What is ELASTIC DEFORMATION ?

- Material Properties (thermal, optical, mechanical, physical, chemical, and nuclear) **connected to material structure.**
- Material structure is **result of** synthesis and processing.
- In general, the materials are divided to:
  1. Metals
  2. Ceramics
  3. Polymers
  4. Composite materials

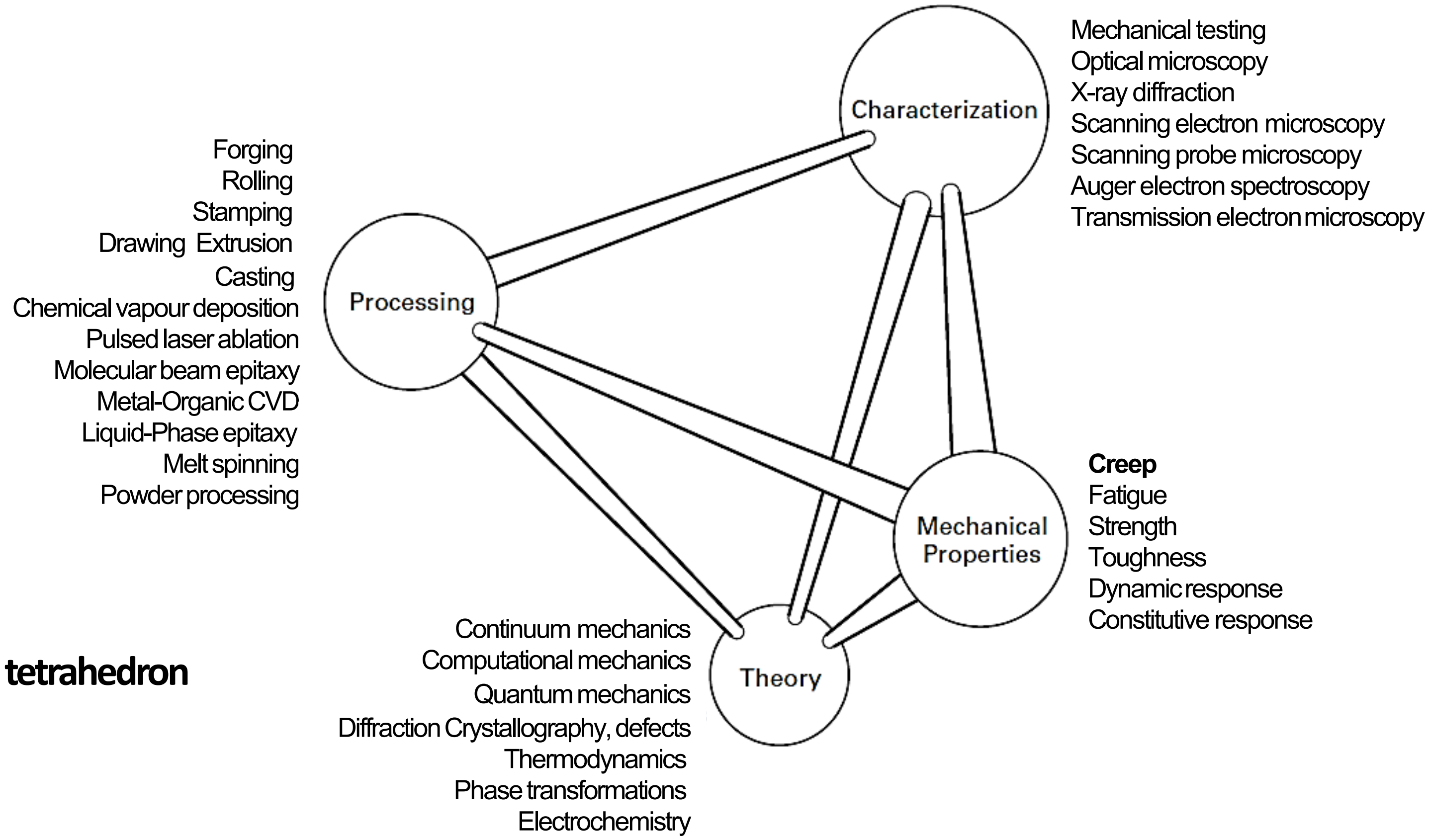


Fig. 1.1. Thomas's iterative tetrahedron

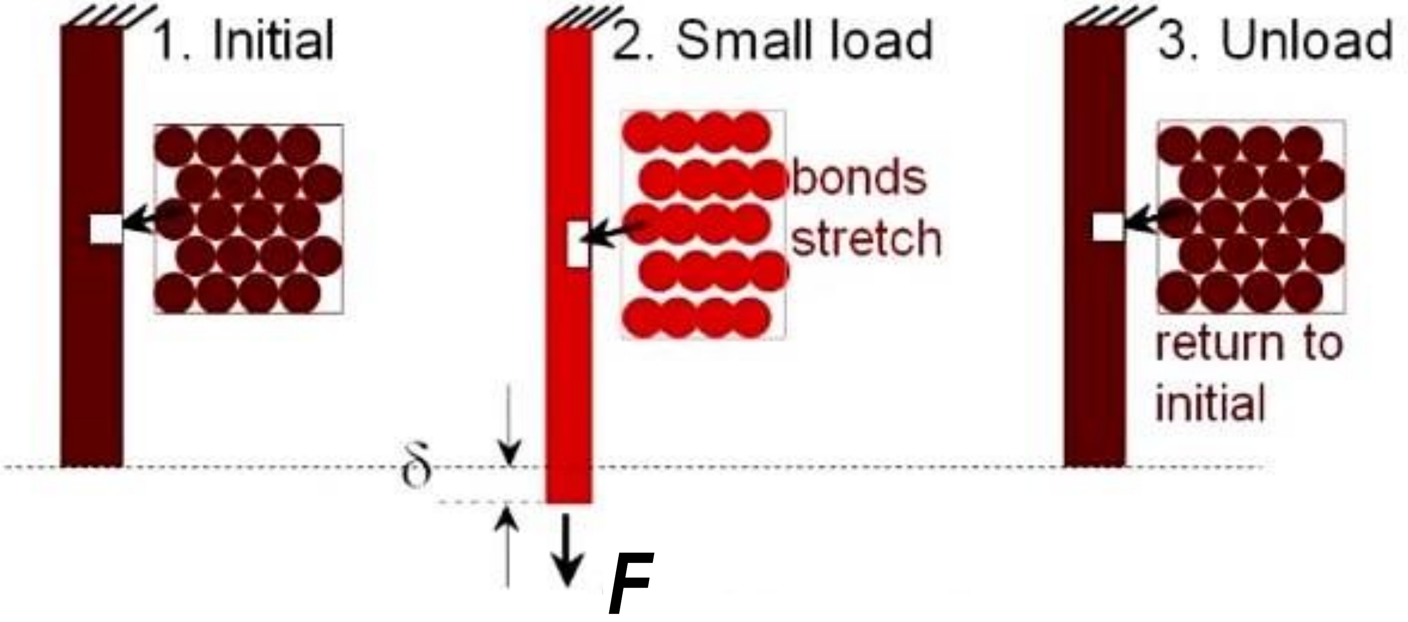
- According to the deformation characteristics, the materials can be classified into:

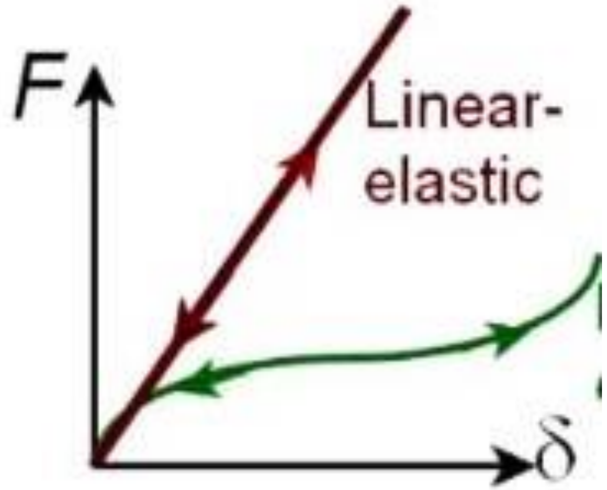
  - **Isotropic material:**
    Symmetrical properties in relation to **any plane** containing the material point under consideration or by other words these materials show the same behavior in all directions (**do not depend** on the direction) such as in metal and glass.

