What is a Composite Material

- The composite material consists of two or more physically and/or chemically distinct phases, suitably arranged or distributed. The main function of this combination is to exploit favourable properties of each.
- The **continuous** phase is referred to as the **matrix**, while the **distributed** phase is called the **reinforcement**. Three items determine the characteristics of a composite: the **reinforcement**, the **matrix**, and the **interface** between them.
- Composites may be **classified** on the basis of the **type of matrix** employed in them -- for example, **polymer** matrix composites (PMCs), **metal** matrix composites (MMCs), and **ceramic** matrix composites (CMCs).
- Composites can be also classified according to the **type of reinforcement** they employ (see figure 6.1)

1. Particle reinforced composites.
2. Short fiber, or whisker reinforced, composites.
3. Continuous fiber, or sheet reinforced, MMCs.
4. Laminate composite.

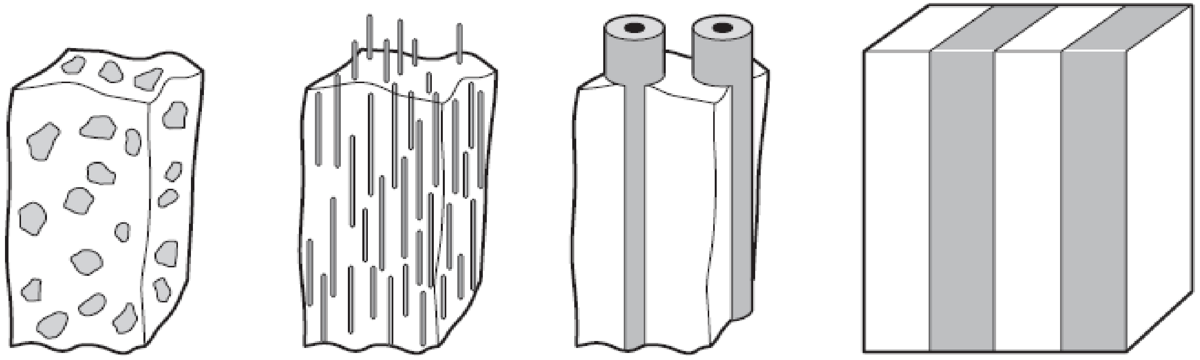


Figure 6.1

Composites used today are characterised by the following properties:

- A strengthening second phase is **embedded** in a continuous matrix.
- The strengthening second phase and the matrix are initially separate materials and are joined during processing – the second phase is thus not produced by internal processes like precipitation.
- The **particles** of the second phase have a size of **several micrometres** at least.
- The strengthening effect of the second phase is at least partially caused by **load transfer**.
- The volume fraction of the strengthening second phase is at least approximately **10%**.

Table 6.1: Some of the important ones.

Materials (Fibers)	Tensile Modulus (GPa)	Tensile Strength (GPa)	Density (g/cm^3)
Alumina	350–380	1.7	3.9
Boron	415	3.5	2.5–2.6
SiC	300–400	2.8	2.8
E-Glass	71	1.8–3.0	2.5
Carbon P100 (pitch-based)	725	2.2	2.15
Carbon M60J (PAN-based)	585	3.8	1.94
Aramid	125	3.5	1.45
Polyethylene	110	3	0.97

Microstructural Aspects and Importance of the Matrix

- The **differential thermal expansion** between the reinforcement and the metal matrix can introduce a **high dislocation density** in a metallic matrix, especially in the near-interface region of the matrix.
- In the case of a ceramic matrix composite, the **brittle matrix** can undergo **cracking** in response to such thermal stresses.
- Such is not commonly the case with the reinforcement, however; only in rare instances of very high temperature processing, as, for example, in the case of a CMC, can the reinforcement also undergo a **change in its microstructure.**
- These **microstructural changes** in the matrix, in turn, can **affect** the mechanical and physical **behavior** of the composite.
- The final matrix microstructure is a function of the **type**, **diameter**, and **distribution of the fiber**, as well as conventional solidification parameters.
- **Porosity** is a critical defect that is likely to be present in the matrix. The main sources of porosity are any gas evolution, shrinkage occurring upon solidification, and, in the case of CMCs, incomplete elimination of any binder material

- A **low** quantity of voids is necessary for **improved interlaminar shear strength**.
- Depending on the thermal expansion coefficients of the components, there is also the possibility of developing **hydrostatic tensile stresses** in the matrix that will counter the driving force for sintering.
- The following are some of the common **structural defects** in composites:
  - Matrix-rich (fiber-poor) regions.
  - Voids.
  - Microcracks (which may form due to thermal mismatch between the components, curing stresses, or the absorption of moisture during processing).
  - Debonded regions.
  - Delaminated regions.
  - Variations in fiber alignment.

