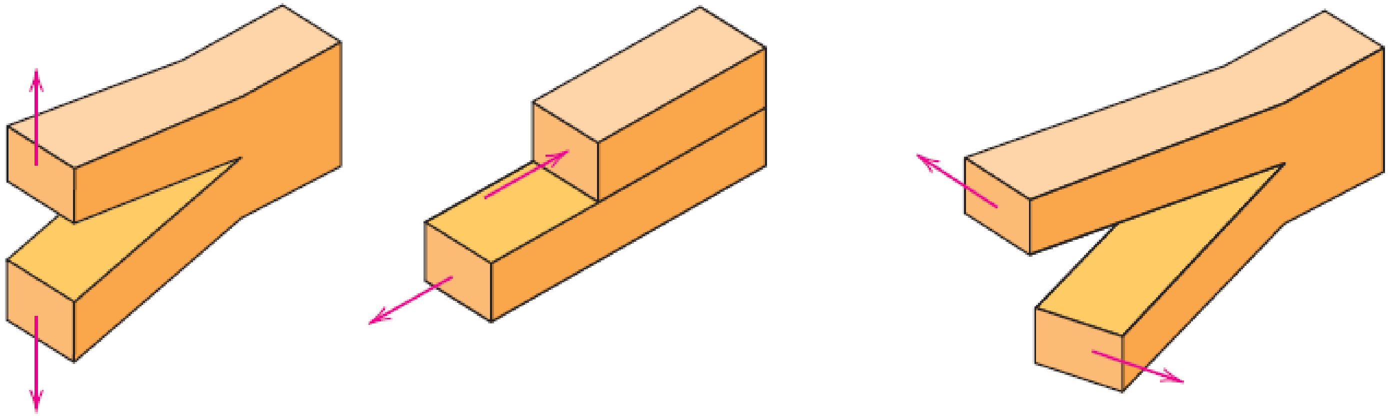
Introduction

- The **separation** or **fragmentation** of a solid body into two or more parts, under the action of stresses, is called fracture.
- Fracture of a material by **cracking** can occur in many ways, principally the following:
  1. Slow application of external loads.
  2. Rapid application of external loads (impact).
  3. Cyclic or repeated loading (fatigue).
  4. Time-dependent deformation (creep).
  5. Internal stresses, such as thermal stresses caused by anisotropy of the thermal expansion coefficient or temperature differences in a body.
  6. Environmental effects (stress corrosion cracking, hydrogen embrittlement, liquid metal embrittlement, etc.).

The **process of fracture** can, in most cases, be subdivided into the following categories:

1. Damage accumulation (associated to material properties such as atomic structure, prior loading history).
2. Nucleation of one or more cracks or voids.
3. Growth of cracks or voids. (This may involve a coalescence of the cracks or voids.)

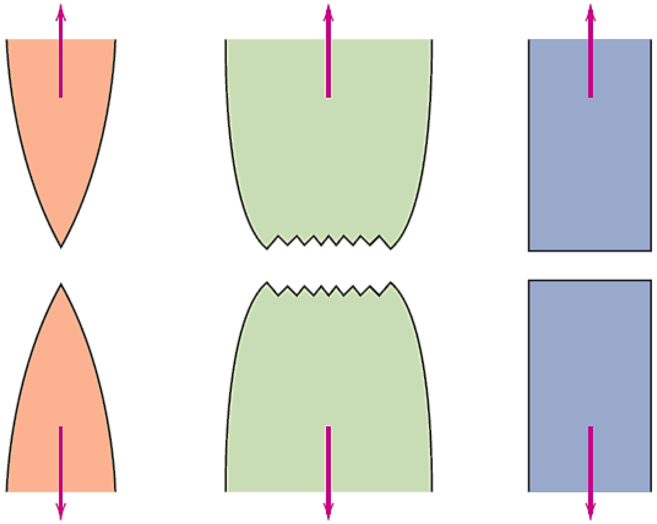
- When the local **strength** or **ductility** is **exceeded**, a crack (two free surfaces) is formed.
- Linear elastic fracture mechanics (**LEFM**) applies the theory of **linear elasticity** to the phenomenon of fracture mainly, the propagation of cracks.
- If we define the **fracture toughness** of a material as its **resistance to crack propagation**, then we can use LEFM to provide us with a quantitative measure of fracture toughness.
- The three modes of fracture are shown in the figure.



