_2 = 1 + 0.2 \log\left(\frac{5}{0.1}\right) = 1.34$$

Hence,

$$S_e = C_1, C_2(\bar{q} - q)\sum_{0}^{2B}\frac{I_z}{E_s}\cdot\Delta z$$

$$= (0.9)(1.34)[160 - (17.8 \times 1.5)](1.55 \times 10^{-4})$$

$$= 249.2 \times 10^{-4}\,\text{m} \approx \mathbf{24.9\ mm}$$

4.10 Settlement of Foundation on Sand Based on Standard Penetration Resistance

4.10.1 Meyerhof's Method

- Meyerhof (1956) proposed a correlation for the *net bearing pressure* for foundations with the standard penetration resistance, N_{60}. The net pressure has been defined as

$$q_{net} = \overline{q} - \gamma D_f$$

where $\overline{q}$ = stress at the level of the foundation.

$$q_{net}(\text{kip/ft}^2) = \frac{N_{60}}{4} \quad (\text{for } B \leq 4 \text{ ft}) \qquad 4.21$$

and

$$q_{net}(\text{kip/ft}^2) = \frac{N_{60}}{6}\left(\frac{B+1}{B}\right)^2 \quad (\text{for } B > 4 \text{ ft}) \qquad 4.22$$

- Bowles (1977) proposed that the modified form of the bearing equations be expressed as

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m}) \qquad 4.23$$

and

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m}) \qquad 4.24$$

where B is in meters and S_e is in mm. Hence,

$$S_e(\text{mm}) = \frac{1.25 q_{net}(\text{kN/m}^2)}{N_{60} F_d} \quad (\text{for } B \leq 1.22 \text{ m}) \qquad 4.25$$

and

$$S_e(\text{mm}) = \frac{2 q_{net}(\text{kN/m}^2)}{N_{60} F_d}\left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22 \text{ m}) \qquad 4.26$$

- The N_{60} referred to in the preceding equations is the standard penetration resistance between the bottom of the foundation and $2B$ below the bottom.

Consolidation Settlement

4.11 Primary Consolidation Settlement Relationships

As mentioned before, consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by construction of the foundation. (See Figure 4.20.) On the basis of the one-dimensional consolidation settlement equations, we write

$$S_{c(p)} = \int \varepsilon_z dz$$

where

ε_z = vertical strain

$= \dfrac{\Delta e}{1 + e_o}$

Δe = change of void ratio

$= f(\sigma'_o, \sigma'_c, \text{ and } \Delta\sigma')$

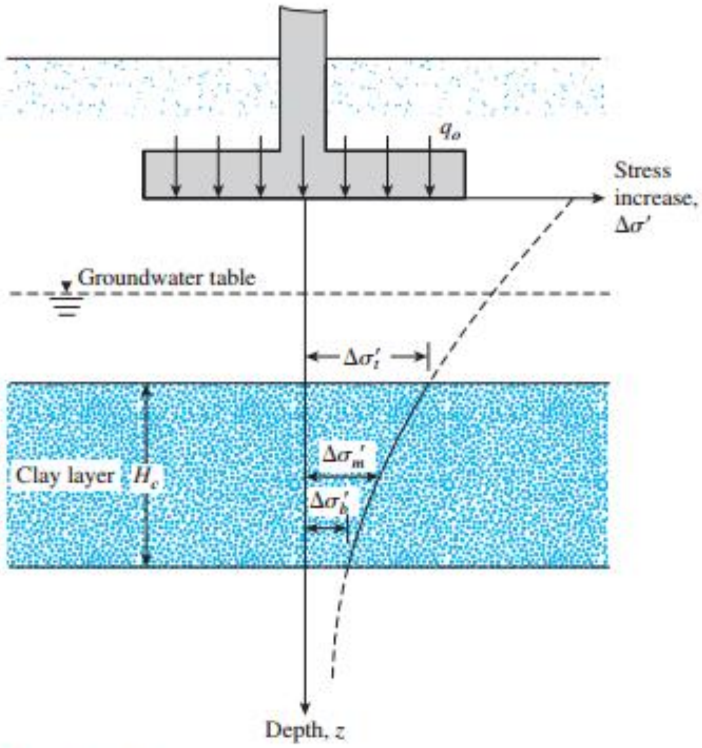

Figure 4.20 Consolidation settlement calculation

So,

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$ (for normally consolidated clays)

