OHM'S LAW

One of the most important electrical characteristics of a solid material is the ease with which it transmits an electric current. **Ohm's law** relates the current *I*-or time rate of charge passage-to the applied voltage *V* as follows:

$$V = IR$$

Where *R* is the resistance of the material through which the current is passing. The units for *V*, *I*, and *R* are, respectively, volts (J/C), amperes (C/s), and ohms (V/A). The value of *R* is influenced by specimen configuration, and for many materials is independent of current. The **resistivity** is independent of specimen geometry but related to *R* through the expression

$$\rho = \frac{RA}{l}$$

Where *l* is the distance between the two points at which the voltage is measured, and *A* is the cross-sectional area perpendicular to the direction of the current. The units for ρ are ohm-meters (Ω -m). From the expression for Ohm's law resistivity can expressed as

$$\rho = \frac{VA}{Il}$$

ELECTRICAL CONDUCTIVITY

Sometimes, **electrical conductivity σ** is used to specify the electrical character of a material. It is simply the reciprocal of the resistivity, or

$$\sigma = \frac{1}{\rho}$$

and is indicative of the ease with which a material is capable of conducting an electric current. The units for σ are reciprocal ohm-meters $[(\Omega\text{-m})^{-1}$ or mho/m]. The following discussions on electrical properties use both resistivity and conductivity. In addition to above Equation, Ohm's law may be expressed as

$$J = \sigma \xi$$

In which *J* is the current density, the current per unit of specimen area *I*/*A*, and ξ is the electric field intensity, or the voltage difference between two points divided by the distance separating them; that is,

$$\xi = \frac{V}{l}$$

Solid materials exhibit an amazing range of electrical conductivities, extending over 27 orders of magnitude; probably no other physical property experiences this breadth of variation. In fact, one way of classifying solid materials is according to the ease with which they conduct an electric current; within this classification scheme there are three groupings: ***conductors***, ***semiconductors***, and ***insulators***. **Metals** are good conductors, typically having conductivities on the order of $[10^7]$ $(\Omega\text{-m})^{-1}$ at the other extreme are materials with very low conductivities,

ranging between [10^{-10} and 10^{-20}] $(\Omega\text{-m})^{-1}$ these are electrical **insulators.** Materials with intermediate conductivities, generally from [$10^{-6}$to 10^{4}] $(\Omega\text{-m})^{-1}$ are termed **semiconductors.**

ENERGY BAND STRUCTURES IN SOLIDS

In all conductors, semiconductors, and many insulating materials, only electronic conduction exists, and the magnitude of the electrical conductivity is strongly dependent on the number of electrons available to participate in the conduction process. However, not all electrons in every atom will accelerate in the presence of an electric field. The number of electrons available for electrical conduction in a particular material is related to the arrangement of electron states or levels with respect to energy, and then the manner in which these states are occupied by electrons. A thorough exploration of these topics is complicated and involves principles of quantum mechanics that are beyond the scope of this lecture; the ensuing development omits some concepts and simplifies others. Concepts relating to electron energy states, their occupancy, and the resulting electron configuration for isolated atoms are known. By way of review, for each individual atom there exist discrete energy levels that may be occupied by electrons, arranged into shells and subshells. Shells are designated by integers (1, 2, 3, etc.), and subshells by letters ***(s, p, d,* and *f)***. For each of *s*, *p*, *d*, and *f* subshells, there exist, respectively, one, three, five, and seven states. The electrons in most atoms fill only the states having the lowest energies, two electrons of opposite spin per state, in accordance with the Pauli Exclusion Principle. The electron configuration of an isolated atom represents the arrangement of the electrons within the allowed states. Let us now make an extrapolation of some of these concepts to solid materials. A solid may be thought of as consisting of a large number, say, *N*, of atoms initially separated from one another, which are subsequently brought together and bonded to form the ordered atomic arrangement found in the crystalline material. At relatively large separation distances, each atom is independent of all the others and will have the atomic energy levels and electron configuration as if isolated. However, as the atoms come within close proximity of one another, electrons are acted upon, or perturbed, by the electrons and nuclei of adjacent atoms. This influence is such that each distinct atomic state may split into a series of closely spaced electron states in the solid, to form what is termed an **electron energy band.** The extent of splitting depends on interatomic separation (Figure 1) and begins with the outermost electron shells, since they are the first to be perturbed as the atoms coalesce. Within each band, the energy states are discrete, yet the difference between adjacent states is exceedingly small. At the equilibrium spacing, band formation may not occur for the electron subshells nearest the nucleus, as illustrated in Figure 2*b*. Furthermore, gaps may exist between adjacent bands, as also indicated in the figure; normally, energies lying within these band gaps are not available for electron occupancy.