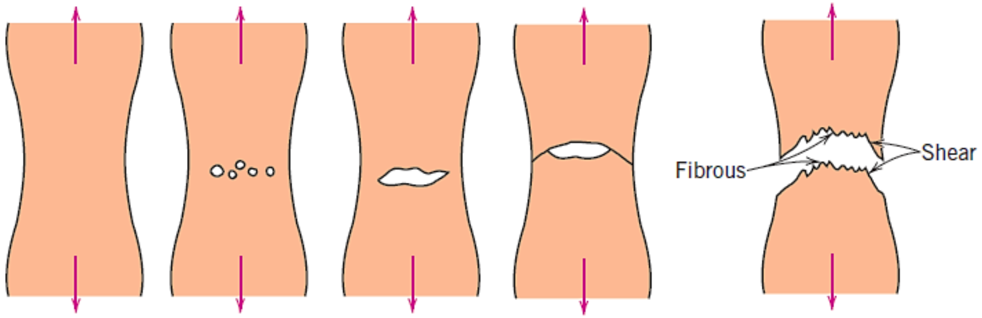
Opening mode Sliding mode Tearing mode

Ductile Fracture

- The figure here shows schematic representations for **two characteristic** macroscopic ductile fracture profiles.
- The **extremely soft** materials neck down to a point fracture, showing virtually **100% reduction in area**.
- The most common type of tensile fracture profile for **ductile metals** show a **moderate amount of necking**.

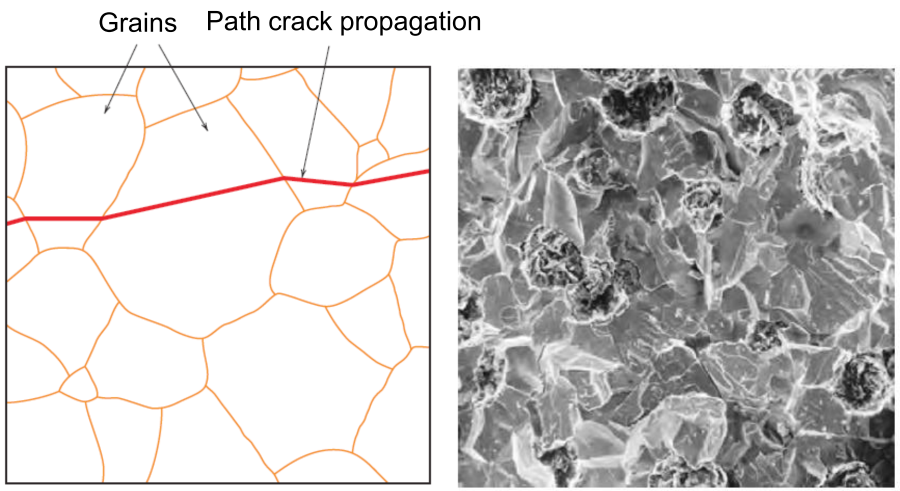


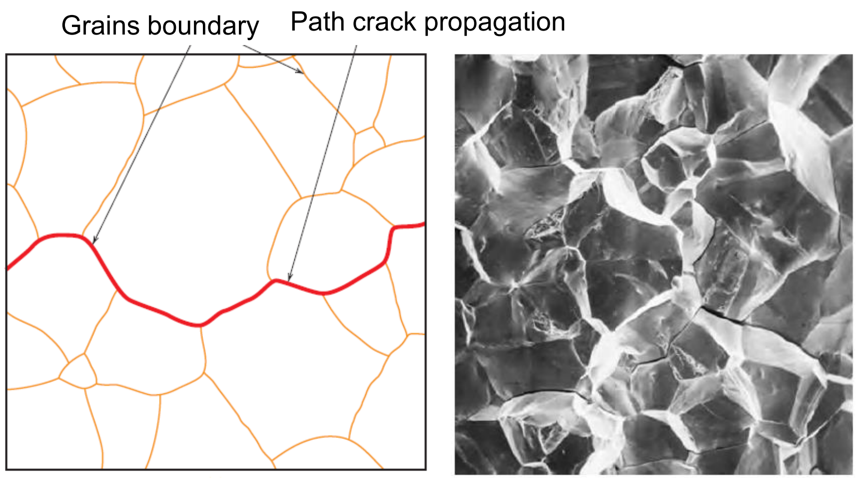
- The fracture process normally occurs in several stages:
  - small cavities, or micro voids, form in the interior of the cross section
  - these micro voids enlarge, come together, and coalesce to form an elliptical crack.
  - The crack continues to grow in a direction parallel to its major axis by this micro void coalescence process
  - fracture ensues by the rapid propagation of a crack around the outer perimeter of the neck by shear deformation at an angle of about 45° with the tensile axis.