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$ (for overconsolidated clays with $\sigma'_o + \Delta\sigma'_{av} < \sigma'_c$)

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c}$$ (for overconsolidated clays with $\sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}$)

where

σ'_o = average effective pressure on the clay layer before the construction of the foundation
$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation
σ'_c = preconsolidation pressure
e_o = initial void ratio of the clay layer
C_c = compression index
C_s = swelling index
H_c = thickness of the clay layer

- Note that the increase in effective pressure, $\Delta\sigma'$, on the clay layer is not constant with depth: The magnitude of $\Delta\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by

$$\Delta\sigma'_{av} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

where $\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ are, respectively, the effective pressure increases at the *top*, *middle*, and *bottom* of the clay layer that are caused by the construction of the foundation.

The method of determining the pressure increase caused by various types of foundation load using Boussinesq's solution is discussed in previous Sections

Example 4.4

A plan of a foundation 1 m × 2 m is shown in Figure 4.23. Estimate the consolidation settlement of the foundation.

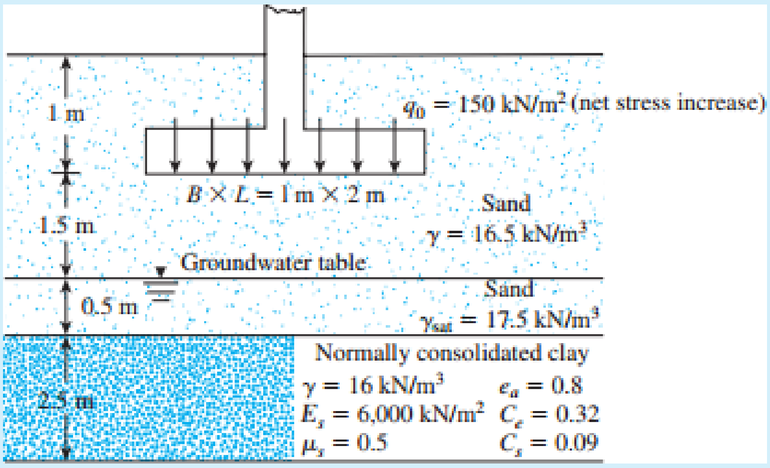

Figure 4.23 Calculation of primary consolidation settlement for a foundation

Solution

The clay is normally consolidated. Thus,

$$S_{c(p)-oed} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

so

$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81)$$

$$= 41.25 + 3.85 + 7.74 = 52.84 \text{ kN/m}^2$$

From Eq. (6.29),

$$\Delta\sigma'_{av} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

Now the following table can be prepared (*Note:* L = 2 m; B = 1 m):

$m_1 = L/B$	z(m)	$z/(B/2) = n_1$	I_c[a]	$\Delta\sigma' = q_o I_c$[b]
2	2	4	0.190	$28.5 = \Delta\sigma'_t$
2	2 + 2.5/2 = 3.25	6.5	≈ 0.085	$12.75 = \Delta\sigma'_m$
2	2 + 2.5 = 4.5	9	0.045	$6.75 = \Delta\sigma'_b$

[a]Table 6.5
[b]Eq. (6.14)

Now,

$$\Delta\sigma'_{av} = \tfrac{1}{6}(28.5 + 4 \times 12.75 + 6.75) = 14.38 \text{ kN/m}^2$$

so

$$S_{c(p)-oed} = \frac{(0.32)(2.5)}{1 + 0.8} \log\left(\frac{52.84 + 14.38}{52.84}\right) = 0.0465 \text{ m}$$

$$= \mathbf{46.5 \text{ mm}}$$

4.12 Settlement Due to Secondary Consolidation

At the end of primary consolidation (i.e., after the complete dissipation of excess pore water pressure) some settlement is observed that is due to the plastic adjustment of soil fabrics. This stage of consolidation is called *secondary consolidation.* A plot of deformation against the logarithm of time during secondary consolidation is practically linear as shown in Figure 4.24. From the figure, the secondary compression index can be defined as

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log (t_2/t_1)}$$

where

C_α = secondary compression index
Δe = change of void ratio
t_1, t_2 = time

The magnitude of the secondary consolidation can be calculated as

$$S_{c(s)} = C'_\alpha H_c \log(t_2/t_1)$$

where

$C'_\alpha = C_\alpha/(1 + e_p)$
e_p = void ratio at the end of primary consolidation
H_c = thickness of clay layer