Interfaces in Composites

- The interface region in a particular composite has a great deal of importance in determining the **ultimate properties** of the composite, essentially for **two reasons**:
  1. The interface occupies a very large area per unit volume in a composite.
  2. The reinforcement and the matrix form a system that is not in thermodynamic equilibrium.
- An interface can be defined as a **boundary surface** between two phases in which a discontinuity in one or more material parameters occurs.
- A very important factor in regard to reinforcement--matrix compatibility has to do with the **mismatch** between the **coefficient of thermal expansion** of the reinforcement and that of the matrix.
- This **thermal mismatch** can lead to thermal stresses large enough to cause **plastic** deformation in a soft metallic matrix and cracking in a brittle ceramic or polymeric matrix.
- **Plastic deformation** in the metallic matrix leads to the introduction of **defects** such as dislocations, vacancies, etc., in the matrix, especially in the region near the interface.
- The researchers observed that the **dislocation density** near the fiber was **much higher** than the dislocation density far away from the fiber.

Properties of Composites

In particular, we present expressions that allow us to predict the properties of composites in terms of the properties of their **components**, their **amounts**, and their **geometric distribution** in the composite.

Density and Heat Capacity

Density and heat capacity are two properties that may be predicted rather accurately by a **rule-of-mixtures** type of relationship, irrespective of the arrangement of one phase in another.

Density

The density of a composite is given by the **rule-of-mixture** equation

$$\boxed{\rho_c = \rho_m \rho f_m + \rho_f f_f} \qquad \textbf{(6.1)}$$

Where ρ designates the density and f represents volume fraction, with the subscripts c, m, and f denoting the composite, matrix, and reinforcement, respectively.

Heat Capacity

The heat capacity of a composite is given by the expression

$$C_c = (C_m \rho_m \rho f_m + C_f \rho_f f_f)/\rho_c \tag{6.2}$$

where C denotes heat capacity and the other symbols have the significance given in the equation for density.

Elastic Moduli

Young's modulus of a fiber composite is determined by the elastic properties of the constituent materials and also depends on the **loading direction** because the fibers are usually **stiffer** than the matrix, the modulus is **larger** in fiber direction than transversally to it.

Loading parallel to the fibers

If the fibres are oriented **parallel** to the loading direction, their arrangement is called parallel connection, (figure 6.2). Similar to springs connected in parallel, the **deformation** in the matrix (subscript 'm') and in the fibre (subscript 'f') must be the **same,** but the stress may differ:

$$\varepsilon_f = \varepsilon_m \qquad \sigma_f \neq \sigma_m \tag{6.3}$$

By inspecting the geometry and defining the volume fractions f_f and f_m, with $f_f + f_m = 1$, we find the (isostrain) rule of mixtures.

$$\sigma_c = \sigma_f f_f + \sigma_m (1 - f_f) \tag{6.4}$$

Using this together with equation (6.1), we find that Young's modulus of the composite is:

$$E_c = E_m f_m + E_f f_f = E_m \left[1 + f_f \left(\frac{E_f}{E_m} - 1 \right) \right] \quad (6.5) \tag{6.5}$$

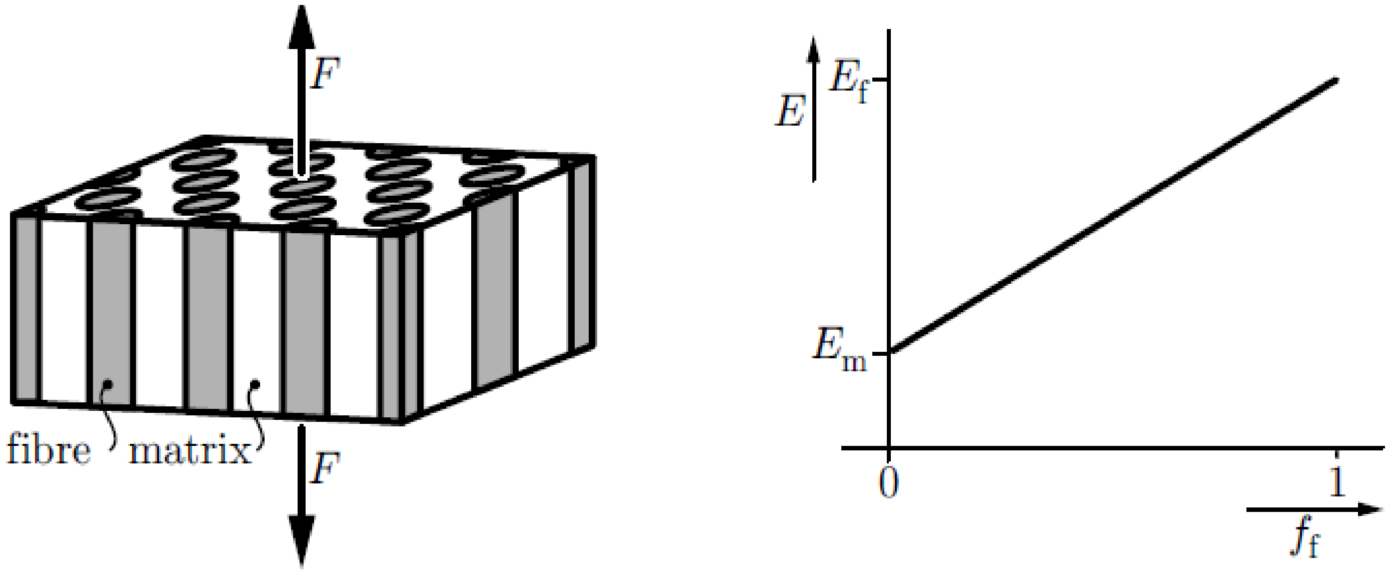


Figure 6.2

Loading Perpendicular to the fibers

- If the fibres are oriented perpendicular to the loading axis, this is called serial connection (figure 6.3).
- In the serial connection, the balance of forces must hold at each fibre-matrix interface:

$$\varepsilon_f = \varepsilon_m \qquad \sigma_f \neq \sigma_m \tag{6.5}$$

The following (isostress) rule of mixtures applies:

$$\varepsilon_c = \varepsilon_f f_f + \varepsilon_m (1 - f_f) \tag{6.6}$$

If we use Hooke's law, we find, after some rearrangement, Young's modulus

$$E_c = \frac{E_m}{1 + f_f\left(\frac{E_m}{E_f} - 1\right)} \tag{6.7}$$

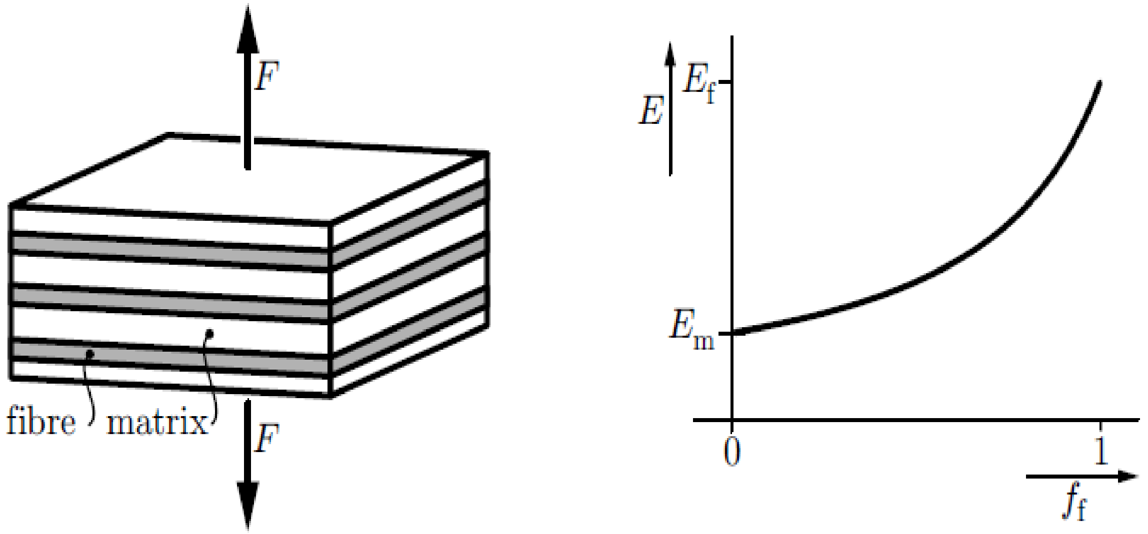


Figure 6.3

Compared to figure 6.1(b), it is apparent that the increase in stiffness perpendicular to the fibre direction is much smaller than in the parallel orientation.

Strength

- Unlike elastic moduli, it is **difficult to predict** the strength of a composite by a simple rule-of-mixture type of relationship, because strength is a **very structure-sensitive property**.
- For the equation 6.7, We can put certain restrictions on f_{f} in order to have real reinforcement. For this, a composite **must** have a **certain minimum-fiber** (continuous) volume fraction, f_{min}.
- The **ultimate strength** of the composite will be attained, ideally, at a strain **equal** to the strain corresponding to the ultimate stress of the fiber. Then, we have

$$\sigma_{\mathrm{cu}} = \sigma_{\mathrm{fu}} f_{\mathrm{f}} + \sigma'_{\mathrm{m}}(1 - f_{\mathrm{f}}) \qquad f_{\mathrm{f}} \geq f_{\mathrm{min}} \tag{6.8}$$

- Where σ_{fu} is the ultimate tensile of stress of the fiber in the composite and σ'_{m} is the matrix stress at the strain corresponding to the fiber's ultimate tensile stress.
- Note that σ'_{m} is to be determined from the stress--strain curve of the matrix alone; that is, it is the matrix flow **stress** at a strain in the matrix **equal** to the **breaking strain** of the fiber.

- Assuming that all the fiber break at the same time, in order to have a real reinforcement effect, one must satisfy the relation

$$\sigma_{cu} = \sigma_{fu} f_f + \sigma'_m (1 - f_f) \geq \sigma_{mu} (1 - f_f) \quad (6.9)$$

Where σ_{mu} is the ultimate tensile stress of the matrix.

- The equality in this expression serves to define the **minimum fiber volume fraction**, f_{min}, that must be surpassed in order to have real reinforcement. In that case,

$$f_{min} = \frac{\sigma_{mu} - \sigma'_m}{\sigma_{fu} + \sigma_{mu} - \sigma'_m} \quad (6.10)$$

The value of f_{min} **increases** with **decreasing** fiber strength **or increasing** matrix strength.