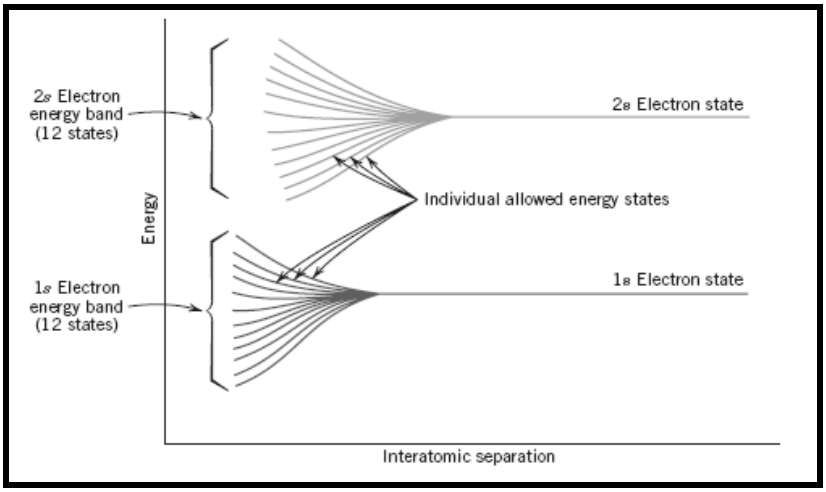

Figure 1

The conventional way of representing electron band structures in solids is shown in Figure 2*a*. The number of states within each band will equal the total of all states contributed by the *N* atoms. For example, an *s* band will consist of *N* states, and a *p* band of 3*N* states. With regard to occupancy, each energy state may accommodate two electrons, which must have oppositely directed spins. Furthermore, bands will contain the electrons that resided in the corresponding levels of the isolated atoms; for example, a 4*s* energy band in the solid will contain those isolated atom's 4*s* electrons. Of course, there will be empty bands and, possibly, bands that are only partially filled. The electrical properties of a solid material are a consequence of its electron band structure—that is, the arrangement of the outermost electron bands and the way in which they are filled with electrons. Four different types of band structures are possible at 0 K.

