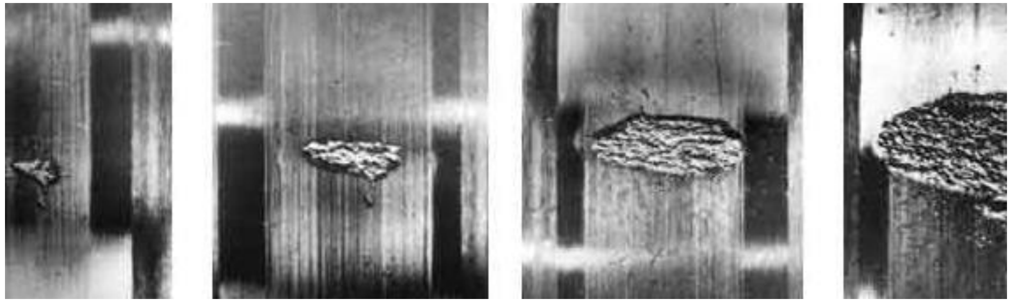
# What is FATIGUE ?

- Most machine elements are subjected to **varying or fluctuating** stresses due to movement such as shafts, gears, bearings, cams and followers, etc.

- Fluctuating loads **repeated over long period of time** will cause a part to fail (fracture) at stress level **much smaller** than the **ultimate strength** or even the **yield strength** in some cases.

- Fatigue failure is somehow similar to brittle fracture where the fracture surfaces are **perpendicular** to the load axis.

- In general, fatigue failure develops in three stages:
  - **Stage1:** development of one or more **micro cracks** (size of two to five grains) due to the cyclic local plastic deformation.
  - **Stage2:** the cracks progress from **micro to macro** scale and keep growing making a smooth plateau-like fracture surface with beach marks.
  - **Stage3:** occurs during the final stress cycle where the remaining material cannot support the load resulting in **sudden fracture**.



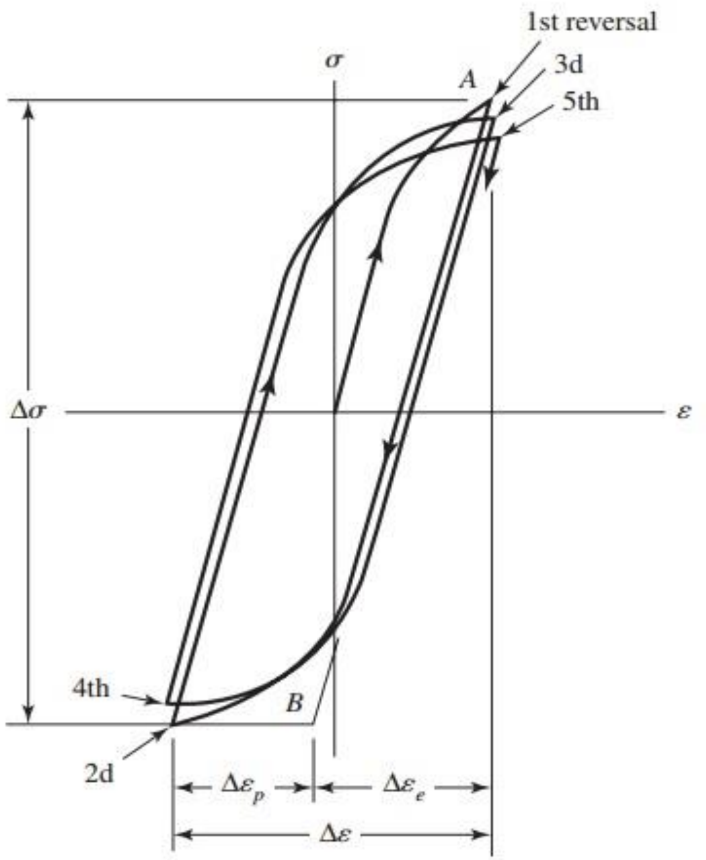
- Fatigue failure is due to crack formation and propagation. Fatigue cracks usually initiate at location with high stresses such as **discontinuities** (hole, notch, scratch, sharp corner, etc.)
- Fatigue cracks can also initiate at surfaces having **rough surface** finish or due to the presence of tensile **residual stresses**. Thus all parts subjected to fatigue loading are **heat treated and polished** in order to increase the fatigue life.