Brittle Fracture

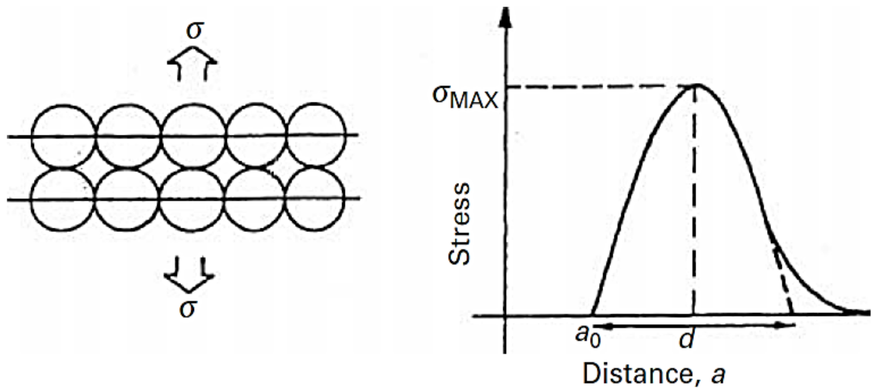
- Fracture surfaces of materials that failed in a brittle manner will have their own **distinctive patterns.**
- For most brittle crystalline materials, crack propagation corresponds to the successive and repeated breaking of atomic bonds along specific crystallographic planes such a process is termed **cleavage.**
- The cleavage fracture is also known **transgranular** (or **transcrystalline**), because the fracture cracks pass through the grains.
- In some alloys, crack propagation is along grain boundaries this fracture is termed **intergranular.**





Theoretical Tensile Strength

- A material is said to **cleave** when it breaks under normal stress and the fracture path is perpendicular to the applied stress.
- Orowan method assumes that **no stress concentrations** at the tip of the crack are assumed; instead, it is assumed that all atoms separate simultaneously once their separation reaches a critical value



- The work of deformation (area under curve) **cannot be lower** than the energy of the two new surfaces created by the cleavage.
- The stress dependence on plane separation is then given by the following equations, admitting a sine function and assuming a periodicity of 2d:

$$\sigma = K \sin\frac{2\pi}{2d}(a - a_o) \quad (1)$$

K is a constant that can be determined by assuming that the material responds linearly to the applied loads (Hookean behavior) when a is close to a_o.

$$\frac{da}{a_o} = d\varepsilon \quad \Rightarrow \quad \frac{d\sigma}{d\varepsilon} = E \xrightarrow{yields} \frac{d\sigma}{da/a_o} = E \quad \Rightarrow \quad a_o \frac{d\sigma}{da} = E$$

Taking the derivative of equation (1) and substituting into the equation above for $a = a_o$

$$a_o \frac{d\sigma}{da} = K \frac{\pi}{d} a_o \cos \frac{\pi}{d}(a - a_o) = E \quad \Rightarrow \quad \therefore K = \frac{E}{\pi} \frac{d}{a_o} \qquad (2)$$

However, d is not known; to determine d, the area under the curve has to be equated to the energy of the two surfaces created:

$$\int_{a_o}^{a_o+d} \sigma da = 2\gamma$$

Substituting equation (1) here, we get

$$\int_{a_o}^{a_o+d} K \sin \frac{2\pi}{2d}(a - a_o) da = 2\gamma$$

However, using the standard form; $\int \sin axdx = \frac{1}{a} \cos ax$ we have $a - a_o$= y; therefore, da = dy, and

$$K\int_0^d \sin\frac{\pi}{d}y\,dy = 2\gamma \quad \Rightarrow \quad \therefore K\frac{d}{\pi} = \gamma \xrightarrow{yields} d = \frac{\pi\gamma}{K} \qquad (3)$$