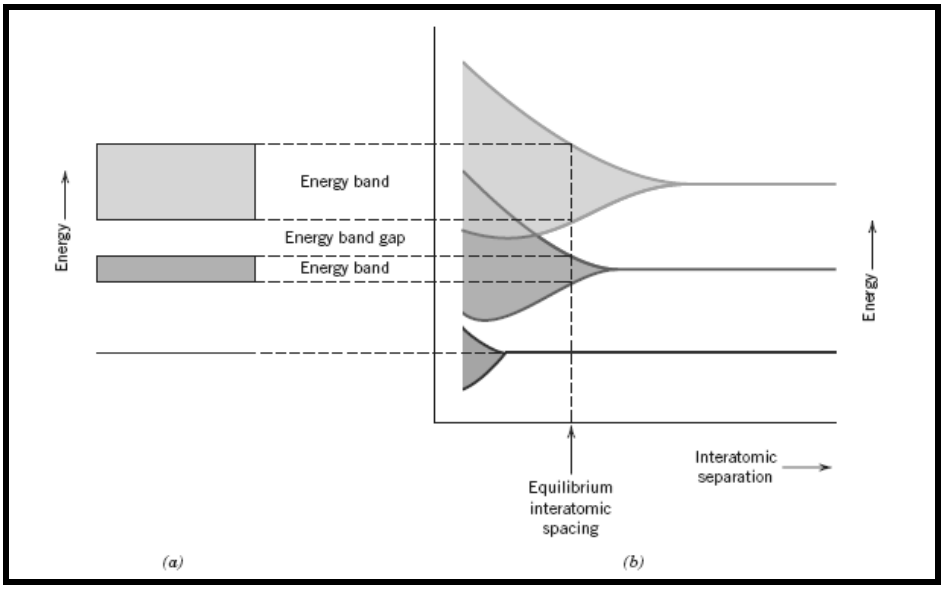

Figure 2

In the first (Figure 3*a*), one outermost band is only partially filled with electrons. The energy corresponding to the highest filled state at 0 K is called the **Fermi energy** as indicated. This energy band structure is typified by some metals, in particular those that have a single *s* valence electron (e.g., copper). Each copper atom has one 4*s* electron; however, for a solid comprised of *N* atoms, the 4*s* band is capable of accommodating 2*N* electrons. Thus only half the available

electron positions within this 4*s* band are filled. For the second band structure, also found in metals (Figure 3*b*), there is an overlap of an empty band and a filled band. Magnesium has this band structure. Each isolated Mg atom has two 3*s* electrons. However, when a solid is formed, the 3*s* and 3*p* bands overlap. In this instance and at 0 K, the Fermi energy is taken as that energy, below which, for *N* atoms, *N* states are filled, two electrons per state. The final two band structures are similar; one band (the **valence band**) that is completely filled with electrons is separated from an empty **conduction band,** and an **energy band gap** lies between them.

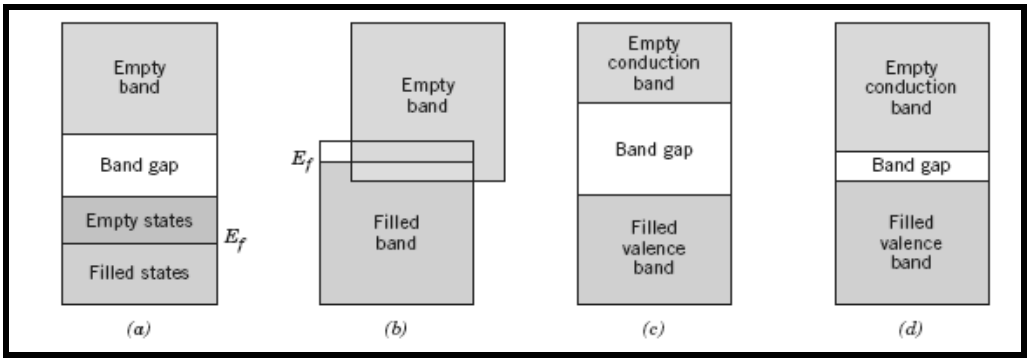

Figure 3

For very pure materials, electrons may not have energies within this gap. The difference between the two band structures lies in the magnitude of the energy gap; for materials that are insulators, the band gap is relatively wide (Figure 3*c*), whereas for semiconductors it is narrow (Figure3*d*). The Fermi energy for these two band structures lies within the band gap—near its center.

CONDUCTION IN TERMS OF BAND AND ATOMIC BONDING MODELS

At this point in the discussion, it is vital that another concept be understood— namely, that only electrons with energies greater than the Fermi energy may be acted on and accelerated in the presence of an electric field. These are the electrons that participate in the conduction process, which are termed **free electrons.** Another charged electronic entity called a **hole** is found in semiconductors and insulators. Holes have energies less than E_f and also participate in electronic conduction. As the ensuing discussion reveals, the electrical conductivity is a direct function of the numbers of free electrons and holes. In addition, the distinction between conductors and nonconductors (insulators and semiconductors) lies in the numbers of these free electron and hole charge carriers.