## Fatigue Life Methods

- There are three major fatigue life methods: the **stress-life** method, the **strain-life** method and the **linear elastic fracture mechanics** method. Each one of these methods is more accurate for types of loading or for some materials.
- The fatigue life is usually classified according to the **number of loading cycles** into:
  - Low cycle fatigue (**$1 \leq N \leq 1000$**) and for this low number of cycles, designers sometimes **ignore** fatigue effects and just use static failure analysis.
  - High cycle fatigue (**$N > 1000$**):
    i. **Finite life**: from $10^3$ to $10^6$ cycles
    ii. **Infinite life**: more than $10^6$ cycles

# The strain-Life Method

- The fatigue life is related to the **amount of plastic strain** suffered by the part during the repeated loading cycles.
- If the stress direction is reversed (from tension to compression), the yield strength in the reversed direction will be smaller than its initial value which means that the material has been **softened** in the reverse direction.
- The strain-life is applicable to **Low-cycle fatigue**.

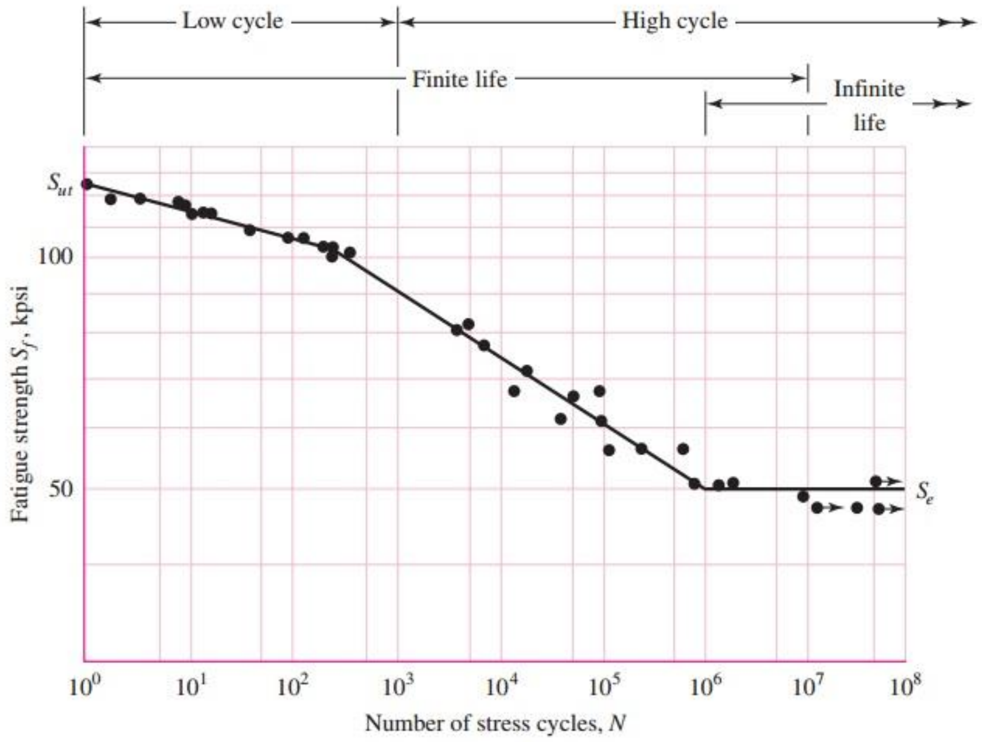



# The Linear Elastic Fracture Mechanics Method

- This method assumes that a crack initiates in the material and it keeps growing until failure occurs (the three stages described before)
- The LEFM approach assumes that a small crack **already exists** in the material, and it **calculates** the number of loading cycles required for the crack to grow enough to cause complete fracture.
- This method is more **applicable** to High-cycle fatigue.

# The stress-Life Method

- This method relates the **fatigue life to the alternating stress level** causing failure but it dose not provide explanation to why failure failure happens. .
- The stress-life relation is obtained experimentally using Moore high speed **rotating beam test**.
  - The data obtained from the tests is used to generate the **fatigue strength vs. fatigue life** diagram which is known **S-N diagram**.
  - The first point is the ultimate strength which corresponds to **failure in half a cycle**.
  - The alternating stress amplitude is **reduced** below the ultimate strength and the test run until failure. The stress level and number of cycles until failure give a data point on the chart.
  - for steel alloys, the low cycle fatigue and the high cycle fatigue (finite and infinite) can be recognized as having different slopes.
  - for steel, with reducing the stress amplitude, we will reach to a stress for which the specimen will never fail, and this stress value is known as the Endurance Limit ($S_e$)



- – The number of stress cycles associated with the **Endurance Limit** defines the boundary between **Finite** life and **infinite** life, and it is usually between $10^6$ to $10^7$ cycles.

