University of Anbar
Engineering College
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CHAPTER THREE

SHALLOW FOUNDATIONS

ULTIMATE BEARING CAPACITY

LECTURE
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3.1. Introduction

To perform satisfactorily, shallow foundations must have two main characteristics:

1. They have to be safe against overall shear failure in the soil that supports them.

2. They cannot undergo excessive displacement, or settlement. (The term ***excessive*** is relative, because the degree of settlement allowed for a structure depends on several considerations.)

The load per unit area of the foundation at which shear failure in soil occurs is called the ***ultimate bearing capacity***, which is the subject of this chapter. In this chapter, we will discuss the following:

- Fundamental concepts in the development of the theoretical relationship for ultimate bearing capacity of shallow foundations subjected to centric vertical loading
- Effect of the location of water table and soil compressibility on ultimate bearing capacity
- Bearing capacity of shallow foundations subjected to vertical eccentric loading and eccentrically inclined loading.

3.2 General Concept

3.2.1 Types of Shear Failure

Shear Failure: Also called "Bearing capacity failure" and it's occur when the shear stresses in the soil exceed the shear strength of the soil.

There are three types of shear failure in the soil:

1. General Shear Failure (Fig. 3.1)

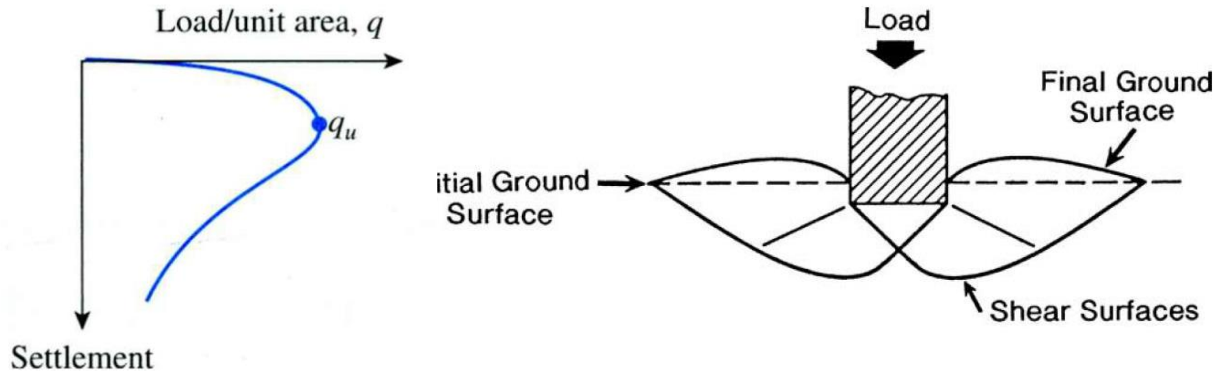


Fig. 3.1 General shear failure.

The following are some characteristics of general shear failure:

- Occurs over dense sand or stiff cohesive soil.
- Involves total rupture of the underlying soil.
- There is a continuous shear failure of the soil from below the footing to the ground surface (solid lines on the figure above).
- When the (load / unit area) plotted versus settlement of the footing, there is a distinct load at which the foundation fails (Q_u)
- The value of (Q_u) divided by the area of the footing is considered to be the ultimate bearing capacity of the footing(q_u).
- For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes **a sudden** catastrophic failure of the foundation.
- As shown in the above **Fig. 3.1**, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).
- However, for actual failures on the field, the soil is often pushed up on **only one side** of the footing with **subsequent tilting** of the structure as shown in **Fig. 3.2** below:

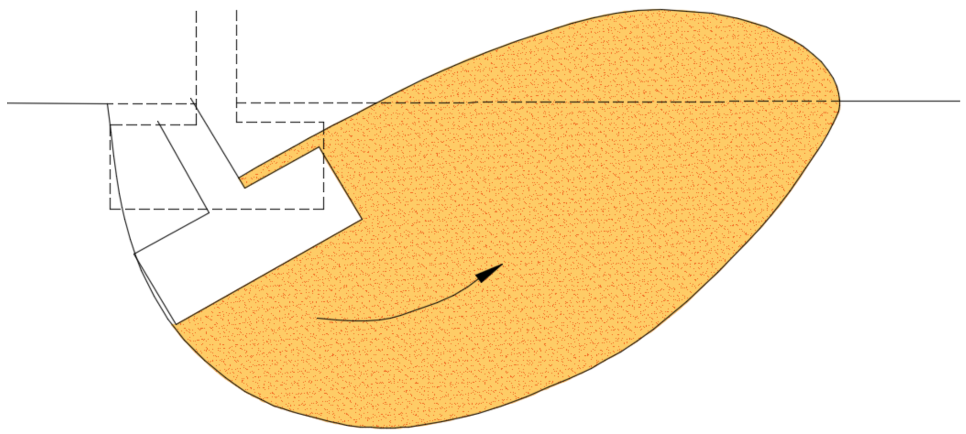


Fig. 3.2 Actual general shear failure.

2. Local Shear Failure (Fig. 3.3):

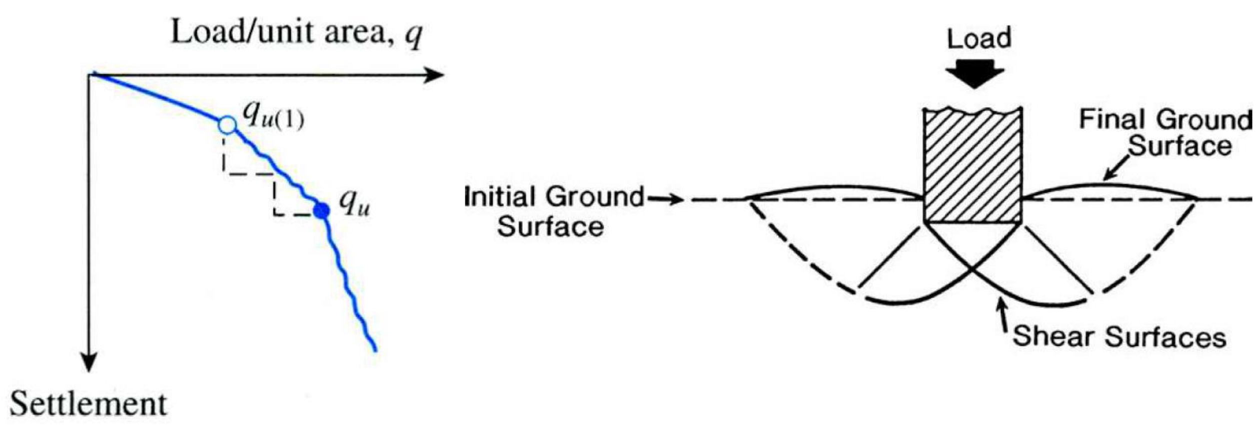


Fig. 3.3 Local shear failure.