The maximum value of σ is equal to the theoretical cleavage stress. From Equation (1), and making the sine equal to 1, we have:

$$\sigma_{max} = K = \frac{E}{\pi}\frac{d}{a_o} \qquad (4)$$

Substituting equation (3) into equation (4) yields

$$K = \sigma_{max} = \frac{E}{a_o}\frac{\gamma}{K} \xrightarrow{yields} K^2 = (\sigma_{max})^2 = \frac{E\gamma}{a_o}$$

$$\therefore \sigma_{max} = \sqrt{\frac{E\gamma}{a_o}}$$

According to Orowan's model, the surface energy is given by

$$\gamma = \frac{Kd}{\pi} = \frac{E}{a_o}\left(\frac{d}{\pi}\right)^2 \quad \Rightarrow \quad \gamma = \frac{E a_o}{10} \quad \text{and} \quad \sigma_{max} \cong \frac{E}{\pi}$$

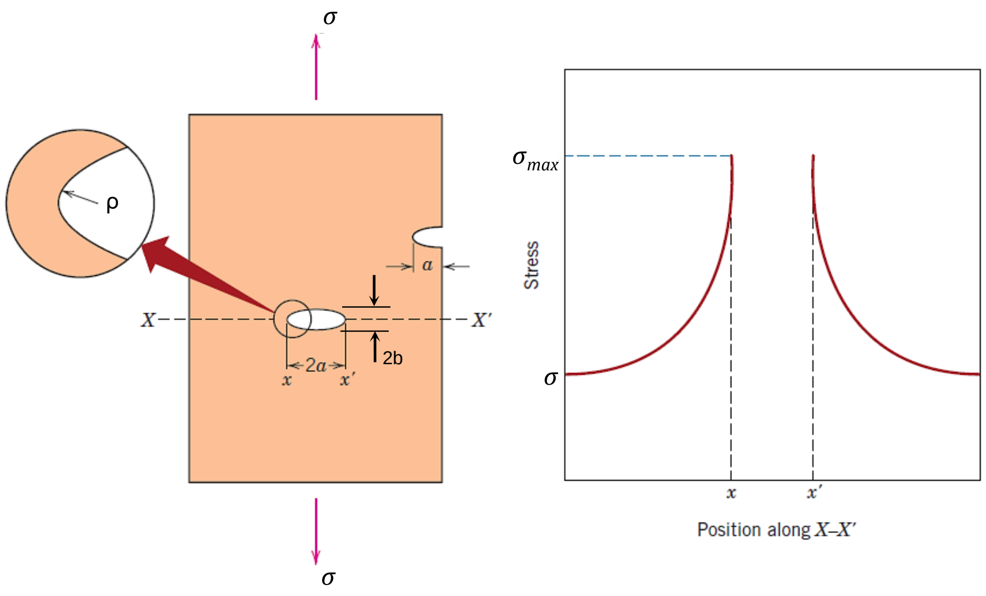
Principles of Fracture Mechanics

Fracture mechanics allows quantification of the relationships between **material properties**, **stress level**, the **presence of crack**-producing flaws, and crack propagation mechanisms. Design engineers are now better equipped to anticipate, and thus prevent, structural failures.

Stress Concentration

- The **theoretical** fracture stress of a solid is on the order E/10, but **practically** it is significantly **less** than this value due to the presence of very small, microscopic flaws or cracks.
- The first attempt to explain this discrepancy was introduced by **Grifth**. His analytical model was based on the **elastic solution of a cavity elongated in the form of an ellipse**.
- In the figure shown here, the maximum stress occurs at the **ends of the major axis** of the cavity and is given by Inglis's formula;