In case we require that the composite strength should surpass the matrix ultimate stress, we can define a critical fiber volume fraction, f_{crit} that **must be exceeded**. f_{crit} is given by the equation:

$$f_{crit} = \frac{\sigma_{mu} - \sigma'_m}{\sigma_{fu} - \sigma'_m} \quad (6.11)$$

- f_{crit} **increases** with increasing degree of matrix work-hardening $(\sigma_{mu} - \sigma'_m)$.

Anisotropic Nature of Fiber Reinforced Composites

- Fiber reinforced composites are highly **anisotropic**; in particular their mechanical properties are strongly dependent on direction. We derive an expression for the variation in the Young's modulus with the orientation of the fiber for a unidirectionally aligned composite.

- The generalized Hooke's law may be written as:

$$\varepsilon_i = S_{ij}.\sigma_j \tag{6.12}$$

Where ε_i is the strain, σ_j is the stress, S_{ij} is the compliance matrix, and i and j take values from 1 to 6, with summation indicated by a repeated suffix.

- The compliances S_{11}, S_{22} and S_{33} are reciprocals of the generalized stiffness moduli, and it can be shown (see Chapter 2) that they transform with rotation about a principal axis, say, the x_3 -axis, according to relations of the type:

$$S'_{11} = m^4 S_{11} + n^4 S_{22} + m^2 n^2 (2S_{12} + S_{66}) + 2mn(m^2 S_{16} + n^2 S_{26}) \tag{6.13}$$

- For an **orthotropic** sheet material, such as a prepreg, for which the x_3-axis is normal to the plane of the sheet, we have S16=S26=0; then, assuming that the properties in the directions 1 and 2 are the same, Equation 15.13a becomes

(6.14)

$$S'_{11} = m^4 S_{11} + n^4 S_{22} + m^2 n^2 (2S_{12} + S_{66}) = \frac{1}{2} S_{11} + \left(\frac{1}{2} S_{12} + \frac{1}{4} S_{66}\right) + \left[\frac{1}{2} S_{11} \left(\frac{1}{2} S_{12} + \frac{1}{4} S_{66}\right)\right] cos^2 2\theta$$

- Now let E_o and E_{45} be young's modulus for $\theta = 0$ and $\theta = 45$ respectively. Then $S_{11} = 1/E_o$ and $(1/2)S_{11} + (1/2)S_{12} + (1/4)S_{66} = 1/E_{45}$. using these three relationships, we get

$$S' = \frac{1}{E_o} = \frac{1}{E_{45}} - \left(\frac{1}{E_{45}} - \frac{1}{E_0}\right) cos^2 2\theta \quad \textbf{(6.15)}$$

where E_0 is the modulus of the composite when the loading direction makes an angle with the fiber direction.

- We can also write the compliances S_{12} and S_{66} in terms of the shear modulus G and Poisson's ratio v for stresses applied in the plane of the sheet in the directions 1 and 2. From this, we obtain the relationship.

$$\frac{1}{2G} = \frac{1}{E_{45}} - \frac{1}{E_0}(1 - v) \quad \textbf{(6.16)}$$

Toughness

- The toughness of a given composite **depends** on the following factors:
  - Composition and microstructure of the matrix.
  - Type, size, and orientation of the reinforcement.
  - Any processing done on the composite, in so far as it affects microstructural variables (e.g., the distribution of the reinforcement, porosity, segregation, etc.).

Continuous fiber reinforced composites show **anisotropy in toughness** just as in other properties. The 0^o and 90^o arrangements of fibers result in two extremes of toughness, while the $0^o/90^o$ arrangement (i.e., alternating laminae of 0^o and 90^o) gives a sort of pseudo random arrangement with a reduced degree of anisotropy.

- Using fibers in the form of a **braid** can make the crack propagation toughness **increase** greatly due to extensive matrix deformation, crack branching, fiber bundle debonding, and pullout.

- The tougher the matrix, the **tougher** will be the composite. Thus, a **thermoplastic matrix** would be expected to provide a **higher toughness** than a thermoset matrix.

- The final failure of the composite **does not occur** catastrophically with the passage of a **single** crack; that is, self-similar crack propagation does not occur. Thus, it is difficult to define an unambiguous fracture toughness value, such as a value for K_{Ic}.

- Many researches amply demonstrated that **incorporation** of **continuous fibers** such as carbon, alumina, silicon carbide, and **mullite fibers** in brittle matrix materials (e.g., cement, glass, and glass--ceramic matrix) can **result in toughening**.

Load Transfer from Matrix to Fiber

- The matrix has the important function of **transmitting** the applied load **to the fiber**.
- Recall that we emphasized the idea that in fiber reinforced composites, the **fibers** are the **principal load-carrying members**.
- When the composite is loaded axially, the axial displacements in the fiber and in the matrix **are locally different** due to the different elastic moduli of the components. Macroscopically, the composite is deformed **homogeneously**.
- The difference in the axial displacements in the fiber and the matrix implies that shear deformations are produced on planes **parallel to the fiber axis and in the direction of this axis**. These shear deformations are the means by which the applied load is distributed between the two components.