The following are some characteristics of local shear failure:

- Occurs over sand or clayey soil of medium compaction.
- Involves rupture of the soil only immediately below the footing.
- There is soil bulging on both sides of the footing, but the bulging is not as significant as in general shear. That's because the underlying soil compacted less than the soil in general shear.
- The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines** in **Fig. 3.3.**

- So, local shear failure can be considered as a transitional phase between general shear and punching shear.
- Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the firs failure load ($q_{u(1)}$) which occur at the point which have the first measure nonlinearity in the load/unit area-settlement curve (open circle), or at the point where the settlement starts rabidly increase (q_u) (closed circle).
- This value of (q_u) is the required (load/unit area) to extends the failure surface to the ground surface (dashed lines as in **Fig. 3.3**).
- In this type of failure, the value of (q_u) it's not the peak value so, this failure called (Local Shear Failure).
- The actual local shear failure in field is proceed as shown in **Fig. 3.4**.

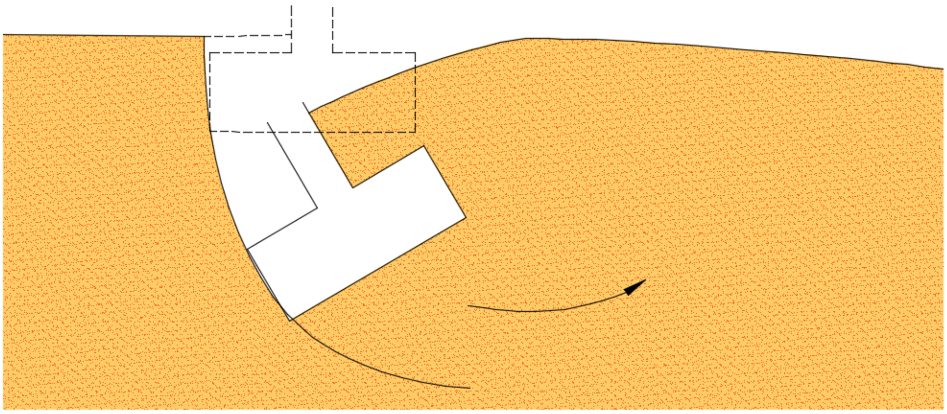


Fig. 3.4 Actual Local shear failure.

3. Punching Shear Failure (Fig. 3.5):

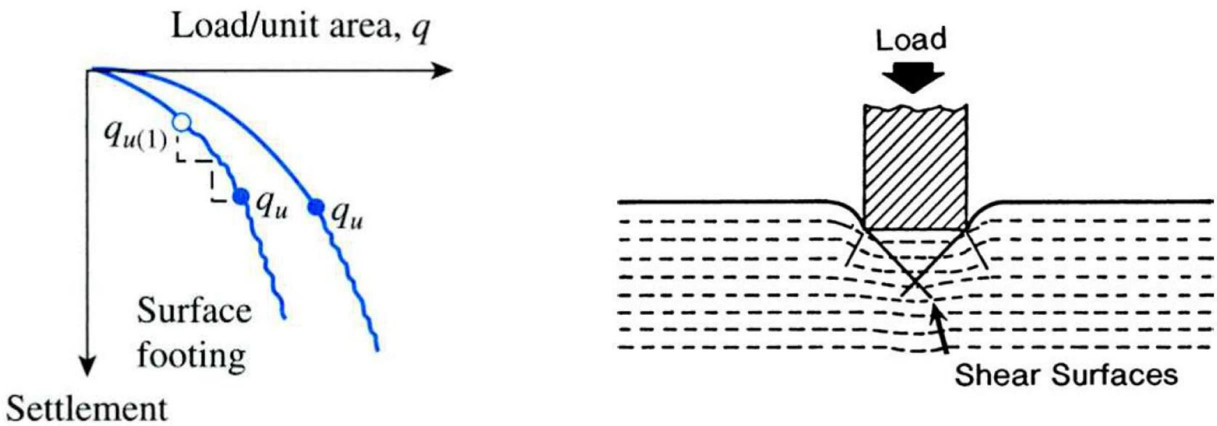


Fig. 3.5 Punching shear failure.

The following are some characteristics of punching shear failure:

- Occurs over fairly loose soil.
- Punching shear failure does not develop the distinct shear surfaces associated with a general shear failure.
- The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- As shown in **Fig. 3.5** above, the (q)-settlement curve does not have a dramatic break, and the bearing capacity is often defined as the first measure nonlinearity in the (q)-settlement curve($q_{u(1)}$).
- Beyond the ultimate failure (load/unit area) ($q_{u(1)}$), the (load/unit area)- settlement curve will be steep and practically linear.
- The actual punching shear failure in field is proceed as shown in **Fig.3.6**:

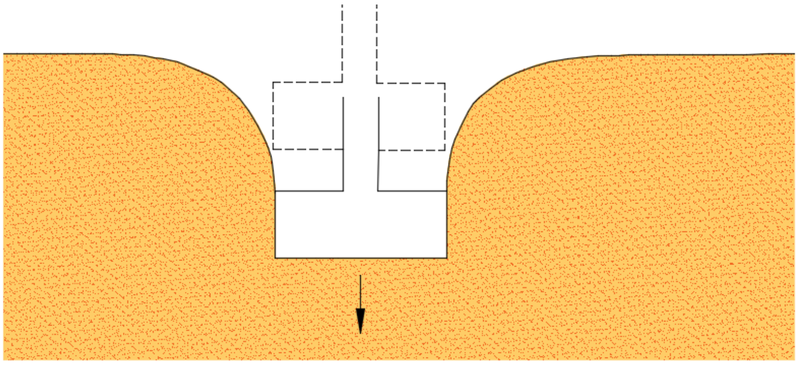


Fig. 3.6 Actual Punching shear failure.

Note:

Ultimate Bearing Capacity (q_u):

It's the **minimum load per unit area** of the foundation that causes shear failure in the underlying soil.

Or, it's the **maximum load per unit area** of the foundation can be resisted by the underlying soil without occurs of shear failure (if this load is exceeded, the shear failure will occur in the underlying soil).

Allowable Bearing Capacity (q_{all})

It's the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure **will not** occur in the underlying soil **till** reaching the ultimate load.

3.3 Terzaghi's Bearing Capacity Theory

Terzaghi (1943) was the first to present a comprehensive theory for evaluation of the ultimate bearing capacity of rough shallow foundation. **This theory is based on the following assumptions:**

1. The foundation is considered to be shallow if ($D_f \leq B$).
2. The foundation is considered to be strip or continuous if (Width to length ratio approaches zero), and the derivation of the equation is to a strip footing.
3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q = \gamma \times D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines ad and cd in the below **Fig. 3.7**)
4. The failure surface of the soil is similar to **general shear failure** (i.e. equation is derived for general shear failure) as shown in **Fig. 3.7**.

Note:

1. In recent studies, investigators have suggested that, foundations are considered to be shallow if [$D_f \leq (3 \rightarrow 4)B$], otherwise, the foundation is deep.
2. Always the value of (q) is the **effective stress** at the bottom of the foundation.