$$\sigma_{max} = \sigma\left(1 + 2\frac{a}{b}\right)$$



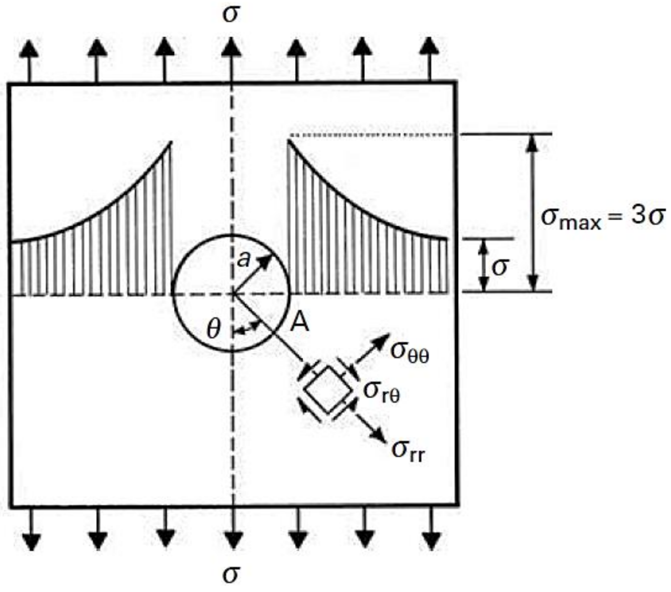
- The value of the stress at the leading edge of the cavity becomes extremely **large** as the ellipse is **flattened**.
- In the case of an **extremely flat** ellipse or a very narrow crack of length $2a$ and having a radius of curvature $\rho = b^2/a$, the former equation can be written as;

$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right) \cong 2\sigma\sqrt{\frac{a}{\rho}} \quad \text{for } \rho \ll a$$

where σ is the magnitude of the nominal applied tensile stress

- Sometimes the ratio is denoted as σ_{max}/σ the stress concentration factor K_t.

$$K_t = \frac{\sigma_{max}}{\sigma} = 2\left(\frac{a}{\rho}\right)^{1/2}$$



- In addition to producing a stress concentration, a **notch** produces a local situation of **biaxial or triaxial stress** (as can be seen in the figure).

$$\sigma_{rr} = \frac{\sigma}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{r\theta} = -\frac{\sigma}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$

- The maximum stress occurs at point A, where $\theta=\pi/2$ and $r=a$. In this case,

$$\sigma_{\theta\theta} = 3\sigma = \sigma_{max}$$

- On the other hand, the **stresses around spherical voids** in perfectly **elastic** materials given by Timoshenko and Goodier can be determined from the methods of elasticity theory. At the equatorial plane ($\theta = \pi/2$), the tangential stress $\sigma_{\theta\theta}$ is equal to

$$\sigma_{\theta\theta} = \left[1 + \frac{4-5v}{2(7-5v)}\frac{a^3}{r^3} + \frac{9}{2(7-5v)}\frac{a^5}{r^5}\right]\sigma$$

For $r = a$, $v = 0.3$, ⇨

$$(\sigma_{\theta\theta})_{max} = \frac{45}{22}\sigma \approx 2\sigma$$

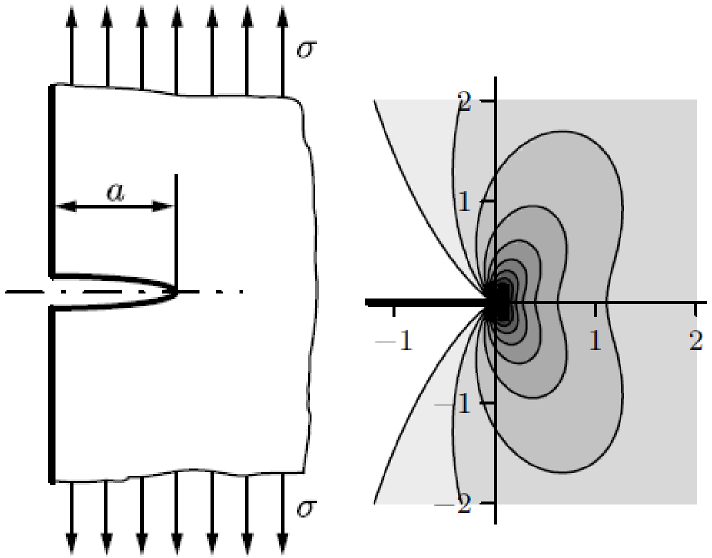
- For $\theta = 0$, Timoshenko and Goodier have the equation:

$$(\sigma_{rr})_{\theta} = (\sigma_{\theta\theta})_{\theta=0} = -\frac{3+15v}{2(7-5v)}\sigma$$

Griffith Criterion

This theory was developed by Andrew A. Griffith in 1921 and states that the **critical stress intensity factor** at which a material will fail due to brittle fracture is **proportional** to the **square root of the surface energy** of the material.