  - **Anisotropic material:**
    Anisotropic behavior means **different properties** in different directions, it is the opposite of isotropic. Wood and composite are good examples of anisotropic materials. Properties of theses materials are **depending on directions**.

  - **Orthotropic material:**
    It is a special case of anisotropic materials. Materials have properties that differ along **three mutually-orthogonal (perpendicular) twofold axes** of rotational symmetry (i.e. material is symmetric with respect to **two plane**) such as in wood and sheet metal.

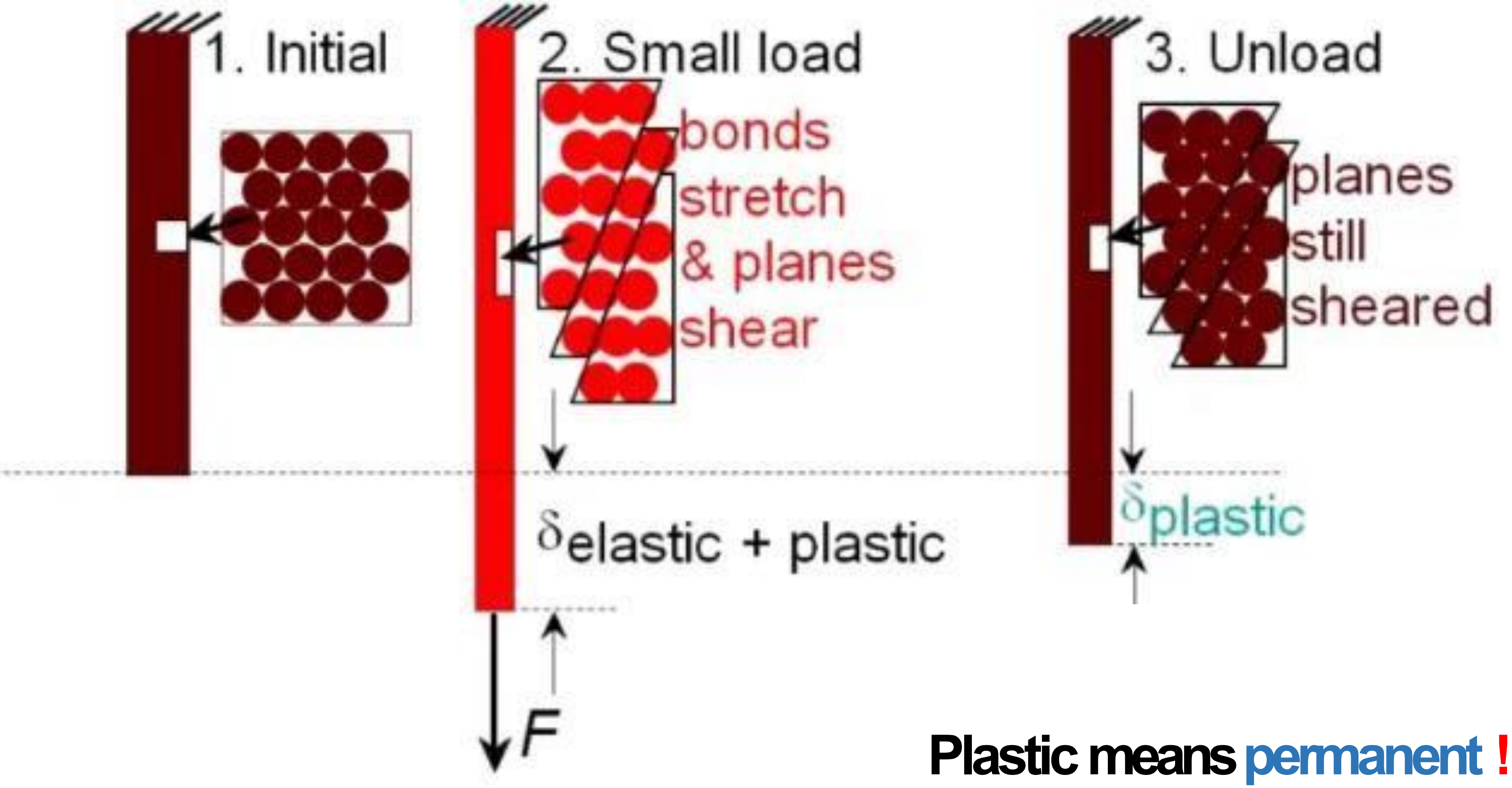
- Elasticity or elastic behaviour deals with elastic stresses and strains, their relationship, and the external forces that cause them.
- **Elastic strain** is defined as a strain that **disappears** instantaneously once the forces that cause it are removed.