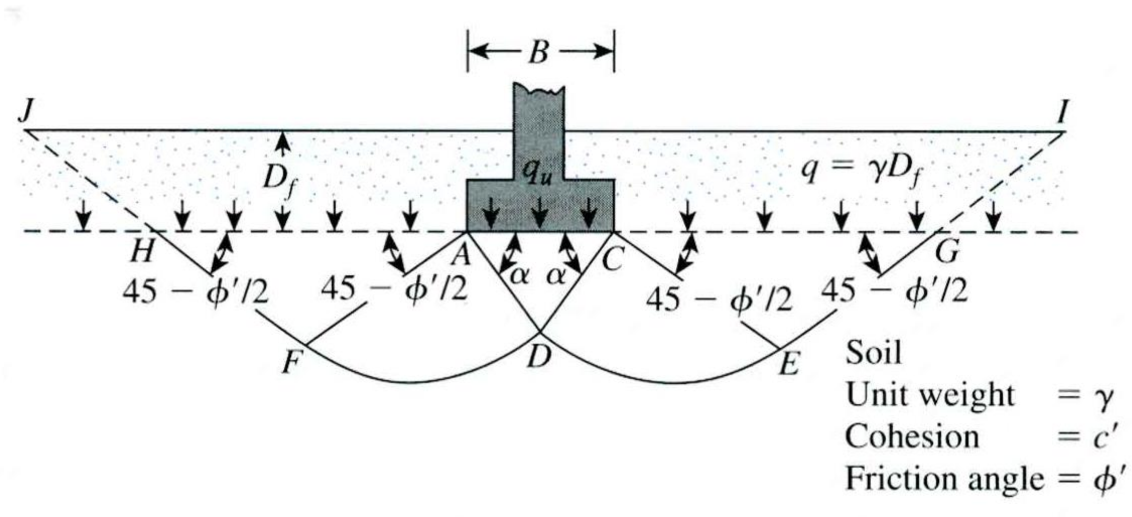


Fig. 3.7 Bearing capacity failure in soil under a rough rigid continous foundation

The failure zone under the foundation can be separated into three parts (see Fig. 3.7):

1. The *triangular zone ACD* immediately under the foundation

2. The *radial shear zones ADF* and *CDE,* with the curves *DE* and *DF* being arcs of a logarithmic spiral

3. Two triangular *Rankine passive zones AFH* and *CEG*

The angles *CAD* and *ACD* are assumed to be equal to the soil friction angle ϕ'. Note that, with the replacement of the soil above the bottom of the foundation by an equivalent surcharge *q*, the shear resistance of the soil along the failure surfaces *GI* and *HJ* was neglected.

Terzaghi's Bearing Capacity Equations

As mentioned previously, the equation was derived for a **strip footing** and **general shear failure,** this equation is:

$q_u = cN_c + qN_q + 0.5B\gamma N_\gamma$ (for continuous or strip footing) (3.1)

Where

q_u=Ultimate bearing capacity of the soil (kN/m^2)

c=Cohesion of soil (kN/m^2)

q=**Efeective** stress at the bottom of the foundation= γD_f (kN/m^2)

N_c, N_q, N_γ=Bearing capacity factors (nondimensional)and are functions **only** of the soil friction angle,ϕ'

The bearing capacity factors N_c, N_q, and N_γ are defined by

$$N_c = \cot\phi'\left[\frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(\frac{\pi}{4}+\frac{\phi'}{2}\right)} - 1\right] = \cot\phi'(N_q - 1) \quad (3.2)$$

$$N_q = \frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(45+\frac{\phi'}{2}\right)} \quad (3.3)$$

and

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2 \phi'} - 1\right)\tan \phi' \tag{3.4}$$

$K_{p\gamma}$= passive pressure coefficient

The variations of the bearing capacity factors shown in **Table 3.1 for general shear failure**.

The above equation (for strip footing) was modified to be useful for both **square** and **circular footings** as following:

$q_u=1.3cN_c+qN_q+0.4B\gamma N_\gamma$ B (square foundation) (3.5)

B=The dimension of each side of the foundation .

$q_u=1.3cN_c+qN_q+0.3B\gamma N_\gamma$ B (circular foundation) (3.6)

B=The diameter of the foundation .

Note:

These two equations are also for **general shear failure,** and all factors in the two equations are the same as explained for strip footing.

Table 3.1 Terzaghi's bearing capacity factors

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

Now for **local shear failure** the above three equations were modified to be useful for local shear failure as following:

$q_u = 2/3c' N_c' + qN_q' + 0.5B\gamma N_\gamma'$ (for continuous or strip footing) (3.7)

$q_u = 0.867\ c' N_c' + qN_q' + 0.4B\gamma N_\gamma'$ (for square footing) (3.8)

$q_u = 0.867\ c' N_c' + qN_q' + 0.3B\gamma N_\gamma'$ (for circular footing) (3.9)

N_c', N_q', N_γ'=Modified bearing capacity factors can be calculated by using the bearing capacity factor equations (for N_c , N_q, and N_γ, respectively) by replacing ϕ' by $\phi'' = \tan^{-1}(2/3 \tan \phi')$. The variation of N_c', N_q', and N_γ' with the soil friction angle ϕ' is given in **Table 3.2**.

Table 3.2 Terzaghi's modified bearing capacity factors N_c', N_q', and N_γ'

ϕ'	N_c'	N_q'	N_γ'	ϕ'	N_c'	N_q'	N_γ'
0	5.70	1.00	0.00	26	15.53	6.05	2.59
1	5.90	1.07	0.005	27	16.30	6.54	2.88
2	6.10	1.14	0.02	28	17.13	7.07	3.29
3	6.30	1.22	0.04	29	18.03	7.66	3.76
4	6.51	1.30	0.055	30	18.99	8.31	4.39
5	6.74	1.39	0.074	31	20.03	9.03	4.83
6	6.97	1.49	0.10	32	21.16	9.82	5.51
7	7.22	1.59	0.128	33	22.39	10.69	6.32
8	7.47	1.70	0.16	34	23.72	11.67	7.22
9	7.74	1.82	0.20	35	25.18	12.75	8.35
10	8.02	1.94	0.24	36	26.77	13.97	9.41
11	8.32	2.08	0.30	37	28.51	15.32	10.90
12	8.63	2.22	0.35	38	30.43	16.85	12.75
13	8.96	2.38	0.42	39	32.53	18.56	14.71
14	9.31	2.55	0.48	40	34.87	20.50	17.22
15	9.67	2.73	0.57	41	37.45	22.70	19.75
16	10.06	2.92	0.67	42	40.33	25.21	22.50
17	10.47	3.13	0.76	43	43.54	28.06	26.25
18	10.90	3.36	0.88	44	47.13	31.34	30.40
19	11.36	3.61	1.03	45	51.17	35.11	36.00
20	11.85	3.88	1.12	46	55.73	39.48	41.70
21	12.37	4.17	1.35	47	60.91	44.45	49.30
22	12.92	4.48	1.55	48	66.80	50.46	59.25
23	13.51	4.82	1.74	49	73.55	57.41	71.45
24	14.14	5.20	1.97	50	81.31	65.60	85.75
25	14.80	5.60	2.25				

Terzaghi's bearing capacity equations have now been modified to take into account the effects of the foundation shape *(B/L)*, depth of embedment (D_f) , and the load inclination. This is given in Section 3.6. Many design engineers, however, still use Terzaghi's equation, which provides **fairly good results** considering the uncertainty of the soil conditions at various sites.