- Griffith pointed out that two things happen when a crack propagates:
  1. Elastic strain energy is **released** in **a volume** of material
  2. **Two new crack surfaces** are created, which represent a surface-energy term



- According to Griffith, an **existing** crack will propagate if the **elastic strain energy** released by doing so is **greater** than the surface energy created by the two new crack surfaces.

- The elastic energy per unit volume in a solid under stress is given by $\sigma^2/2E$ which is multiplied by the volume of the material in which this energy is released to get the total strain energy.

  The volume in which this energy is released = The ellipse area x The plate thickness $\mathrm{V} = 2\pi a^2 \times t$

The total strain energy released is thus,

$$\left(\frac{\sigma^2}{2E}\right)(2\pi a^2 t) = \frac{\pi\sigma^2 a^2 t}{E}$$

Or the strain energy per unit thickness

$$U_e = \frac{\pi\sigma^2 a^2}{E}$$

- The **decrease** in strain energy, U_e, when a crack propagates is balanced by an **increase** in the surface energy, U_s. However, The increase in surface energy equals

$$U_s = (2at)(2\gamma)$$

here γ is the specific surface energy, i.e., the **energy per unit area**.

- for the change in potential energy ΔU of the plate,

$$\Delta U = U_s - U_e \quad \Rightarrow \quad \Delta U = 4a\gamma - \frac{\pi\sigma^2 a^2}{E}$$

- The crack becomes **stable** when these energy components (strain energy and surface energy) **balance** each other. If they are not in balance, we have an **unstable crack** (i.e., the **crack will grow**). The equilibrium conditions can be obtained through;

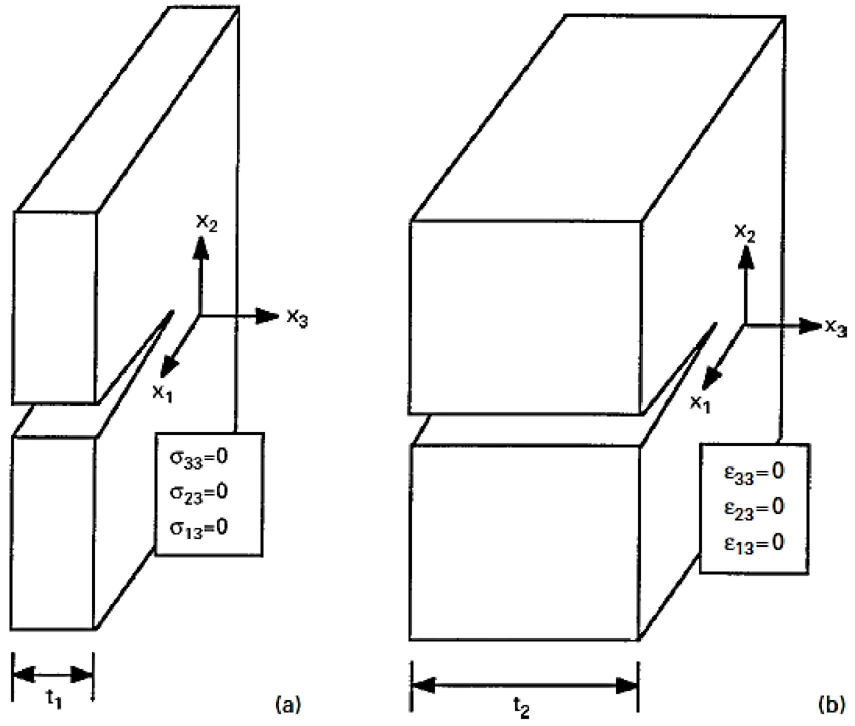
$$\frac{\Delta U}{\partial a} = 4\gamma - \frac{\pi\sigma^2 a}{E} = 0 \quad \Rightarrow \quad 2\gamma = \frac{\pi\sigma^2 a}{E}$$

- The **critical stress** required for the crack to propagate in the **plane-stress** situation,

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}} \quad \text{(plane stress)}$$

For the **plane-strain** situation, we will have the factor (**$1 - \nu^2$**) in the denominator because of the confinement in the direction of thickness

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a(1-\nu^2)}} \quad \text{(plane strain)}$$