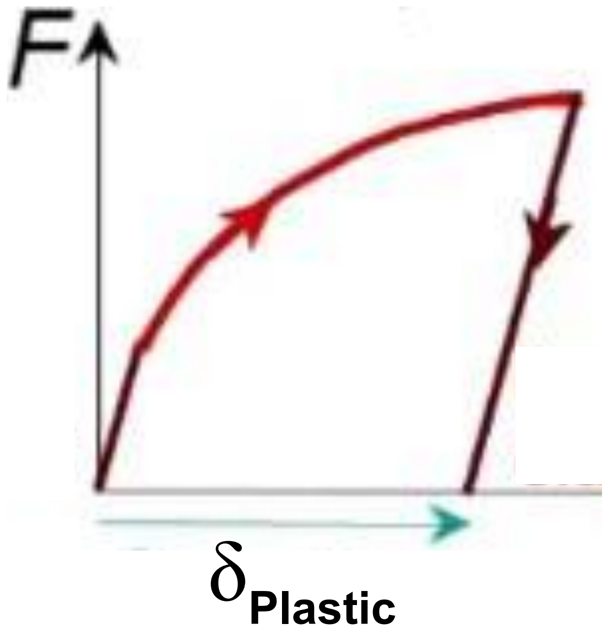


Elastic means reversible !

Plastic deformation



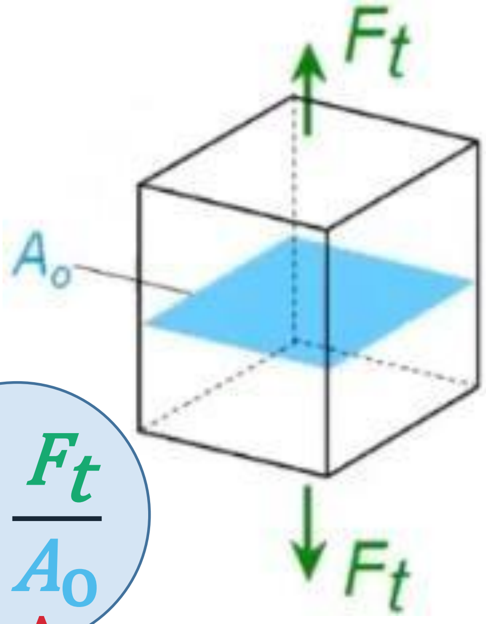
Plastic means permanent !



State of Stresses

- Force is not important unless we know **on how big an area** it is acting on.
- Definition: force per area.

Tensile stress (σ)

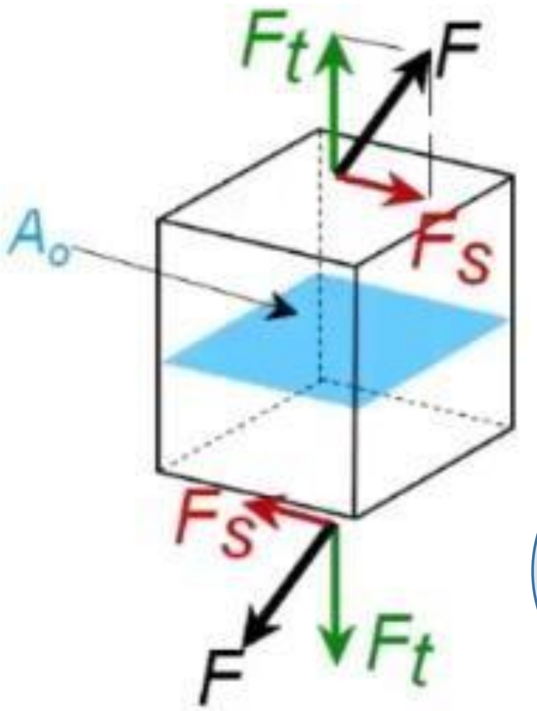


$$\sigma = \frac{F_t}{A_0}$$

Original area before loading

Stress has units:
N/m^2 OR lb/in^2

Shear stress (τ)