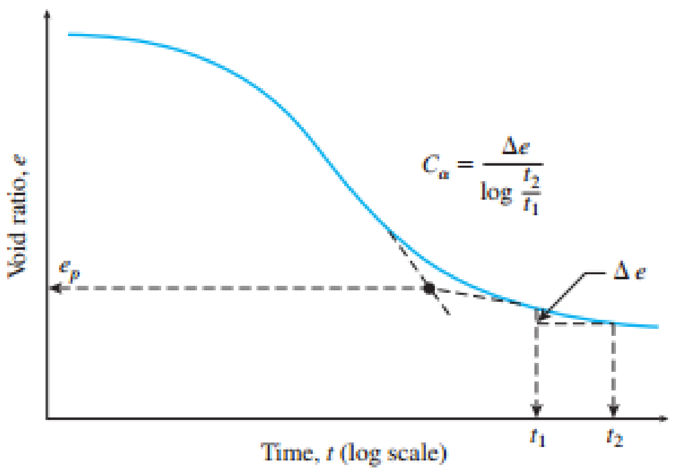

Figure 4.24 Variation of e with log t under a given load increment, and definition of secondary compression index

Mesri (1973) correlated C'_α with the natural moisture content (w) of several soils, from which it appears that

$$C'_\alpha \approx 0.0001w$$

where w = natural moisture content, in percent. For most overconsolidated soils, C'_α varies between 0.0005 to 0.001.

Secondary consolidation settlement is more important in the case of all organic and highly compressible inorganic soils. In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

There are several factors that might affect the magnitude of secondary consolidation, some of which are not yet very clearly understood (Mesri, 1973). The ratio of secondary to primary compression for a given thickness of soil layer is dependent on the ratio of the stress increment, $\Delta\sigma'$, to the initial effective overburden stress, σ'_o . For small $\Delta\sigma' / \sigma'_o$ ratios, the secondary-to-primary compression ratio is larger.

Example 4.5

Refer to Example 7.10. Given for the clay layer: $C_\alpha = 0.02$. Estimate the total consolidation settlement five years after the completion of the primary consolidation settlement. (*Note:* Time for completion of primary consolidation settlement is 1.3 years).

Solution

From Eq. (2.53),

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)}$$

For this problem, $e_1 - e_2 = \Delta e$.

Referring to Example 4.4, we have

$$\sigma_2' = \sigma_o' + \Delta\sigma' = 52.84 + 14.38 = 67.22 \text{ kN/m}^2$$

$$\sigma_1' = \sigma_o' = 52.84 \text{ kN/m}^2$$

$$C_c = 0.32$$

Hence,

$$\Delta e = C_c \log\left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right) = 0.32 \log\left(\frac{67.22}{52.84}\right) = 0.0335$$

Given: $e_o = 0.8$. Hence,

$$e_p = e_o - e = 0.8 - 0.0335 = 0.7665$$

From Eq. (7.71),

$$C_\alpha' = \frac{C_\alpha}{1 + e_p} = \frac{0.02}{1 + 0.7665} = 0.0113$$

From Eq. (7.70),

$$S_{c(s)} = C_\alpha' H_c \log\left(\frac{t_2}{t_1}\right)$$

Note: $t_1 = 1.3$ years; $t_2 = 1.3 + 5 = 6.3$ years.
Thus,

$$S_{c(s)} = (0.0113)(2.5 \text{ m}) \log\left(\frac{6.3}{1.3}\right) = 0.0194 \text{ m} = 19.4 \text{ mm}$$

Total consolidation settlement is

$$\underbrace{36.3 \text{ mm}}_{\substack{\uparrow \\ \text{Example 7.10} \\ \text{(Primary} \\ \text{consolidation} \\ \text{settlement)}}} + 19.4 = \mathbf{55.7 \text{ m}}$$

4.13 Field Load Test

The ultimate load-bearing capacity of a foundation, as well as the allowable bearing capacity based on tolerable settlement considerations, can be effectively determined from the field load test, generally referred to as the *plate load test.* The plates that are used for tests in the field are usually made of steel and are 25 mm (1 in.) thick and 150 mm to 762 mm (6 in. to 30 in.) in diameter. Occasionally, square plates that are 305 mm × 305 mm (12 in. × 12 in.) are also used.