3.4 Factor of Safety

Calculating the gross *allowable load-bearing capacity* of shallow foundations requires the application of a factor of safety (FS) to the gross ultimate bearing capacity, or

$$q_{all} = q_u / FS \quad (3.10)$$

However, some practicing engineers prefer to use a factor of safety such that

$$\text{Net stress increase on soil} = (\text{net ultimate bearing capacity}) / FS \quad (3.11)$$

The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level. If the difference between the unit weight of concrete used in the foundation and the unit weight of soil surrounding is assumed to be negligible, then

$$q_{net(u)} = q_u - q \quad (3.12)$$

where

$q_{net(u)}$ = net ultimate bearing capacity

$q = \gamma D_f$

So

$$q_{all(net)} = (q_u - q) / FS \quad (3.13)$$

The factor of safety as defined by Eq. (3.13) should be at least 3 in all cases.

3.5 Modification of Bearing Capacity Equations

Equations (3.1) and (3.5) through (3.6) give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure 3.8.)

Case I. If the water table is located so that $0 \leq D_1 \leq D_f$, the factor q in the bearing capacity equations takes the form

$$q = \text{effective surcharge} = D_1\gamma + D_2\,(\gamma_{sat} - \gamma_w) \quad (3.14)$$

where

γ_{sat} = saturated unit weight of soil

γ_w = unit weight of water

Also, the value of γ in the last term of the equations has to be replaced by $\gamma' = (\gamma_{sat} - \gamma_w)$.

Case II. For a water table located so that $0 \leq d \leq B$,

$$q = \gamma D_f \quad (3.15)$$

In this case, the factor γ in the last term of the bearing capacity equations must be replaced by the factor

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

The preceding modifications are based on the assumption that there is no seepage force in the soil.

Case III. When the water table is located so that $d \geq B$, the water will have no effect on the ultimate bearing capacity.

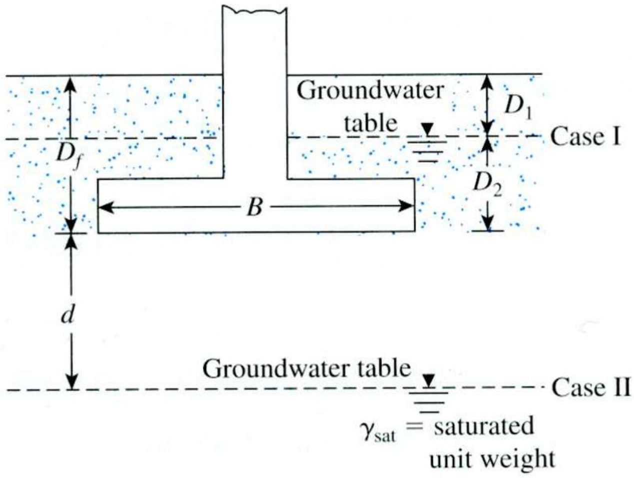


Figure 3.8 **Modification of bearing capacity equations for water table**

3.6 The General Bearing Capacity Equation (Meyerhof Equation)

Terzagi's equations shortcomings:

- They don't deal with rectangular foundations (0<B/L<1).
- The equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation (as mentioned previously).
- The inclination of the load on the foundation is not considered (if exist).on the foundation may be inclined.

To account for all these shortcomings, **Meyerhof (1963)** suggested the following form of the general bearing capacity equation:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \quad (3.15)$$

Where

c=Cohesion of the underlying soil

q=Effective stress at the level of the bottom of the foundation. γ=unit weight of the underlying soil

B=Width of footing (=diameter for a circular foundation).

N_c, N_q, N_γ=Bearing capacity factors

F_{cs}, F_{qs}, $F_{\gamma s}$=Shape factors

F_{cd}, F_{qd}, $F_{\gamma d}$=Depth factors

F_{ci}, F_{qi}, $F_{\gamma i}$=Inclination factors

Notes:

1. This equation is valid for both general and local shear failure.
2. This equation is similar to original equation for ultimate bearing capacity (Terzaghi's equation) which derived for continuous foundation . The shape, depth, and load inclination factors are empirical factors based on experimental data.

Bearing Capacity Factors:

The basic nature of the failure surface in soil suggested by Terzaghi now appears to have been borne out by laboratory and field studies of bearing capacity (Vesic, 1973). However, the angle **α** shown in **Fig. 3.6** is closer to $45+\phi'/2$ than to ϕ'. If this change is accepted, the values of N_c, N_q, and N_γ for a given soil friction angle will also change from those given in **Table 3.1**. With $\alpha = 45+\phi'/2$, it can be shown that

$$N_q = \tan^2\left(45 + \frac{\phi'}{2}\right)e^{\pi \tan \phi'} \qquad (3.16)$$

and

$$N_c = (N_q - 1)\cot \phi' \qquad (3.17)$$

Equation (3.17) for N_c was originally derived by Prandtl (1921), and Eq. (316) for N_q was presented by Reissner (1924). Caquot and Kerisel (1953) and Vesic (1973) gave the relation for N_γ as

$$N_\gamma = 2(N_q + 1)\tan \phi' \qquad (3.18)$$

Table 3.3 shows the variation of the preceding bearing capacity factors with soil friction angles.

Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Shape, Depth, and Inclination Factors

Commonly used shape, depth, and inclination factors are given in Table 3.4.

Table 3.4 Shape, Depth and Inclination Factors [DeBeer (1970); Hansen (1970); Meyerhof (1963); Meyerhof and Hanna (1981)]

Factor	Relationship	Reference
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$	DeBeer (1970)
	$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$	
	$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$	
Depth	$\frac{D_f}{B} \le 1$	Hansen (1970)
	For $\phi = 0$:	
	$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$	
	$F_{qd} = 1$	
	$F_{\gamma d} = 1$	
	For $\phi' > 0$:	
	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$	
	$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$	
	$F_{\gamma d} = 1$	
	$\frac{D_f}{B} > 1$	
	For $\phi = 0$:	
	$F_{cd} = 1 + 0.4 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$	
	$F_{qd} = 1$	
	$F_{\gamma d} = 1$	
	For $\phi' > 0$:	
	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$	
	$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$	
	$F_{\gamma d} = 1$	
Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$	Meyerhof (1963); Hanna and Meyerhof (1981)
	$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$	
	β = inclination of the load on the foundation with respect to the vertical	

3.7 Eccentrically Loaded Foundation

If the load applied on the foundation is in the center of the foundation without eccentricity, the bearing capacity of the soil will be uniform at any point under the foundation (as shown in **Fig. 3.9**) because there is no any moments on the foundation, and the general equation for stress under the foundation is:

Stress= $Q/A \pm M_x\, y / I_x \pm M_y\, X / I_y$

In this case, the load is in the center of the foundation and there are no moments so,

Stress= Q/A (uniform at any point below the foundation)

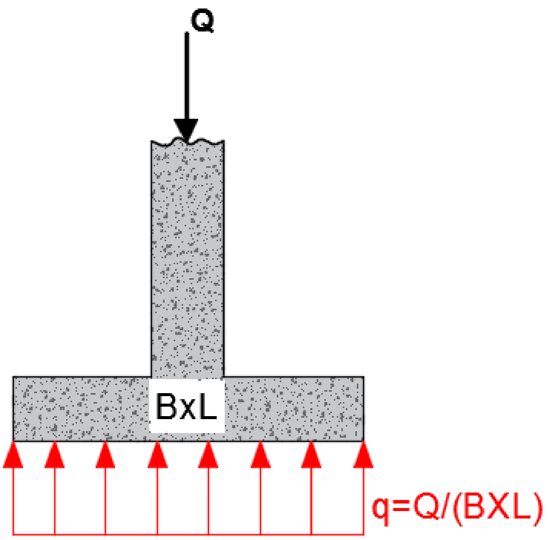


Fig. 3.9 Non-Eccentricity loaded foundations

However, in several cases, as with the base of a retaining wall or neighbor footing, the loads does not exist in the center, so foundations are subjected to moments in addition to the vertical load (as shown **Fig. 3.10**). In such cases, the distribution of pressure by the foundation on the soil is not uniform because there is a moment applied on the foundation and the stress under the foundation will be calculated from the general relation:

Stress= Stress= $Q/A \pm M_x\, y / I_x \pm M_y\, X / I_y$ (in case of two way eccentricity)

But, in this section we deal with (one way eccentricity), the equation will be:

Stress= $Q/A \pm M\, c / I$

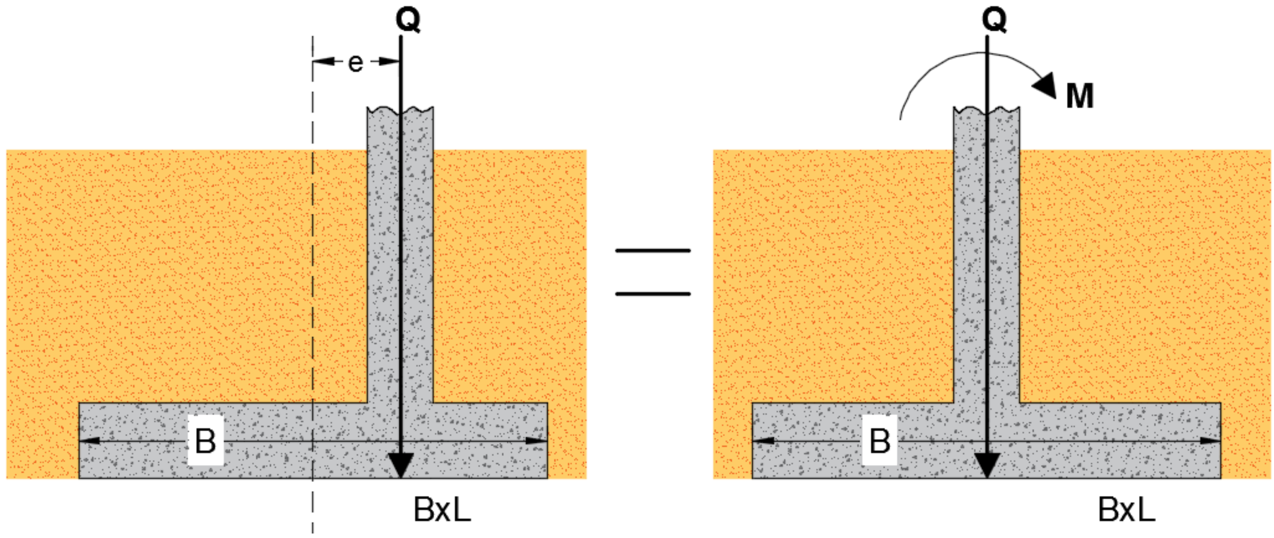
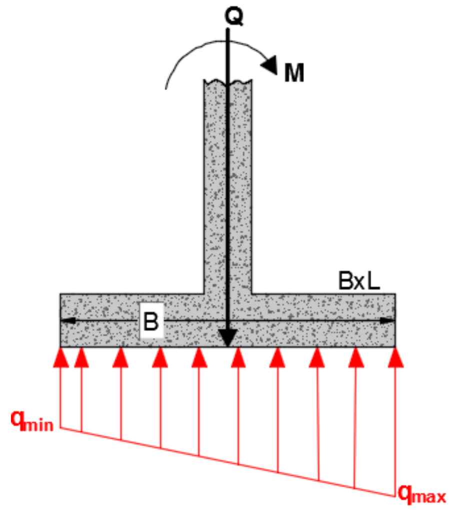


Fig. 3.10 Eccentricity loaded foundation

Since the pressure under the foundation is not uniform, there are maximum and minimum pressures (under the two edges of the foundation) and we concerned about calculating these two pressures.

General equation for calculating maximum and minimum pressure:



Assume the eccentricity is in direction of (B)

Stress=q= Q/A ± M c / I

A= B × L

M= Q × e

c= B/2 (maximum distance from the center)

I= (B^3×L) /12 (I is about the axis that resists the moment)

Substitute in the equation, the equation will be:

q= Q/(B×L) ± (Q×e×B)/((2B^3×L)/12)→q=Q/(B×L)± (6eQ/(B^2L)→

$q = \frac{\ }{\times L}\left(1 \quad \frac{\ }{\ }\right)$ **General Equation**

Now, there are three cases for calculating maximum and minimum pressures according to the values of (e and B/6) **to maintain minimum pressure always**≥0

Case I. (For e<B/6):

$$= \frac{\ }{\times}\left(1 + \frac{\ }{\ }\right)$$

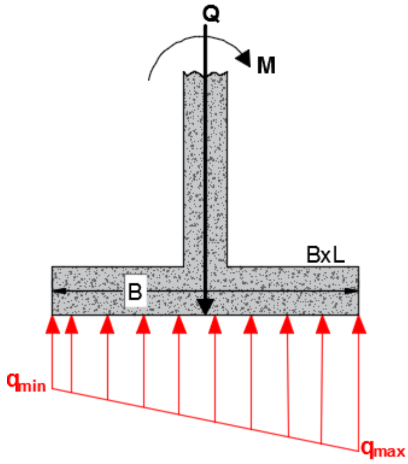
$$= \frac{\quad}{\times} (1 \quad \frac{\quad}{\quad})$$

Note that when e< B/6 the value of q_{min} will be positive (i.e. compression).

If eccentricity in (L) direction: (For e<L/6):

$$= \frac{\quad}{\times} (1 + \frac{\quad}{\quad})$$

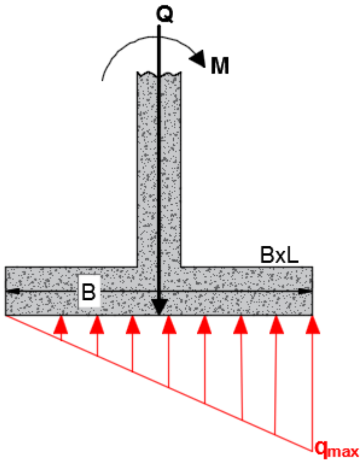
$$= \frac{\quad}{\times} (1 \quad \frac{\quad}{\quad})$$