Tensile loading with continuous fibers

- To simplify the discussion, we start by considering the idealised case from figure 6.2(a), the fibres are loaded directly by the external load, so **no load** transfer between fibre and matrix needs to be considered.

- Failure starts when the stress in matrix or fibre reaches the yield or tensile strength. Similar to the elastic case, we can apply an **isostrain rule of mixtures** (equation (6.4)) to calculate the stress in the composite.

- With σ_{f} and σ_{m} being the stresses in fibre and matrix, and f_{f} being the volume fraction of the fibres. In contrast to equation (6.4), we are now interested in the **failure stress** of the composite. Therefore, **at least one** of the stresses in the equation above is a yield or tensile strength.

- The yield strength of the matrix can be affected by the **presence of the fibres** in different ways: Adding fibres to a matrix may cause **thermal stresses** during cooling or other residual stresses, it may render the stress state **triaxial** if Poisson's ratios of fibre and matrix differ or it may change the microstructure of a metallic matrix.

- we have to distinguish two cases, depending on whether failure **occurs first** in matrix or fibre.

Failure strain in the matrix larger than in the fibre

- If the **failure strain** in the matrix is **larger** than in the fibre, the fibres **fracture before** the matrix fails. This is frequently the case in composites with metallic or polymeric matrix.

- **increasing** the strain, the strengthening fibres fracture, and the stress-strain curve drops to a small stress value that lies below that of the pure matrix material because of the **reduced volume**.

- The **fracture strain** is **smaller** than in a pure matrix material. This is due to damage **initiated** by the breaking fibres and the triaxial stress state in a composite.

- If the fraction of fibres is **so large** that the matrix **cannot** bear a given load after the fibres have fractured (this is the case in figure 6.4), the failure stress is given by the isostrain rule of mixtures, equation (6.4), where σ_f is the failure stress of the fibres and σ_m the stress in the matrix at the failure strain of the fibres.

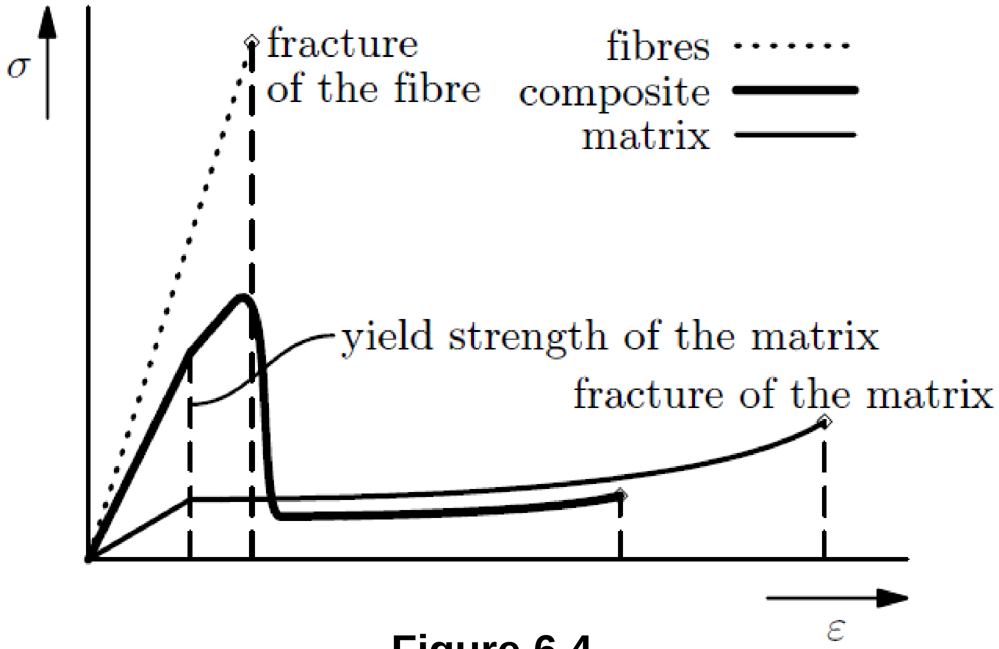


Figure 6.4

- If the **volume fraction** of the fibres is **small**, the matrix can still **bea**r the load even after the fibres have fractured. In this case, the failure stress of the composite is $\sigma_{\mathrm{m}}(1- f_{\mathrm{f}})$, with σ_{m} being the failure stress of the matrix.

Failure strain in the fibre larger than in the matrix

- The failure strain of the fibre may also be larger than that of the matrix in some cases, for example in carbon-fibre reinforced duromers or in ceramic matrix composites..
- After the strain has exceeded the failure strain of the matrix, the complete load has to be borne by the fibres.
- If it is sufficiently large, the fibres do not break but can take a load of $f_{\mathrm{f}}\sigma_{\mathrm{f}}$.
- If the volume fraction is too small, the maximum stress is again determined by the isostrain rule of mixtures, equation (6.4), but now taking σ_{m} as failure stress of the matrix and σ_{f} as the stress in the fibre at the failure strain in the matrix.