$$\tau = \frac{F_s}{A_0}$$

State of Stresses

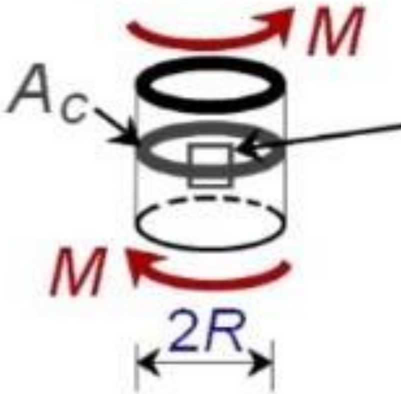
- **Simple tension: cable**

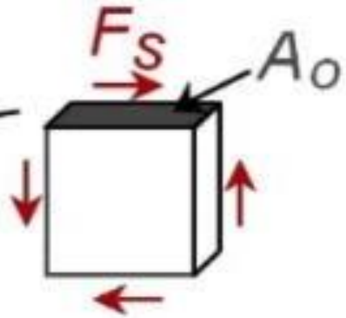
A_o = cross sectional area (when unloaded)

$$\sigma = \frac{F}{A_o}$$

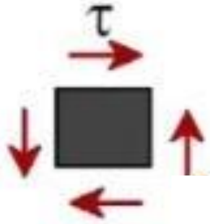
Ski lift (photo courtesy P.M. Anderson)

- **Torsion (a form of shear): drive shaft**





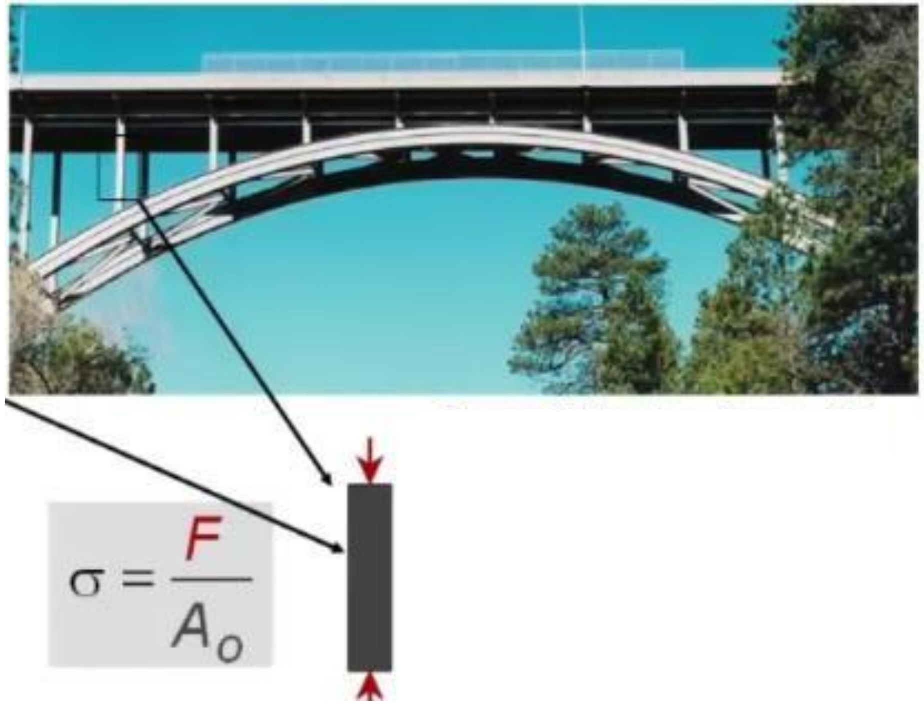
$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_cR$