Case II. (For e=B/6):

$$= \frac{\quad}{\times} (1 + \frac{\quad}{\quad})$$

$$= \frac{\quad}{\times} (1 \quad 1) = 0$$

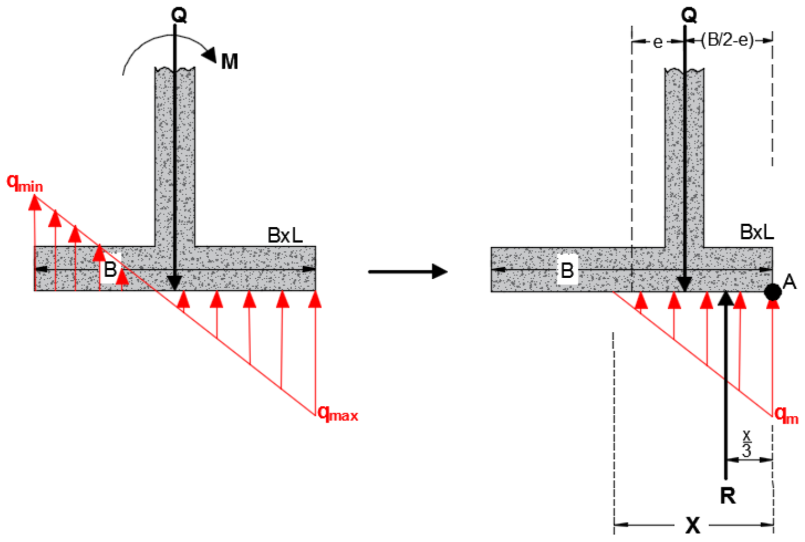


Case III. (For e >B/6):

For $e > B/6$, q_{min} will be negative, which means that tension will develop. Because soil cannot take any tension, there will then be a separation between the foundation and the soil underlying it. The nature of the pressure distribution on the soil will be as shown in Figure below. The value of q_{max} is then

$$q_{max} = \frac{4Q}{3L(B - 2e)}$$

The exact distribution of pressure is difficult to estimate.



3.8 Ultimate Bearing Capacity under Eccentric Loading - One-Way Eccentricity

Effective Area Method (Meyerhoff, 1953)

In 1953, Meyerhof proposed a theory that is generally referred to as the *effective area method*.

The following is a step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

Step 1. Determine the effective dimensions of the foundation (**Fig. 3.10**):

B' = effective width = $B - 2e$

L' = effective length = L

Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to L - 2e. The value of B' would equal B. The smaller of the two dimensions (i.e., L' and B') is the effective width of the foundation.

Step 2. Use Eq. (3.15) for the ultimate bearing capacity:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B'\gamma N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

To evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$, use the relationships given in Table 3.4 with *effective* *length* and *effective width* dimensions instead of L and B, respectively. To determine F_{cd}, F_{qd}, and $F_{\gamma d}$, use the relationships given in Table 3.4. However, do not replace B with B'.

Step 3. The total ultimate load that the foundation can sustain is

$$Q_{ult} = q'_u \overbrace{(B')(L')}^{A'} \quad (3.19)$$

where A'= effective area.

Step 4. The factor of safety against bearing capacity failure is

$$FS = \frac{Q_{ult}}{Q}$$

Step 5. Check the factor of safety against q_{max} , or FS= q'_u / q_{max}.

It is important to note that q'_u is the ultimate bearing capacity of a foundation of width $B'= B - 2e$ with a centric load (**Fig. 3.10a**). However, the actual distribution of soil reaction at ultimate load will be of the type shown in **Fig. 3.10b**. In Figure **3.10b**, $q_{u(e)}$ is the average load per unit area of the foundation. Thus,

$q_{u(e)}$ = [q'_u(B-2e)] / B

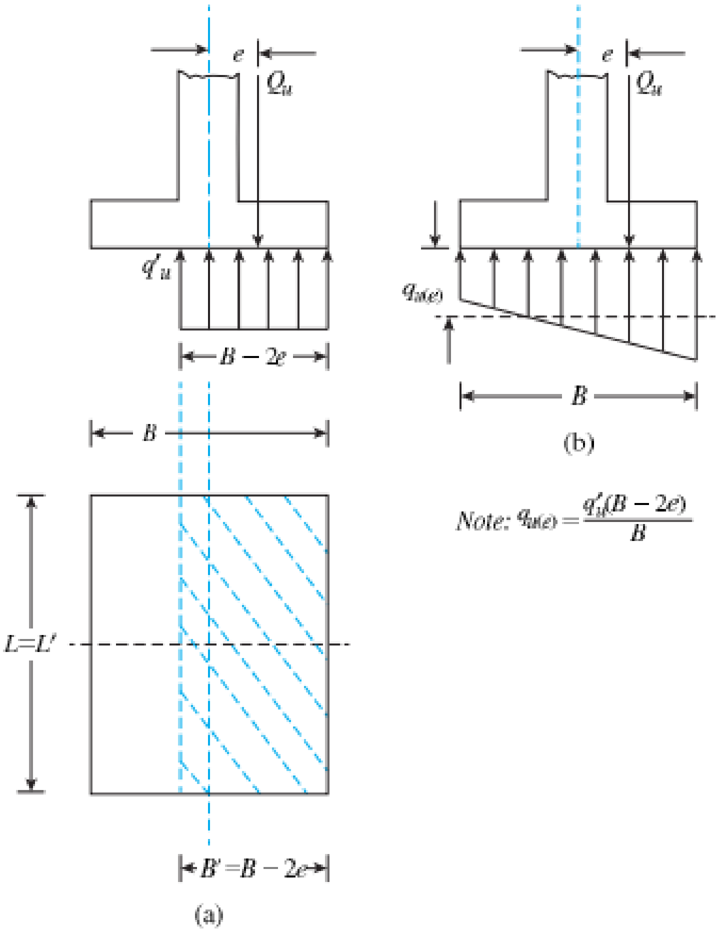


Fig. 3.10 Definition of q'_u and $q_{u(e)}$

3.9 Two- Way Eccentricity:

Consider a situation in which a foundation is subjected to a vertical ultimate load Q_{ult} and a moment M, as shown in Figures 3.12a and b. For this case, the components of the moment M about the x- and y-axes can be determined as M_x and M_y, respectively. (Figure 3.12c.). This condition is equivalent to a load Q_u placed eccentrically on the foundation with $x = e_B$ and $y = e_L$ (Figure 3.12d). Note that

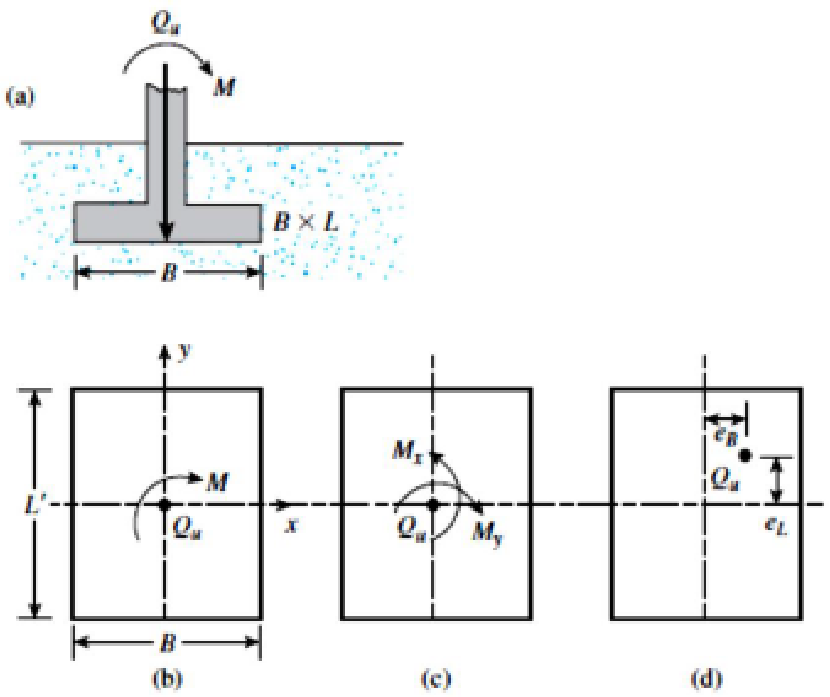


Figure 3.12 Analysis of foundation with two-way eccentricity

$$e_B = \frac{M_y}{Q_u}$$

and

$$e_L = \frac{M_x}{Q_u}$$

If Q_u is needed, it can be obtained from Equation that is,

$$Q_u = q'_u A'$$

where,

$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

and

$$A' = \text{effective area} = B'L'$$

As before, to evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$ (Table 4.3), we use the effective length L' and effective width B' instead of L and B, respectively. To calculate F_{cd}, F_{qd}, and $F_{\gamma d}$, we do not replace B with B'. In determining the effective area A', effective width B', and effective length L',