Load transfer between matrix and fibre

- If the fibres are completely embedded within the matrix, the load transfer between matrix and fibre is crucial in determining the strengthening effect.
- If a fibre composite is loaded in tension, the deformation within the material is inhomogeneous (Figure 6.5)

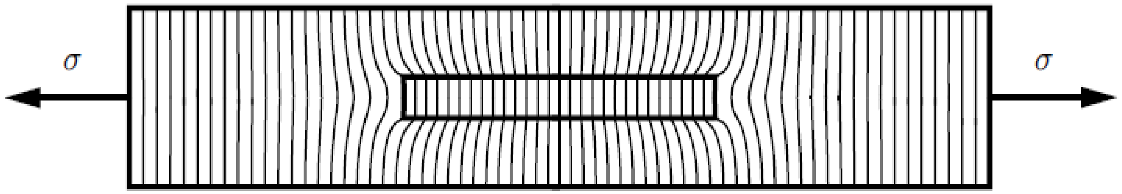


Figure 6.5

- Because the fibre resists the strain more strongly than the matrix, the strain in the matrix increases accordingly. This has two consequences: On the one hand, the strain in the matrix to the left and right of the fibre is larger than besides it. If the total strain is prescribed, the matrix is strained more heavily in this region than the material on average.
- On the other hand, shear stresses occur near the ends of the fibre at the fibre-matrix interface, increasing the strain in this region.

Load transfer between matrix and fibre

- The load transfer between fibre and matrix is mainly due to friction caused by interface roughness or to adhesion between fibre and matrix on the lateral surface of the fibre.
- Although the strains at the front and back side of the fibre can be large, only small loads are transferred there because these sides are much smaller.
- The maximum interfacial shear stress is determined by different factors in different composites.
- In a polymer or ceramic matrix, the adhesion strength between fibre and matrix is the determining factor because it is usually smaller than the yield strength of the polymer or the strength of the ceramic.
- In a metal matrix composite, the maximum interfacial shear stress is usually determined by the yield strength of the matrix which is smaller than the adhesion strength in most cases.
- To estimate the stress σ_f within the fibre, we consider an infinitesimal fibre segment with a constant interfacial shear stress $-\tau_i$ acting on its surface (Figure 6.7).

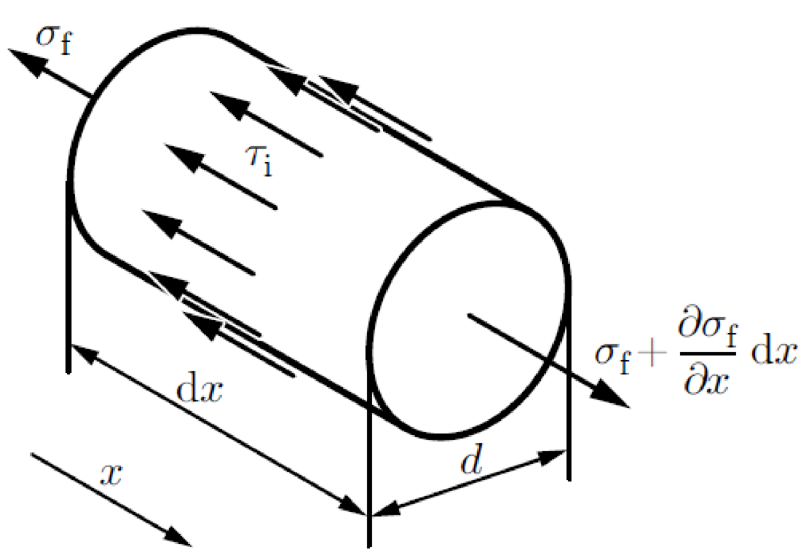


Figure 6.6

- The forces within the fibre are $-\sigma_f.\pi d^2/4$ at position x and $(\sigma_f + \partial\sigma_f/\partial x.dx).\pi d^2/4$ at position $x + dx$, with d denoting fiber diameter. The force equilibrium:

$$-\pi d \mathrm{d}x \tau_\mathrm{i} - \sigma_\mathrm{f}\pi\frac{d^2}{4} + \left(\sigma_\mathrm{f} + \frac{\partial\sigma_\mathrm{f}}{\partial x}\mathrm{d}x\right)\pi\frac{d^2}{4} = 0 \tag{6.17}$$

This yields

$$\frac{\partial\sigma_\mathrm{f}}{\partial x} = \frac{4}{d}\tau_\mathrm{i} \tag{6.18}$$

- The stress changes linearly with position if τ_i is constant
- The maximum stress within the fibre is limited by the strain in the fibre (index ‘f’) which can never exceed the matrix (index ‘m’) strain. The maximum possible fibre stress is thus

$$\sigma_{\mathrm{f,\,max}} = E_\mathrm{f}.\varepsilon_\mathrm{m} \tag{6.19}$$

- If we assume that the **maximum stress value** $E_{\mathrm{f}}.\varepsilon_{\mathrm{m}}$ is **not reached**, the maximum stress actually occurring in the fibre is

$$\sigma_{\mathrm{f,\,max}} = \int_0^{l/2} \frac{4}{d}\tau_{\mathrm{i}} dx = 2\frac{l}{d}\tau_{\mathrm{i}} \qquad \textbf{(6.20)}$$