Crack Propagation with Plasticity

- If the material in which a crack is propagating can deform **plastically**, the form of the **crack tip changes** (**sharp crack tip will be blunted**) because of plastic strain.
- The amount of plastic deformation that can occur at the crack tip will depend on **how fast** the crack is moving.
- In the case of **localized plastic deformation** at and around the crack tip is produced, a certain amount of **plastic work** γ_p is done during crack propagation, in addition to the **elastic work** done in the creation of two fracture surfaces.
- The work done in the propagation of a crack per unit area of the fracture surface is **increased** from γ_s to $(\gamma_s + \gamma_p)$. Consequently, the Griffith criterion is modified to:

$$\sigma_c = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi a}} \quad \text{(plane stress)}$$

$$\sigma_c = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi a(1 - v^2)}} \quad \text{(plane strain)}$$

- These equation are difficult to use practically, therefore, Irwin proposed that **fracture occurs at a stress that corresponds to a critical value of the crack extension force**

$$G = \frac{1}{2}\frac{\partial U_e}{\partial a} = \text{rate of change of energy with crack length}$$

G is sometimes called the strain energy release rate.

thus

$$G = \frac{\pi a \sigma^2}{E}$$

At fracture $G = G_c$, and

$$\sigma_c = \sqrt{\frac{2EG_c}{\pi a}} \quad \text{(plane stress)}$$

$$\sigma_c = \sqrt{\frac{2EG_c}{\pi a(1-\nu^2)}} \quad \text{(plane strain).}$$

Therefore, we can concluded

$$G_c = 2(\gamma + \gamma_p)$$

Effects of Cracks on Strength

- The crack may **suddenly** grow and cause the member to fail by fracturing in a brittle manner—that is, with **little plastic deformation** and a useful quantity called the **stress intensity factor** (K) can be therefore defined.
- Specifically, K is a measure of the severity of a crack situation as affected by crack size, stress, and geometry.
- In defining K, the material is assumed to behave in a **linear-elastic** manner, so that the approach being used is called **linear-elastic fracture mechanics** (LEFM).
- A given material can resist a crack without brittle fracture occurring as long as this K is below a critical value K_c , called the **fracture toughness**.
- Thicker members have lower K_c values until a worst-case value is reached, which is denoted K_{Ic} and called the **plane strain fracture toughness**.
- K_{Ic} defined as the critical stress intensity factor under **plane strain** conditions and **mode I** loading and at which a crack of a given size starts to grow in an **unstable** manner.

Hypothesis of Linear Elastic Fracture Mechanics (LEFM)

- **Brittleness** term refers to the onset of **instability** under an applied stress smaller than the stress corresponding to plastic yielding of the material.

- LEFM provides a **quantitative measure** of the resistance of a brittle material to **unstable** crack propagation. The basic hypothesis of LEFM are:

1. Cracks are inherently present in a material.
2. A crack is a free, internal, plane surface in a linear elastic stress field. With this hypothesis, linear elasticity furnishes us stresses near the crack tip as:

$$\sigma_{r\theta} = \frac{K}{\sqrt{2\pi r}} f(\theta)$$

where r and θ are polar coordinates and K is the stress intensity factor (SIF)

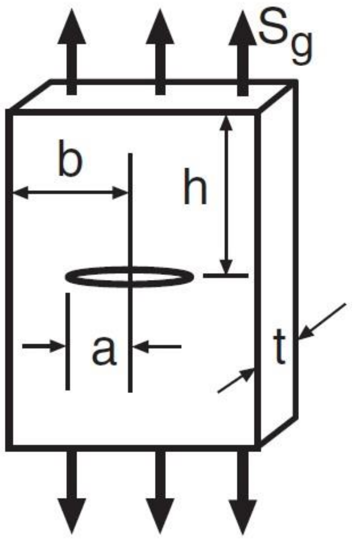
3. The growth of the crack leading to the failure of the structural member is then predicted in terms of the tensile stress acting at the crack tip.