3.10 Bearing Capacity of Layered Soils: Stronger Soil Underlain by Weaker Soil (c'-ϕ' soil)

- The bearing capacity equations presented in preceding sections involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth.
- The cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis. However, in practice, layered soil profiles are often encountered. In such instances, the failure surface at ultimate load may extend through two or more soil layers, and a determination of the ultimate bearing capacity in layered soils can be made in only a limited number of cases.
- This section features the procedure for estimating the bearing capacity for layered soils proposed by Meyerhof and Hanna (1978) and Meyerhof (1974) in c'-ϕ' soil.
- .Figure 3.11 shows a shallow, continuous foundation supported by a stronger soil layer, underlain by a weaker soil that extends to a great depth.
- For the two soil layers, the physical parameters are as follows:

	Soil properties		
Layer	**Unit weight**	**Friction angle**	**Cohesion**
Top	γ_1	ϕ'_1	c'_1
Bottom	γ_2	ϕ'_2	c'_2

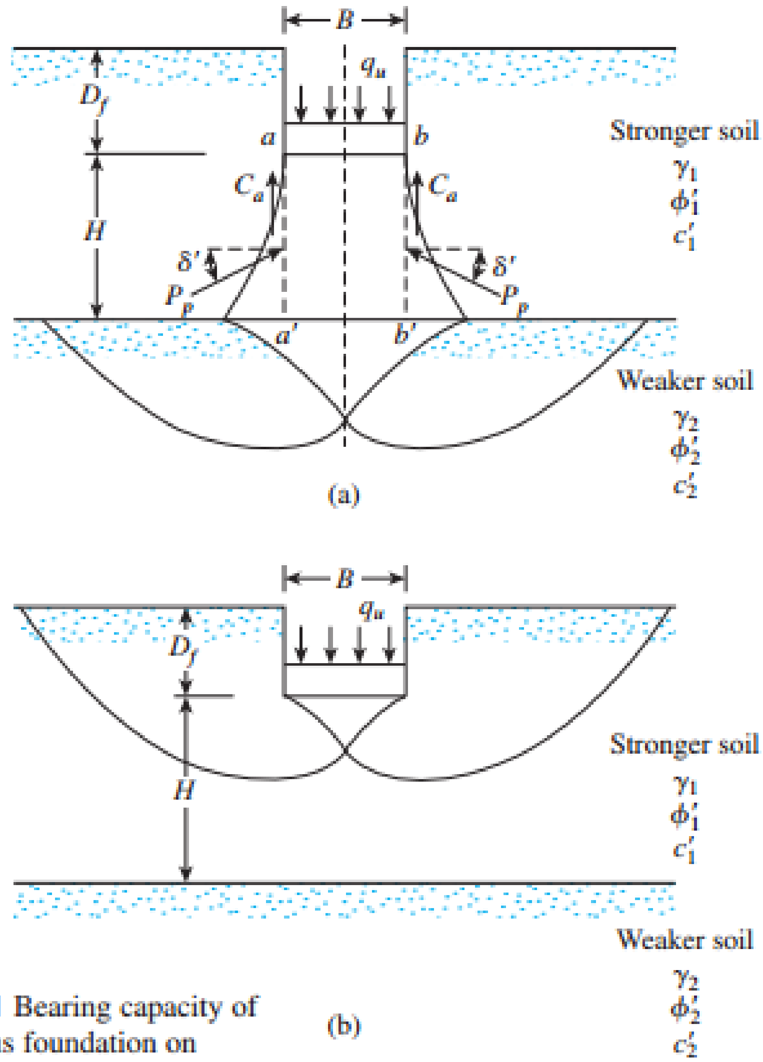


Figure 3.11 Bearing capacity of a continuous foundation on layered soil

- At ultimate load per unit area (q_u) , the failure surface in soil will be as shown in the figure.
- If the depth H is relatively small compared with the foundation width B, a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer. This is shown in Figure 3.11a.
- If the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity. This is shown in Figure 3.11b.
- The ultimate bearing capacity for this problem, as shown in Figure 3.11a, can be given as

$$q_u = q_b + \frac{2(C_a + P_p \sin \delta')}{B} - \gamma_1 H$$

where

B = width of the foundation
C_a = adhesive force
P_p = passive force per unit length of the faces aa' and bb'
q_b = bearing capacity of the bottom soil layer
δ' = inclination of the passive force P_p with the horizontal

Note that, in the above Equation,

$$C_a = c'_a H$$

where c'_a = adhesion.

Equation above can be simplified to the form

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_{pH} \tan \delta'}{B} - \gamma_1 H$$

where K_{pH} = horizontal component of passive earth pressure coefficient.

However, let

$$K_{pH} \tan \delta' = K_s \tan \phi'_1$$

where K_s = punching shear coefficient. Then,

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan \phi'_1}{B} - \gamma_1 H$$

The punching shear coefficient, K_s, is a function of q_2/q_1 and ϕ'_1, or, specifically,

$$K_s = f\left(\frac{q_2}{q_1}, \phi'_1\right)$$

Note that q_1 and q_2 are the ultimate bearing capacities of a continuous foundation of width B under vertical load on the surfaces of homogeneous thick beds of upper and lower soil, or

$$q_1 = c'_1 N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

and

$$q_2 = c'_2 N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

where

$N_{c(1)}, N_{\gamma(1)}$ = bearing capacity factors for friction angle ϕ'_1 (Table 3.3)
$N_{c(2)}, N_{\gamma(2)}$ = bearing capacity factors for friction angle ϕ'_2 (Table 3.3)

Observe that, for the top layer to be a stronger soil, q_2/q_1 should be less than unity.

The variation of K_s with q_2/q_1 and ϕ'_1 is shown in Figure 3.12. The variation of c'_a/c'_1 with q_2/q_1 is shown in Figure 3.13. If the height H is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil layer (Figure3.11b). For this case,

$$q_u = q_t = c'_1 N_{c(1)} + q N_{q(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}.$$

where $N_{c(1)}$, $N_{q(1)}$, and $N_{\gamma(1)}$ = bearing capacity factors for $\phi' = \phi'_1$ (Table 3.3) and $q = \gamma_1 D_f$.