- Steel and titanium alloys have a clear endurance limit, but this is not true for all materials.
  - – For instance, Aluminium alloys dose not have an endurance limit and for such materials the fatigue strength is reported at $5(10^8)$ cycles.
  - – Also, most polymers do not have an endurance limit

## Endurance Limit

- It is important to **sure** that the fatigue stress level in the element is **below** the endurance limit of the material being used.

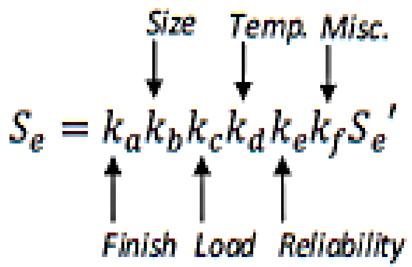
- The relation between the endurance limit and ultimate strength for steel is given as:

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 1400\text{MPa} \\ 700\text{ MPa} & S_{ut} > 1400\text{MPa} \end{cases}$$

The prime (‘) is used to denote that this is the endurance limit value obtained for the test specimen.

# Endurance Limit Modifications Factors

- Some **modifications factors** are used to correlate the endurance limit for a given mechanical element to the value obtained from tests.




$S_e$ : the endurance limit at the critical location of a **machine element** with the geometry and conditions of use.
$S_e'$ : the endurance limit obtained from the rotating beam test.
$k_a .....k_f$ : Modification factors (obtained experimentally)

## ✓ Surface Condition Factor ($k_a$)

- The rotating beam test specimens are highly polished. A **rough surface** finish will **reduce the endurance limit** because there will be a higher potential for crack initiation.

- The surface condition modification factor depends on the **surface finish** of the part (ground, machined, as forged, etc.) and on the tensile strength of the material. It is given as:

$$k_a = aS_{ut}^b$$

Constants $a$ and $b$ depend on surface condition and are given in the below table.

| Surface Finish | Factor $a$: $S_{ut}$, kpsi | Factor $a$: $S_{ut}$, MPa | Exponent $b$ |
|---|---|---|---|
| Ground | 1.34 | 1.58 | −0.085 |
| Machined or cold-drawn | 2.70 | 4.51 | −0.265 |
| Hot-rolled | 14.4 | 57.7 | −0.718 |
| As-forged | 39.9 | 272. | −0.995 |

## ✓ Size Factor ($k_b$)

- The rotating beam specimens have a specific small diameter. Parts of larger size are more likely to contain flaws and to have more non-homogeneities. The size factor is given as:

For bending and torsion

$$k_b = \begin{cases} 1.24\, d^{-0.107} & 2.79 \le d \le 51mm \\ 1.51\, d^{-0.157} & 51 < d \le 254mm \end{cases}$$

❖ where d is the diameter

For axial loading

$$k_b = 1$$

*for rotating shaft* (applies to both cases above)

- When a circular shaft is **not rotating** we use an **effective diameter** value instead of the actual diameter, where

$$d_e = 0.37\, d$$

- A rectangular section of dimensions $h \times b$ has $d$ value as:

$$d = 0.808(hxb)^{1/2}$$

❖ and for nonrotating rectangular shaft, the effective diameter strategy is utilized

## ✓ Loading Factor ($k_c$)

- The rotating beam specimen is loaded in **bending**. Other types of loading will have a different effect.
- The load factor for the **different types of loading** is:

$$k_c = \begin{Bmatrix} 1 & bending \\ 0.85 & axial \\ 0.59 & torsion \end{Bmatrix}$$

## ✓ Temperature Factor ($k_d$)

- When the operating temperature is **below room temperature**, the material becomes **more brittle**. When the **temperature is high** the yield strength decreases and the material becomes **more ductile** and creep may occur.
- For steel, the endurance limit slightly increases as temperature rises, then it starts to drop. Thus, the temperature factor is given as:

$$k_d = 0.9887 + 0.6507(10^{-3})T_c - 0.3414(10^{-5})T_c^2 + 0.5621(10^{-8})T_c^3 - 6.426(10^{-12})T_c^4$$