State of Stresses

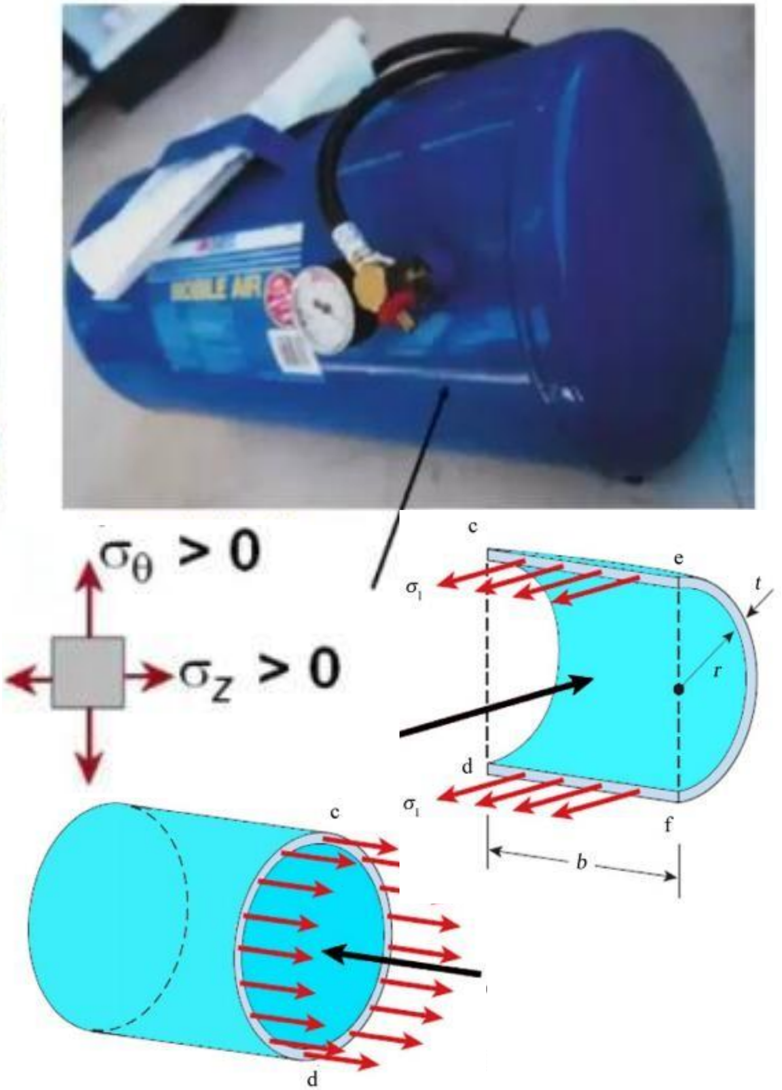
Simple compression



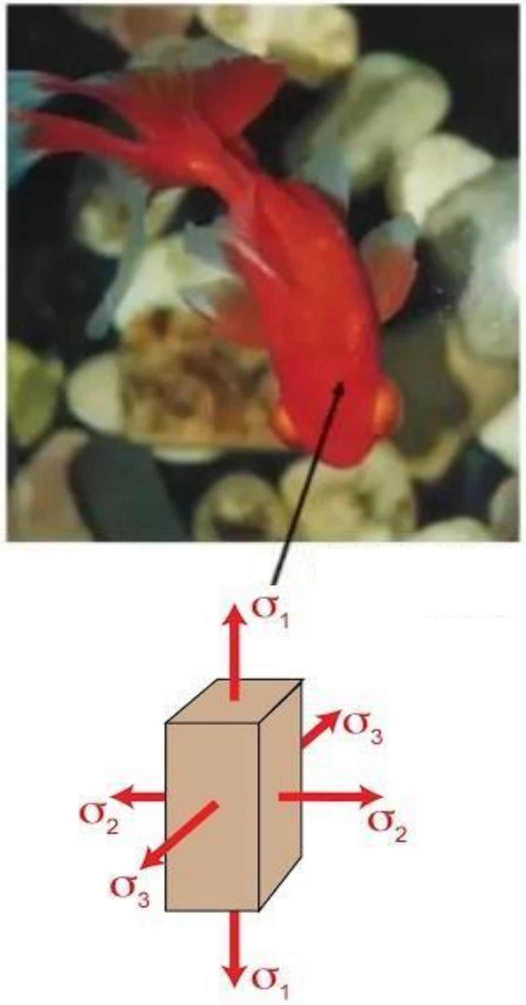
Note: compressive structure member

(σ<(0

Bi-axial tension



Hydrostatic compression



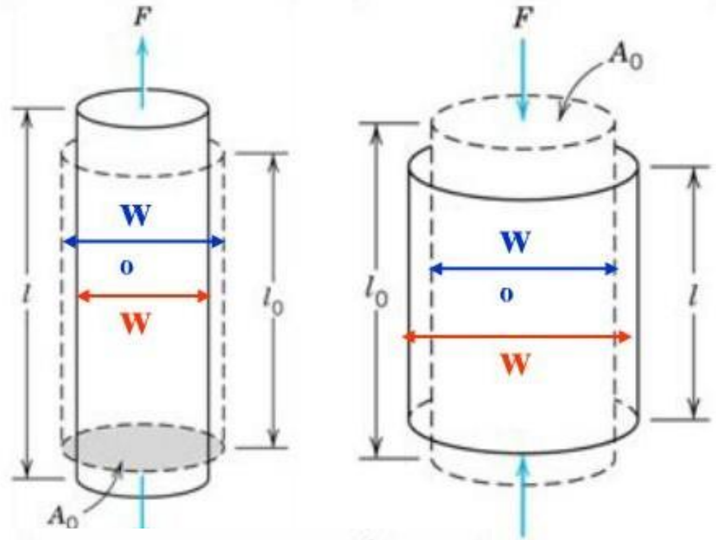
$0= \sigma_h<_3= \sigma_2= \sigma_1\sigma$