Combining Equations above yields

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan \phi'_1}{B} - \gamma_1 H \leq q_t$$

For rectangular foundations, the preceding equation can be extended to the form

$$q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c'_a H}{B}\right) + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan \phi'_1}{B}\right) - \gamma_1 H \leq q_t$$

where

$$q_b = c'_2 N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H)\, N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}$$

and

$$q_t = c'_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer

$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer

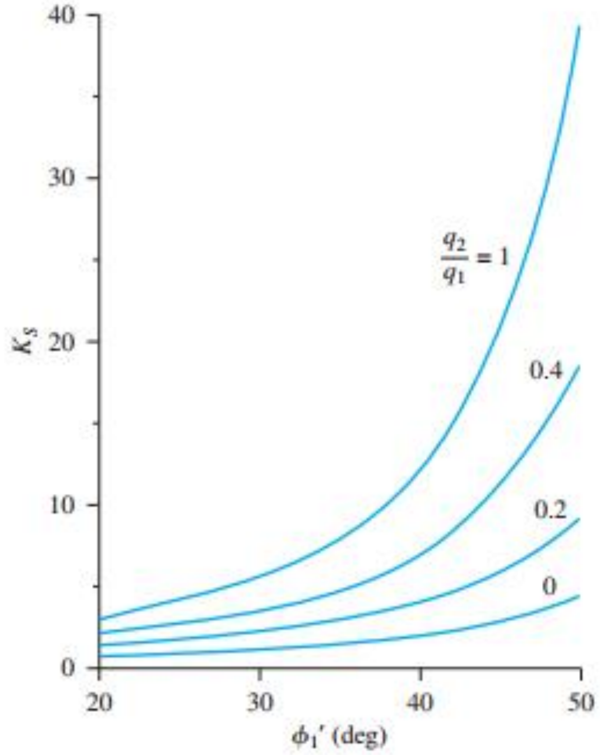


Figure 3.12 Meyerhof and Hanna's punching shear coefficient K_s

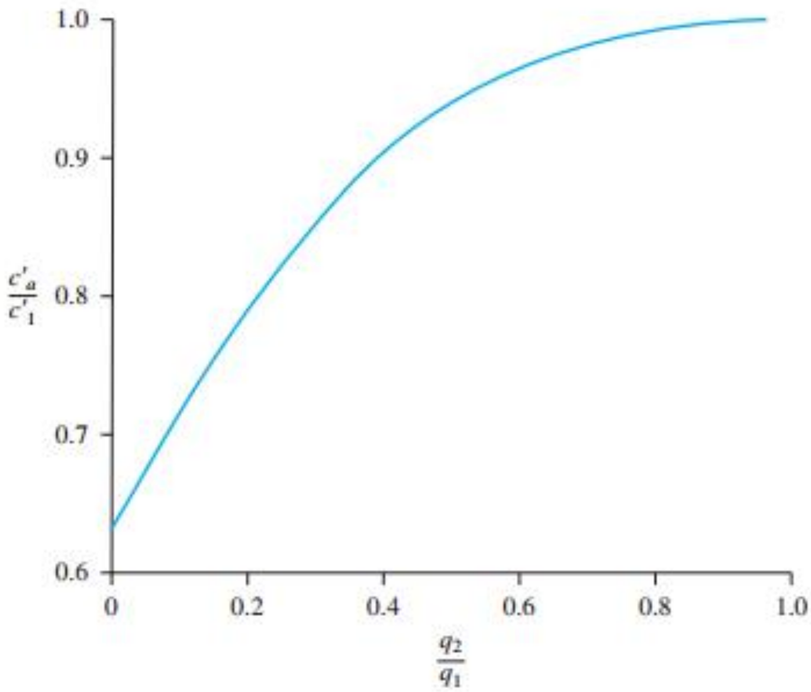


Figure 3.13 Variation of c'_a/c'_1 with q_2/q_1 based on the theory of Meyerhof and Hanna (1978)

Special Cases

1. *Top layer is strong sand and bottom layer is saturated soft clay* ($\phi_2 = 0$).

$$q_b = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1(D_f + H)$$

and

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

Hence,

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} + \gamma_1 D_f \leq \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

For a determination of K_s from Figure **3.12**

$$\frac{q_2}{q_1} = \frac{c_{u(2)} N_{c(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{5.14c_{u(2)}}{0.5\gamma_1 B N_{\gamma(1)}}$$

2. *Top layer is stronger sand and bottom layer is weaker sand* ($c'_1 = 0$, $c'_2 = 0$). The ultimate bearing capacity can be given as

$$q_u = \left[\gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}\right] + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} - \gamma_1 H \leq q_t$$

where

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2}\gamma_2 B N_{\gamma(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}}$$

3. *Top layer is stronger saturated clay ($\phi_1 = 0$) and bottom layer is weaker saturated clay ($\phi_2 = 0$).* The ultimate bearing capacity can be given as

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leq q_t$$

where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}}$$

3.10 Foundations on Rock

- On some occasions, shallow foundations may have to be built on rocks, as shown in Figure 3.14.
- For estimation of the ultimate bearing capacity of shallow foundations on rock, we may use Terzaghi's bearing capacity equations with the bearing capacity factors given here (Stagg and Zienkiewicz, 1968; Bowles, 1996):

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right)$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right)$$

$$N_\gamma = N_q + 1$$

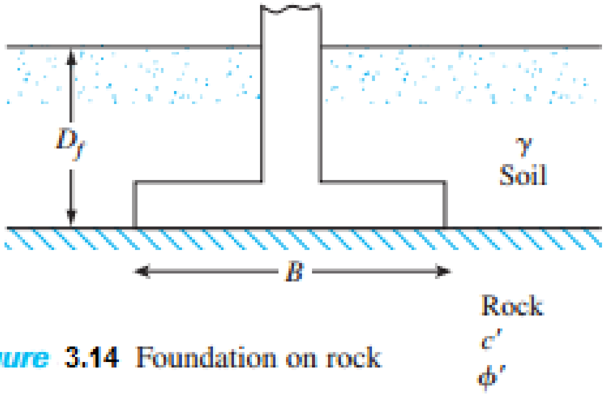


Figure 3.14 Foundation on rock

For rocks, the magnitude of the cohesion intercept, c', can be expressed as

$$q_{uc} = 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

where

q_{uc} = unconfined compression strength of rock
ϕ' = angle of friction

- The unconfined compression strength and the friction angle of rocks can vary widely. Table 3.5 gives a general range of q_{uc} for various types of rocks. It is important to keep in mind that the magnitude of q_{uc} and ϕ' (hence c') reported from laboratory tests are for intact rock specimens. It does not account for the effect of discontinuities.
- To account for discontinuities, Bowles (1996) suggested that the ultimate bearing capacity q_u should be modified as

$$q_{u(\text{modified})} = q_u(\text{RQD})^2$$

where RQD = rock quality designation

In any case, the upper limit of the allowable bearing capacity should not exceed f_c' (28-day compressive strength of concrete).

Table 3.5 Range of the Unconfined Compression Strength of Various Types of Rocks

Rock type	q_{uc} MN/m^2	q_{uc} kip/in^2	ϕ' (deg)
Granite	65–250	9.5–36	45–55
Limestone	30–150	4–22	35–45
Sandstone	25–130	3.5–19	30–45
Shale	5–40	0.75–6	15–30