- The same value calculated by the equation are also presented in the below table where :.

$$k_d = \frac{S_T}{S_{RT}}$$

| Temperature, °C | $S_T/S_{RT}$ | Temperature, °F | $S_T/S_{RT}$ |
|---|---|---|---|
| 20 | 1.000 | 70 | 1.000 |
| 50 | 1.010 | 100 | 1.008 |
| 100 | 1.020 | 200 | 1.020 |
| 150 | 1.025 | 300 | 1.024 |
| 200 | 1.020 | 400 | 1.018 |
| 250 | 1.000 | 500 | 0.995 |
| 300 | 0.975 | 600 | 0.963 |
| 350 | 0.943 | 700 | 0.927 |
| 400 | 0.900 | 800 | 0.872 |
| 450 | 0.843 | 900 | 0.797 |
| 500 | 0.768 | 1000 | 0.698 |
| 550 | 0.672 | 1100 | 0.567 |
| 600 | 0.549 | | |

## ✓ Reliability Factor ($k_e$)

- The **mean** endurance limit is shown to be $Se'/S_{ut}$ =0.5. Most **endurance strength** data are reported as **mean values**.
- Data presented by Haugen and Wirching show standard **deviations** of endurance strengths of less than 8%.
- For other values of reliability, $k_e$ is found from the table given here.

| Reliability, % | Transformation Variate $z_a$ | Reliability Factor $k_e$ |
|---|---|---|
| 50 | 0 | 1.000 |
| 90 | 1.288 | 0.897 |
| 95 | 1.645 | 0.868 |
| 99 | 2.326 | 0.814 |
| 99.9 | 3.091 | 0.753 |
| 99.99 | 3.719 | 0.702 |
| 99.999 | 4.265 | 0.659 |
| 99.9999 | 4.753 | 0.620 |

## ✓ Miscellaneous Factor ($k_f$)

- It is used to account the reduction in the endurance limit due to all other effects (such as residual stress, corrosion, cyclic frequency, metal spraying, etc.)
- However, these effects are not fully characterized and usually not accounted for. Thus we use $k_f$ =1.

## ✓ Stress concentration and notch sensitivity.

- Under fatigue loading conditions, crack initiation and growth usually starts in locations having high stress concentrations.
- The presence of stress concentration **reduces** the fatigue life of an element and the endurance limit as well and it must be considered in fatigue failure analysis.
- Due to the **difference in ductility**, the effect of stress concentration on fatigue properties is not the same for different materials.
- For materials under fatigue loading, the maximum stress near a notch (hole, fillet, etc.) is:

$$\sigma_{max} = k_f . \sigma_o \qquad or \qquad \tau_{max} = k_{fs} . \tau_o$$

❖ where: $\sigma_o$: the nominal stress.
$k_f$: the fatigue stress concentration factor which is a reduced value of the stress concentration factor ($k_t$) because of the difference in material sensitivity to the presence of notches.

and $k_f$ is defined as:

$$k_f = \frac{\text{max. stress in notched specimen}}{\text{stress in notch-free sepcimen}}$$

- **Notch sensitivity** ($q$) is defined as:

$$q = \frac{k_f - 1}{k_t - 1} \quad or \quad q_{shear} = \frac{k_{fs} - 1}{k_{ts} - 1}$$

- ❖ The values of $q$ ranges from 0 to 1

  $q$=0 ⟶ $k_f$=1 material is not sensitive

  $q$=1 ⟶ $k_f$=$k_t$ material is fully sensitive

- ❖ Thus

$$k_f = 1 + q(k_t - 1) \quad or \quad k_{fs} = 1 + q_s(k_{ts} - 1)$$

- ❖ For steels and Aluminum, the notch sensitivity for **Bending and Axial** loading can be found from figure A and for **Torsion** is found from Figure B.

- ❖ For cast iron, the notch sensitive is very low from 0 to 0.2, but to be conservative it is recommended to use $q$=0.2.