Strain ⇨ **Response** of material to applied stress (measure of deformation.(

- **Tensile Strain:**

$$\varepsilon = \frac{\Delta l}{l_o} = \frac{l - l_o}{l_o}$$

This is called engineering strain

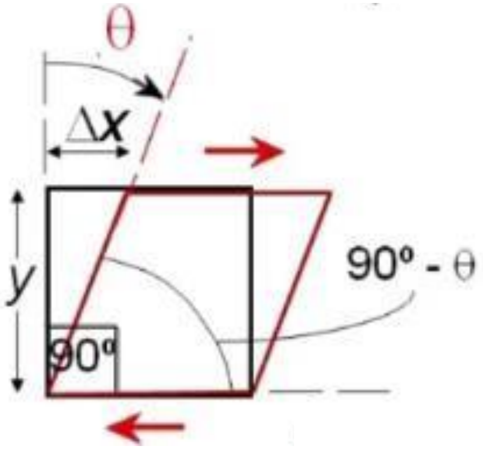


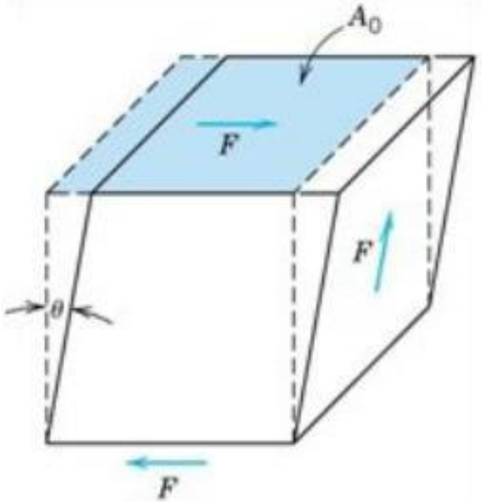
- **Lateral Strain**

$$\varepsilon_w = \frac{\Delta w}{w_o} = \frac{w - w_o}{w_o}$$

- **Shear Strain**

$\gamma = \Delta x / y = \tan \theta$



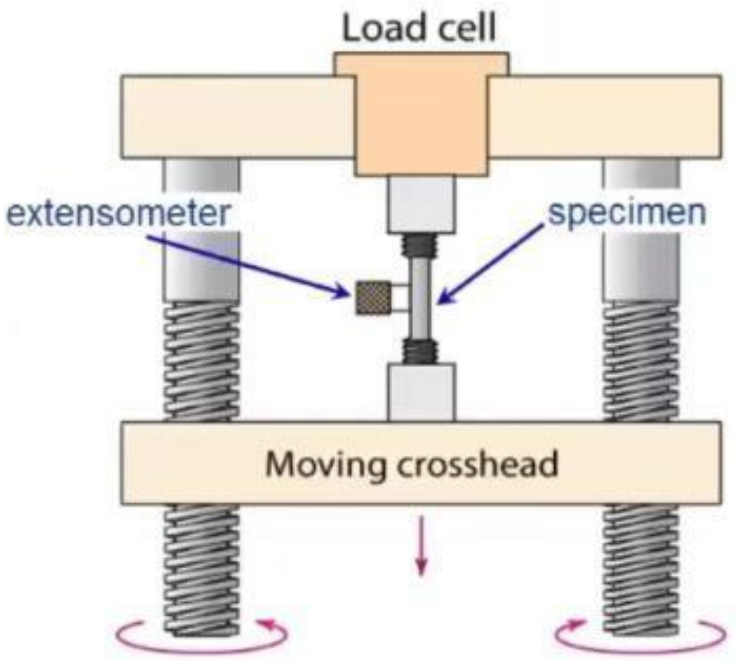


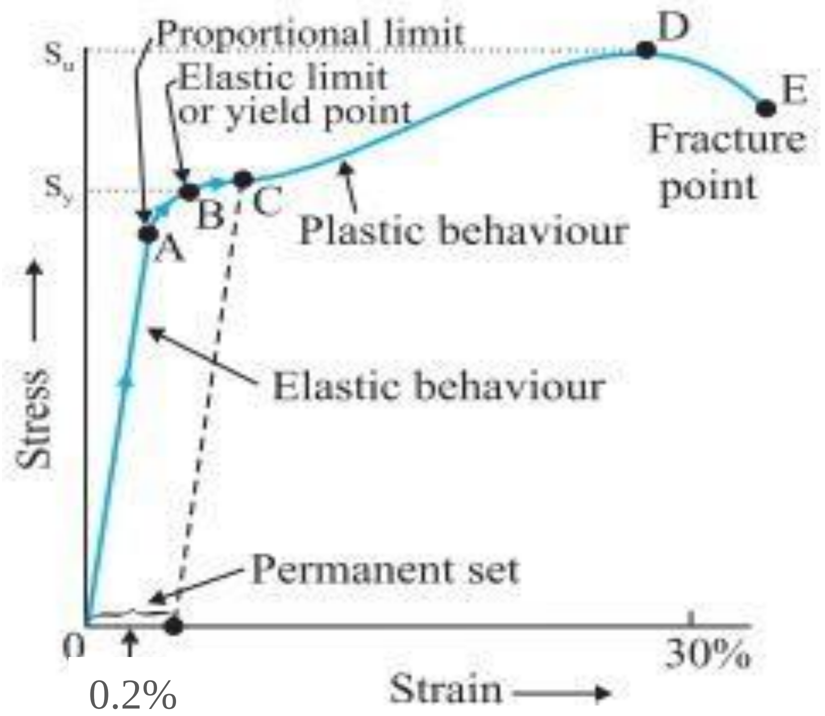
Strain is always dimensionless

Relation between stress and strain

- **Tensile (compressive) testing**
  - Recording load
  - Recording displacement

- **Tensile stress-strain curve**
  - Convert force to stress
  - Convert displacement into strain
  - Plot stress vie strain