If we aim at a **maximum strengthening** effect of the fibres, they have to be sufficiently long to be loaded up to their fracture strength $\sigma_{\mathrm{f,\,B}}$. With the help of the equation above, we can thus define the **critical fibre length**

$$l_c = \frac{d\sigma_{\mathrm{f,\,B}}}{2\tau_i} \qquad \textbf{(6.21)}$$

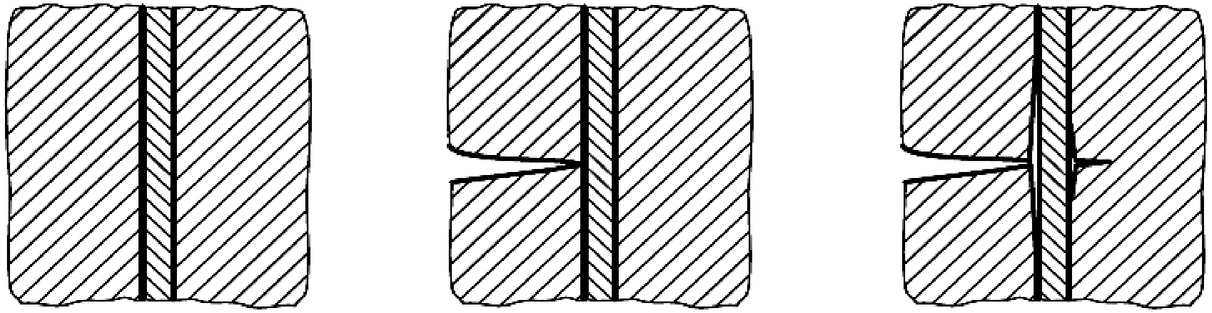
If the fibres are **smaller** than the critical length, the maximum value of the fibre stress is

$$\sigma_{\mathrm{f,\,max}} = 2\frac{l}{d}\tau_{\mathrm{i}} = \frac{l}{l_c}\sigma_{\mathrm{f,\,B}} \qquad \textbf{(6.22)}$$

resulting in a mean fibre stress of $\sigma_{\mathrm{f,\,max}}/2$. If the fibre length **exceeds** the critical length, the **fibres will break** in some places when the load increases, until the fibre fragments have approximately the critical length. These can then bear a maximum stress of $\sigma_{\mathrm{f,\,max}}$. For this reason, it is **reasonable** to use fibres that exceed the critical length because they can bear a load even **after fracture** occurs.

Crack propagation in fibre composites

- In composites with a brittle matrix i. e., mainly in ceramic matrix composites, the aim is not to increase the strength, but the fracture toughness.
- The fracture strain of the matrix is usually smaller than that of the fibre, leading to crack propagation in the matrix when the load increases.
- These cracks propagate within the matrix until they reach a fibre (see figure 9.7).
- To increase the fracture toughness compared to the pure matrix material, the fibre must not fracture when hit by the crack, but the interface between fibre and matrix, causing a detachment between fibre and matrix as shown in figure 6.7.



(a) No crack

(b) The crack reaches the fibre

(c) The crack by-passes the fibre along the interface

Figure 6.7

- The crack can **propagate** along the detached interface and grow further without breaking the fibre.
- It is crucial for the **increase** in fracture toughness that the crack is **bridged** by the fibre after the **crack tip** has propagated beyond the fibre (see figure 6.8).
- The load transfer between the crack surfaces results in a **maximum of the stress** in the part of the fibre that is situated within the crack.
- This stress has to be transferred to the matrix on both sides, in a region whose size is approximately that of the **critical fibre length**.
- Fracture usually occurs at a defect, for example a surface defect or a local reduction in diameter. Because of this, fibre fracture occurs not always directly at the crack surface, but at an arbitrary position between the stress maximum near the crack surface and the region where the fibre stress has decreased markedly.

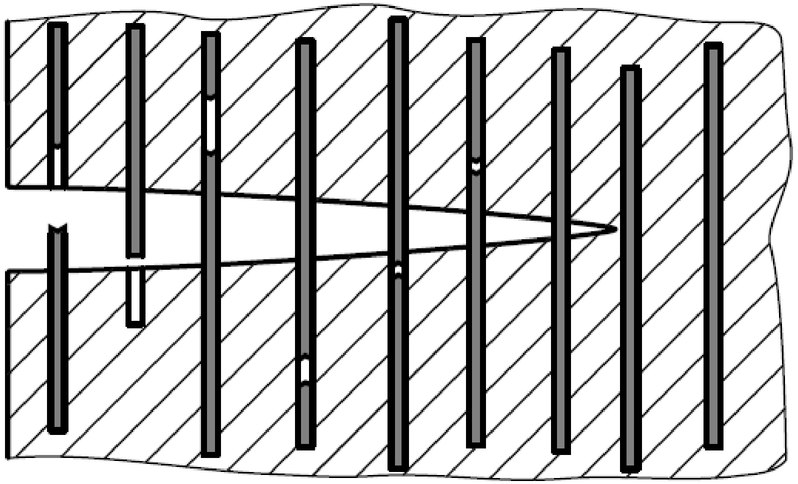


Figure 6.8

- After the fibre has broken, the fragment still remaining in the matrix that **bridges** the crack is shorter than the **critical length**.
- Therefore, the fibre will not break again, but will be **pulled out of the matrix**, doing work against the shear stress τ_i.
- So far, we have **assumed** that the fibre is **longer than the critical length**. If this is not the case, the fibre will not break, but **pull-out** will occur immediately.
- The work done on pull-out **increases** the crack resistance because it impedes crack propagation.
- If the crack has grown a large distance, the fracture toughness approaches a **constant value** because for each fibre entering the process zone another fibre leaves it.
- The crack resistance is the **higher**, the larger the critical fibre length is.
- The energy dissipated on pull-out can be estimated as follows:

The force needed to pull out the fibre is:

$$\boxed{F(x) = \tau_i x \pi d} \qquad \textbf{(6.23)}$$

if the segment of the fibre remaining in the matrix has a length x (see figure 6.9). From this, we can calculate the energy required to pull out the fibre from the matrix by a distance l' on one side of the crack:

$$W_{\mathrm{f},l'} = \int_0^{l'} F\mathrm{d}x = \int_0^{l'} \tau_i x \pi d \mathrm{d}x = \frac{1}{2}\pi d \tau_i l'^2 \qquad \textbf{(6.24)}$$

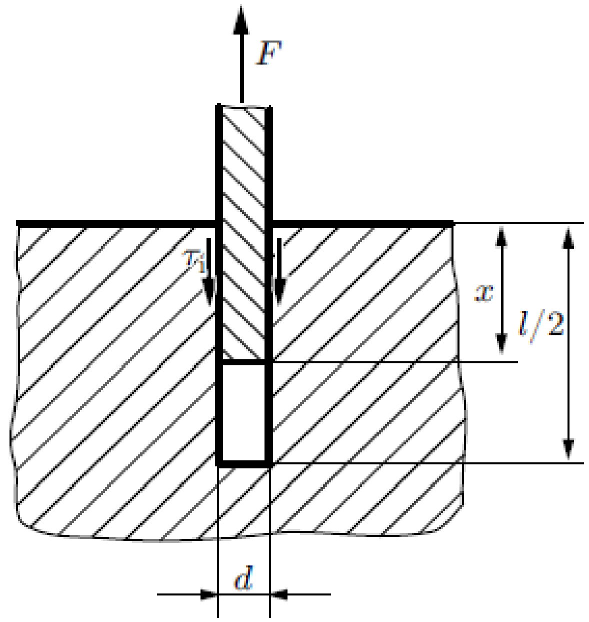


Figure 6.9

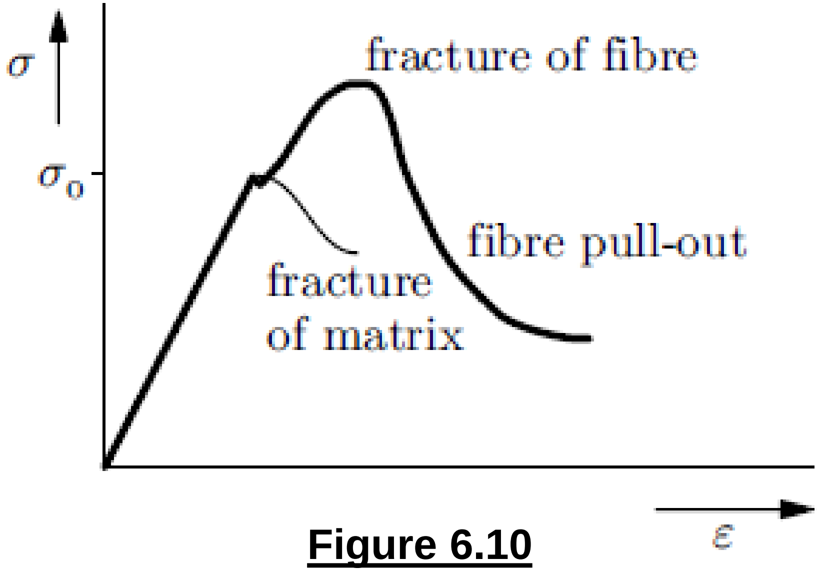


Figure 6.10

In a simple approximation, we can assume that the **pull-out length** varies between 0 and half of the critical length l_c. The mean energy dissipation per fibre is thus

$$W_{\mathrm{f}} = \frac{1}{l_c/2}\int_0^{l_c/2} \frac{1}{2}\pi d\tau_i l'^2 \mathrm{d}l' = \frac{1}{24}\pi d\tau_i l_c^2 \tag{6.25}$$

- The fracture toughness of a fibre composite is **not** determined by the dissipation of **one single fibre**, but by the total dissipation.
- To estimate this, we have to take into account how many fibres **bridge the crack** and can dissipate energy by pull-out. Their number is, if the volume fraction is constant, inversely proportional to the square of the fibre diameter. Using this, the total energy dissipation in the composite is proportional to $\tau_i l_c^2/d$.
- Figure 6.10 schematically shows the stress-strain diagram of a ceramic matrix composite. First cracks in the matrix occur at a stress of σ_0. The load can be increased beyond that because the **bridging fibres** can bear larger loads, until they **finally** fracture.