Effects of Cracks on Strength

- The K depends on the remotely **applied stress** (σ) and the **crack length** (a), measured from the centerline as shown:

$$K = \sigma\sqrt{\pi a} \qquad a \ll b$$

This equation is accurate only if a is small compared with the half-width b of the member



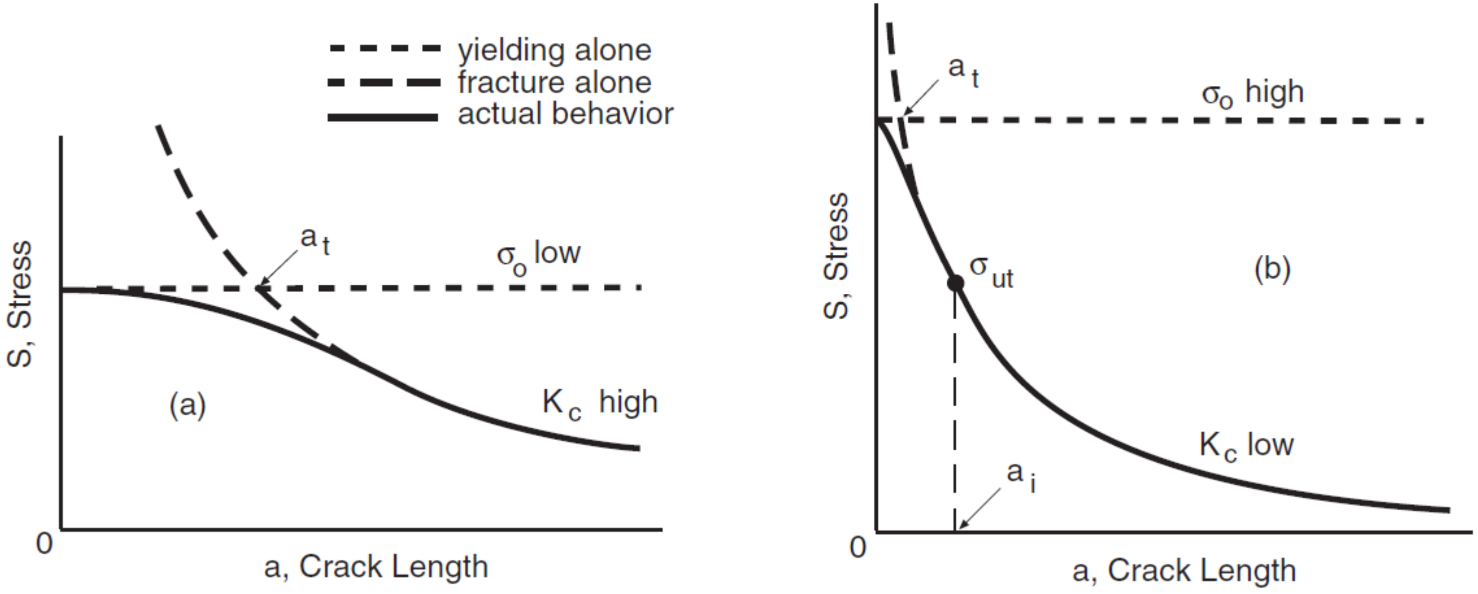
- For a given material and thickness with fracture toughness K_c, the critical value of remote stress necessary to cause fracture is

$$\sigma_c = \frac{K_c}{\sqrt{\pi a}}$$

- when the failure stress (σ_c) predicted by LEFM equals the yield strength (σ_o), the crack length identified as a_t

$$a_t = \frac{1}{\pi}\left(\frac{K_c}{\sigma_o}\right)^2$$

- The cracks **longer** than this **transition crack length** (a_t) will cause the strength to be limited by **brittle fracture**, rather than by yielding.
- If cracks of length **around or greater** than the a_t of a given material are likely to be present, fracture mechanics should be employed in design.
- For crack lengths **below** a_t , **yielding** dominated behavior is expected, so that there will be **little or no strength reduction due to the crack.**
- Consider two materials, one with **low** σ_o and **high** K_c, and the other with, **high** σ_o and **low** K_c. These combinations of properties cause a relatively **large** a_t for the low-strength material, but a **small** a_t for the high-strength one.



- Many **brittle** materials such as glass, natural stone, ceramics, and some cast metals **naturally** contain small cracks or cracklike flaws.

- The **strength** of such materials can never be reached **under tensile loading** because of earlier failure due to small flaws and a low fracture toughness.

- Thus, **brittle materials** have considerably **higher** strengths under compression than under tension, because the **flaws close under compression** and thus have a much **reduced** effect.

  Denoting the inherent flaw size in such a material as (a_i), the ultimate strength in tension:

$$\sigma_{ut} = \frac{K_c}{\sqrt{\pi\, a_i}}$$

- The magnitude of the stress field near the crack tip can be characterized by giving the value of the factor K_{Ic}. It is generally convenient to express this factor as;

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$