To conduct a plate load test, a hole is excavated with a minimum diameter of 4*B* (*B* is the diameter of the test plate) to a depth of D_f , the depth of the proposed foundation. The plate is placed at the center of the hole, and a load that is about one-fourth to one-fifth of the estimated ultimate load is applied to the plate in steps by means of a jack. A schematic diagram of the test arrangement is shown in Figure 4.25a. During each step of the application of the load, the settlement of the plate is observed on dial gauges. At least one hour is allowed to elapse between each application. The test should be conducted until failure, or at least until the plate has gone through 25 mm (1 in.) of settlement. Figure 4.25b shows the nature of the load–settlement curve obtained from such tests, from which the ultimate load per unit area can be determined. Figure 4.26 shows a plate load test conducted in the field.

For tests in clay,

$$q_{u(F)} = q_{u(P)}$$

where

$q_{u(F)}$ = ultimate bearing capacity of the proposed foundation
$q_{u(P)}$ = ultimate bearing capacity of the test plate

Equation above implies that the ultimate bearing capacity in clay is virtually independent of the size of the plate.

For tests in sandy soils,

$$q_{u(F)} = q_{u(P)} \frac{B_F}{B_P}$$

where

B_F = width of the foundation
B_P = width of the test plate

The allowable bearing capacity of a foundation, based on settlement considerations and for a given intensity of load, q_o, is

$$S_F = S_P \frac{B_F}{B_P} \quad \text{(for clayey soil)}$$

and

$$S_F = S_P \left(\frac{2B_F}{B_F + B_P} \right)^2 \quad \text{(for sandy soil)}$$

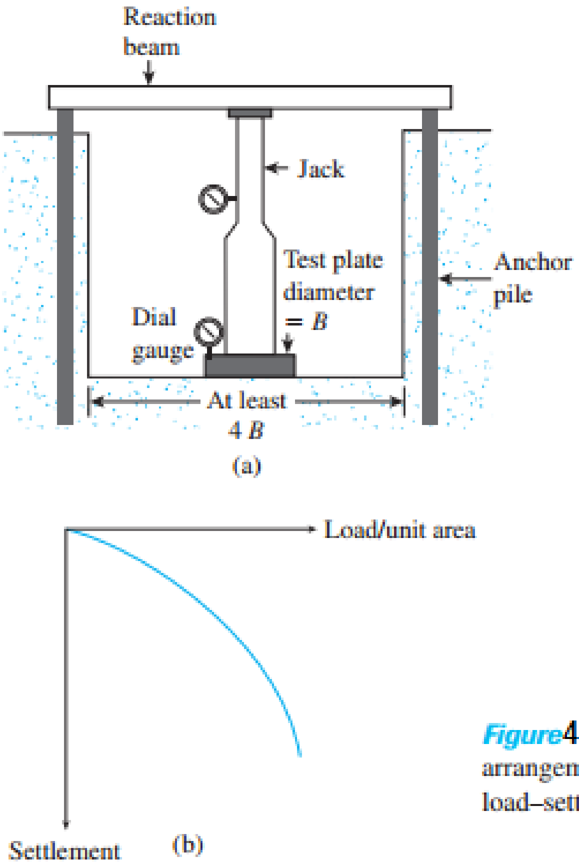

Figure4 .25 Plate load test: (a) test arrangement; (b) nature of load–settlement curve

Figure 4.26 Plate load test in the field (*Courtesy of Braja M. Das, Henderson, Nevada*)

4.14 Tolerable Settlement of Buildings

In most instances of construction, the subsoil is not homogeneous and the load carried by various shallow foundations of a given structure can vary widely. As a result, it is reasonable to expect varying degrees of settlement in different parts of a given building. The ***differential settlement*** of the parts of a building can lead to damage of the superstructure. Hence, it is important to define certain parameters that quantify differential settlement and to develop limiting values for those parameters in order that the resulting structures be safe.