Metals

For an electron to become free, it must be excited or promoted into one of the empty and available energy states above E_f For metals having either of the band structures shown in Figures 3*a* and 3*b*, there are vacant energy states adjacent to the highest filled state at E_f .Thus, very little energy is required to promote electrons into the low-lying empty states, as shown in Figure 4. Generally, the energy provided by an electric field is sufficient to excite large numbers of electrons into these conducting states. For the metallic bonding model, it was assumed that all the valence electrons have freedom of motion and form an "electron gas," which is uniformly distributed throughout the lattice of ion cores. Although these electrons are not locally bound to any particular atom, nevertheless, they must experience some excitation to become conducting electrons that are truly free. Thus, although only a fraction is excited, this

still gives rise to a relatively large number of free electrons and, consequently, a high conductivity.

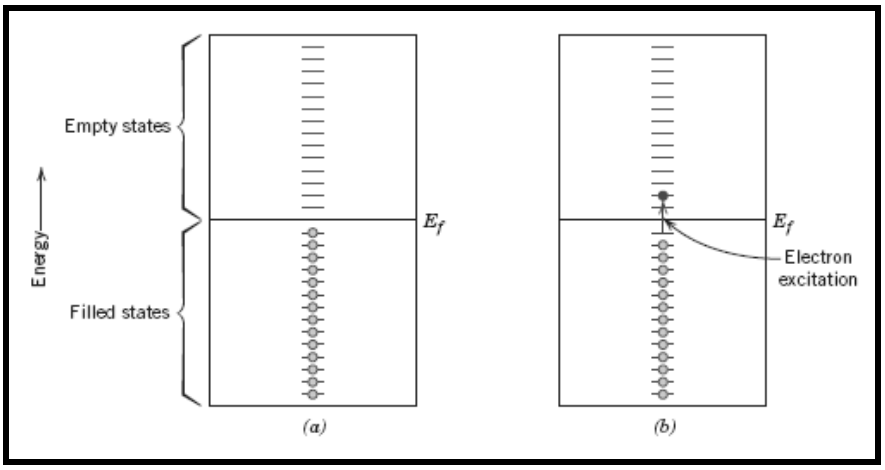

Figure 4

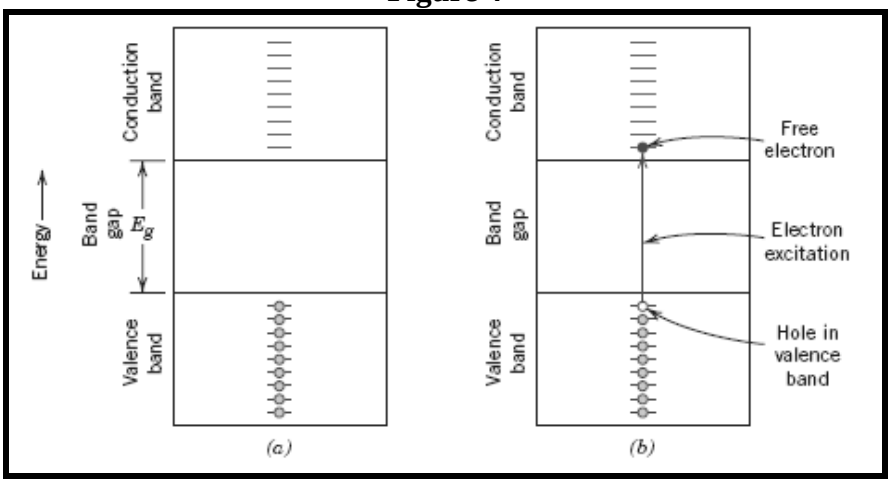

Figure 5

Insulators and Semiconductors

For insulators and semiconductors, empty states adjacent to the top of the filled valence band are not available. To become free, therefore, electrons must be promoted across the energy band gap and into empty states at the bottom of the conduction band. This is possible only by supplying to an electron the difference in energy between these two states, which is approximately equal to the band gap energy E_g .This excitation process is demonstrated in Figure 5. For many materials this band gap is several electron volts wide. Most often the excitation energy is from a nonelectrical source such as heat or light, usually the former. The number of electrons excited thermally (by heat energy) into the conduction band depends on the energy band gap width as well as temperature. At a given temperature, the larger the E_g the lower is the probability that a valence electron will be promoted into an energy state within the conduction band; this results in fewer conduction electrons. In other words, the larger the band gap, the lower is the electrical conductivity at a given temperature. Thus, the distinction between semiconductors and insulators lies in the width of the band gap; for semiconductors it is narrow, whereas for insulating materials it is relatively wide. Increasing the temperature of either a semiconductor or an insulator results in an increase in the thermal energy that is available for electron excitation. Thus, more electrons are promoted into the conduction band, which gives rise to an enhanced conductivity. The conductivity of insulators and semiconductors may also be viewed from the perspective of atomic bonding models. For