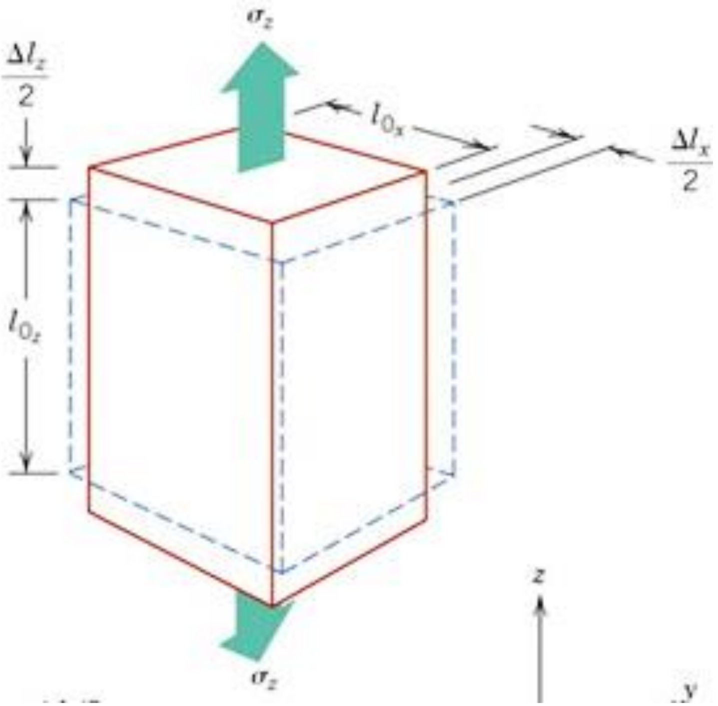
Linear elastic properties

- **Poisson's ratio**

Elongation in the axial direction (z) causes **constructions**, and hence strains, in the lateral directions (x, y).

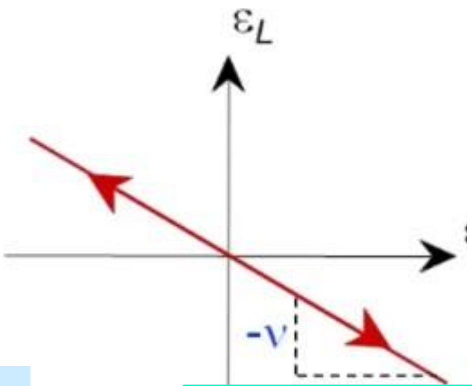
$$v = \frac{-\varepsilon_x}{\varepsilon_z} = \frac{-\varepsilon_y}{\varepsilon_z}$$

isotropic materials



$$\frac{\varepsilon_z}{2} = \frac{\Delta l_z/2}{l_{o_z}}$$

$$-\frac{\varepsilon_x}{2} = \frac{\Delta l_x/2}{l_{o_x}}$$



Metals: $v \simeq 0.3$
Ceramics: $v \simeq 0.25$
Polymers: $v \simeq 0.4$

Units:
E: [GPa] or [psi]
ν: dimensionless

$v = 0.5$ **density increases**
$v < 0.5$ **density decreases (voids form(**

Poisson's ratio and volume conservation

Do you think there is a relation between Poisson's ratio and material compressibility?

Poisson's ratio = **0.5** / **0.3** } **WHY and HOW**

- **Modulus of elasticity (E)**
  - Measure of the resistance of material to elastic deformation
  - The higher the elastic modulus, the stiffer the material (**Stiffness: how resist to elastic deformation**).

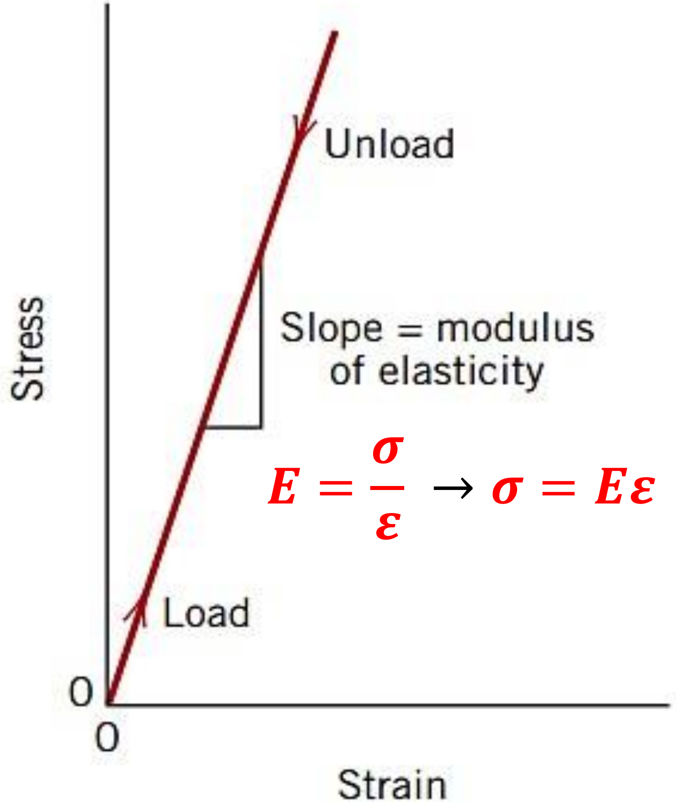
Hooke's law: F=kΔL=kεL$_o$ (define the linear relationship between applied force and deflection within elastic region)

However, **F=σA** to be used in Hook's law

F=σA=kεL$_o$

$$\therefore \sigma = \frac{kL_o}{A}\varepsilon \qquad \because E = \frac{kL_o}{A}$$