Burland and Wroth (1970) summarized the important parameters relating to differential settlement. Figure 4.27 shows a structure in which various foundations, at *A, B, C, D,* and *E,* have gone through some settlement. The settlement at *A* is AA', at *B* is BB', etc. Based on this figure, the definitions of the various parameters are as follows:

S_T = total settlement of a given point
ΔS_T = difference in total settlement between any two points
α = gradient between two successive points

$$\beta = \text{angular distortion} = \frac{\Delta S_{T(ij)}}{l_{ij}}$$

(*Note*: l_{ij} = distance between points i and j)
ω = tilt
Δ = relative deflection (i.e., movement from a straight line joining two reference points)

$$\frac{\Delta}{L} = \text{deflection ratio}$$

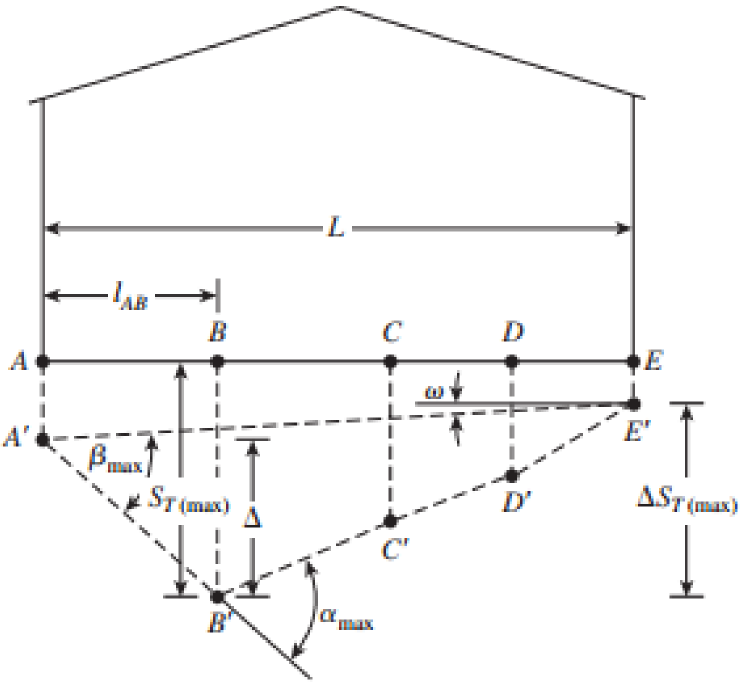

Figure 4.27 Definition of parameters for differential settlement

- In 1956, Skempton and McDonald proposed the following limiting values for maximum settlement and maximum angular distortion, to be used for building purposes:

Maximum settlement, $S_{T(max)}$	
In sand	32 mm
In clay	45 mm
Maximum differential settlement, $\Delta S_{T(max)}$	
Isolated foundations in sand	51 mm
Isolated foundations in clay	76 mm
Raft in sand	51–76 mm
Raft in clay	76–127 mm
Maximum angular distortion, β_{max}	1/300

- Polshin and Tokar (1957) suggested the following allowable deflection ratios for buildings as a function of L / H, the ratio of the length to the height of a building:

$$\Delta/L = 0.0003 \text{ for } L/H \leq 2$$
$$\Delta/L = 0.001 \text{ for } L/H = 8$$

- The 1955 Soviet Code of Practice allowable values are given in Table 4.10.
- Bjerrum (1963) recommended the following limiting angular distortion, β_{max} for various structures, as shown in Table 4.11.

Table 4.10

Type of building	L/H	Δ/L
Multistory buildings and civil dwellings	≤3	0.0003 (for sand) 0.0004 (for clay)
	≥5	0.0005 (for sand) 0.0007 (for clay)
One-story mills		0.001 (for sand and clay)

Table 4.11

Category of potential damage	β_{max}
Safe limit for flexible brick wall ($L/H > 4$)	1/150
Danger of structural damage to most buildings	1/150
Cracking of panel and brick walls	1/150
Visible tilting of high rigid buildings	1/250
First cracking of panel walls	1/300
Safe limit for no cracking of building	1/500
Danger to frames with diagonals	1/600

- If the maximum allowable values of β_{max} are known, the magnitude of the allowable $S_{T(max)}$ can be calculated with the use of the foregoing correlations.
- The European Committee for Standardization has also provided limiting values for serviceability and the maximum accepted foundation movements. (See Table 4.12.)

Table 4.12 Recommendations of European Committee for Standardization on Differential Settlement Parameters

Item	Parameter	Magnitude	Comments
Limiting values for serviceability	S_T	25 mm	Isolated shallow foundation
		50 mm	Raft foundation
(European Committee for Standardization, 1994a)	ΔS_T	5 mm	Frames with rigid cladding
		10 mm	Frames with flexible cladding
		20 mm	Open frames
	β	1/500	—
Maximum acceptable foundation movement	S_T	50	Isolated shallow foundation
	ΔS_T	20	Isolated shallow foundation
(European Committee for Standardization, 1994b)	β	≈1/500	—