electrically insulating materials, interatomic bonding is ionic or strongly covalent. Thus, the valence electrons are tightly bound to or shared with the individual atoms. In other words, these electrons are highly localized and are not in any sense free to wander throughout the crystal. The bonding in semiconductors is covalent (or predominantly covalent) and relatively weak, which means that the valence electrons are not as strongly bound to the atoms. Consequently, these electrons are more easily removed by thermal excitation than they are for insulators.

ELECTRON MOBILITY

When an electric field is applied, a force is brought to bear on the free electrons; as a consequence, they all experience acceleration in a direction opposite to that of the field, by virtue of their negative charge. According to quantum mechanics, there is no interaction between an accelerating electron and atoms in a perfect crystal lattice. Under such circumstances all the free electrons should accelerate as long as the electric field is applied, which would give rise to an electric current that is continuously increasing with time. However, we know that a current reaches a constant value the instant that a field is applied; indicating that there exist what might be termed "frictional forces," which counter this acceleration from the external field. These frictional forces result from the scattering of electrons by imperfections in the crystal lattice, including impurity atoms, vacancies, interstitial atoms, dislocations, and even the thermal vibrations of the atoms themselves. Each scattering event causes an electron to lose kinetic energy and to change its direction of motion, as represented schematically in Figure 6. There is, however, some net electron motion in the direction opposite to the field, and this flow of charge is the electric current. The scattering phenomenon is manifested as a resistance to the passage of an electric current. Several parameters are used to describe the extent of this scattering; these include the ***drift velocity*** and the **mobility** of an electron. The drift velocity v_d represents the average electron velocity in the direction of the force imposed by the applied field. It is directly proportional to the electric field as follows:

$$v_d = \mu_e \xi$$

The constant of proportionality μ_e is called the electron mobility, which is an indication of the frequency of scattering events; its units are square meters per volt-second ($m^2/V.s$). The conductivity σ of most materials may be expressed as

$$\sigma = n|e|\mu_e$$

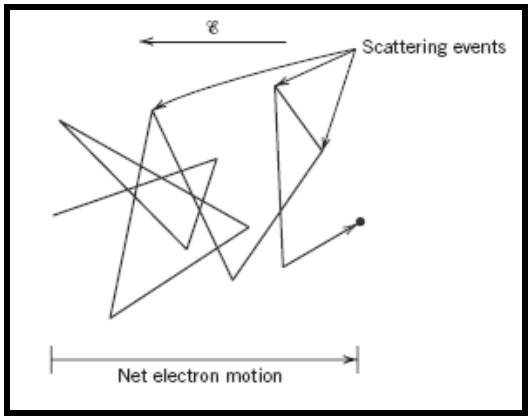

Figure 6

Where n is the number of free or conducting electrons per unit volume (e.g., per cubic meter), and $|e|$ is the absolute magnitude of the electrical charge on an electron (1.6×10^{-19} C) Thus, the electrical conductivity is proportional to both the number of free electrons and the electron mobility. The drift mobility is determined by the mean scattering time τ of the conduction electrons and is given by

$$\mu = e\tau / m_e$$

Where m_e is the mass of the electron (effective mass of a conduction electron in the crystal).