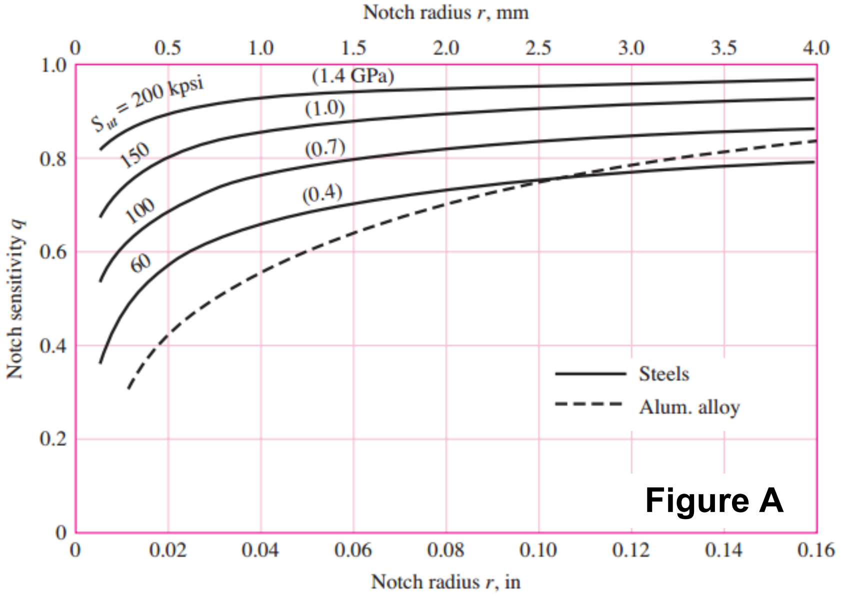


Figure A

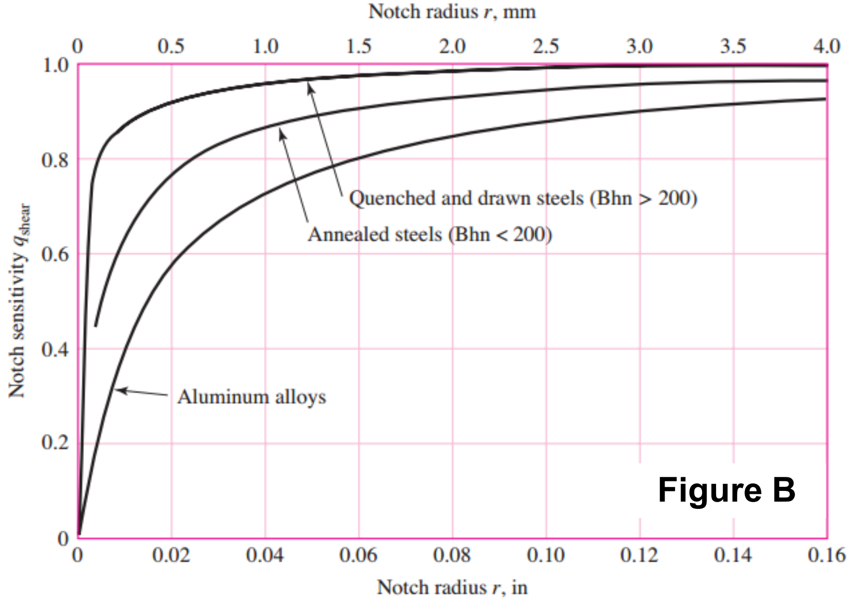


Figure B

- Heywood distinguished between **different types of notches** (hole, shoulder, groove) and according to him, $k_f$ is found as:

$$k_f = \frac{k_t}{1 + \frac{2(k_t - 1)}{k_t}\frac{\sqrt{a}}{\sqrt{r}}}$$

❖ Where, $r$ : radius
$\sqrt{a}$ : constant that depends on the type of the notch.

❖ For steels, $\sqrt{a}$ for different types of notches is given in the table below.

| Notch Type | $\sqrt{a}(\sqrt{in})$, $S_{ut}$ in kpsi | $\sqrt{a}(\sqrt{mm})$, $S_{ut}$ in MPa | Coefficient of Variation $C_{Kf}$ |
|---|---|---|---|
| Transverse hole | $5/S_{ut}$ | $174/S_{ut}$ | 0.10 |
| Shoulder | $4/S_{ut}$ | $139/S_{ut}$ | 0.11 |
| Groove | $3/S_{ut}$ | $104/S_{ut}$ | 0.15 |

- For **simple loading**, $k_f$ can be **multiplied** by the stress value, or the endurance limit can be reduced by **dividing** it by $k_f$. However, for **combined loading** each of stress has to be multiplied by its corresponding $k_f$ value.