$$\therefore \boldsymbol{\sigma = E\varepsilon}$$

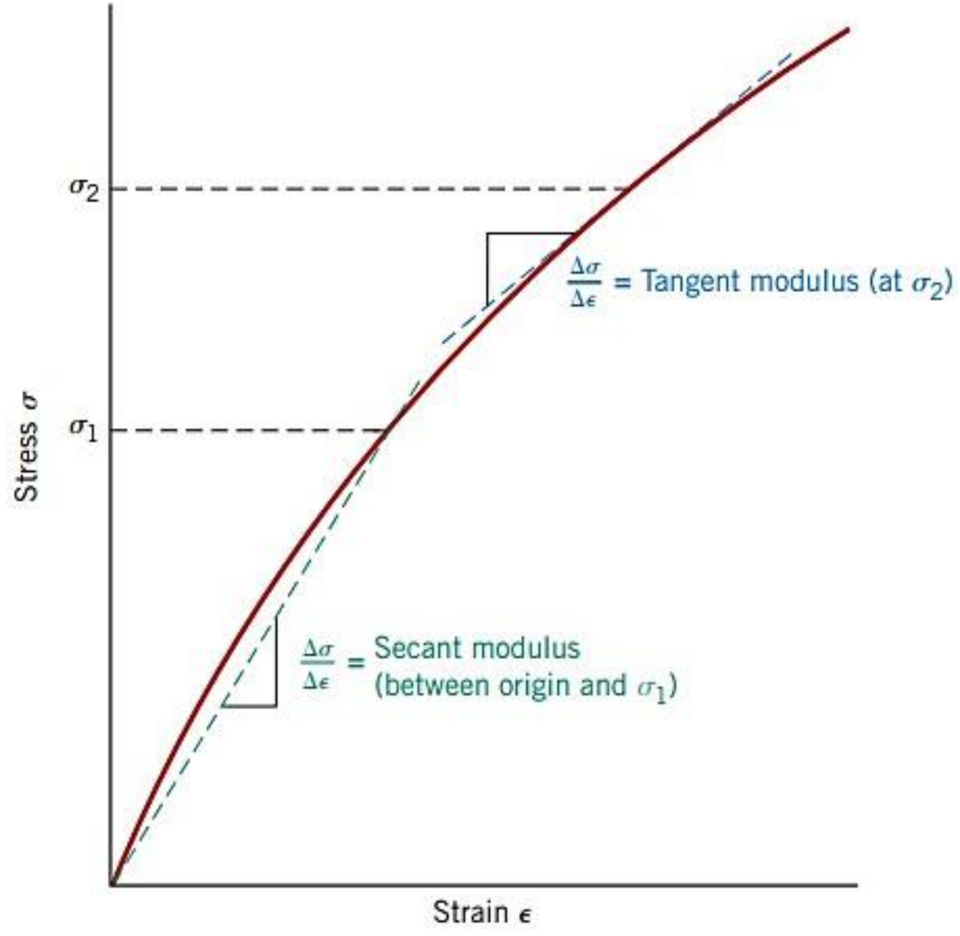


$$E = \frac{\sigma}{\varepsilon} \rightarrow \sigma = E\varepsilon$$

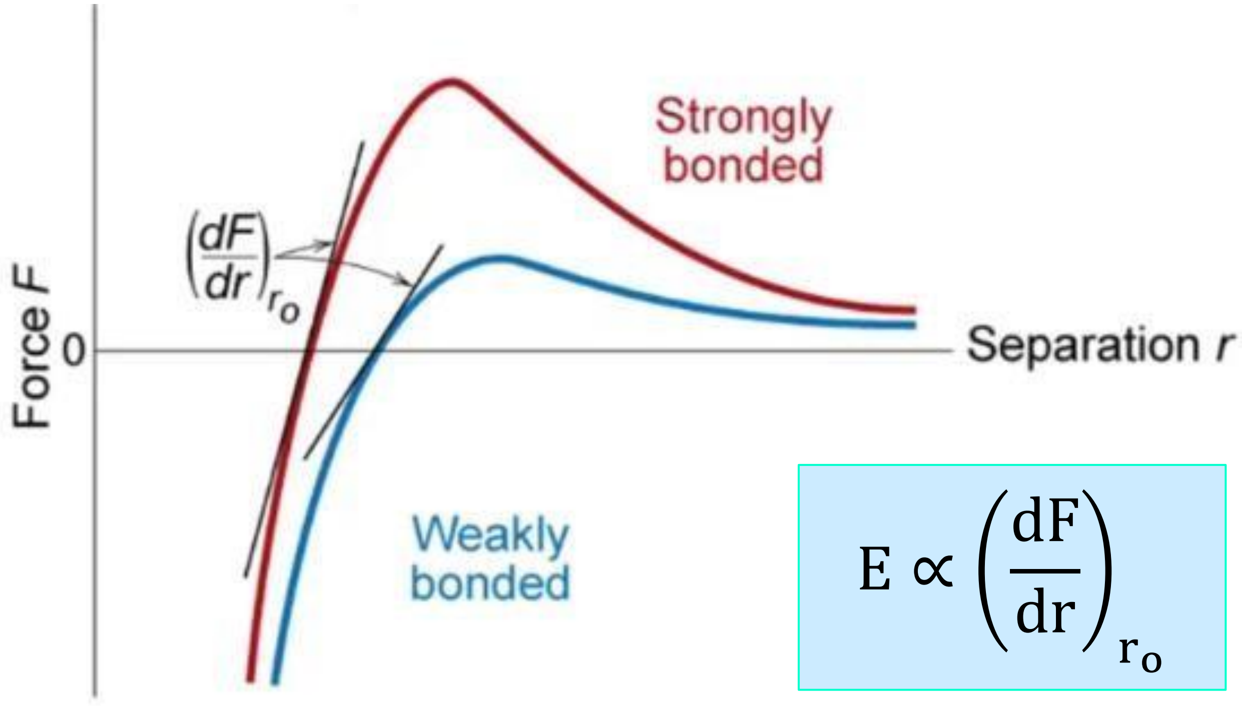
- There are some materials (e.g., gray cast iron, concrete, and many polymers) for which this elastic portion of the stress–strain curve is not linear

- for such non-linear behavior either **tangent** or **secant** modulus is normally used.

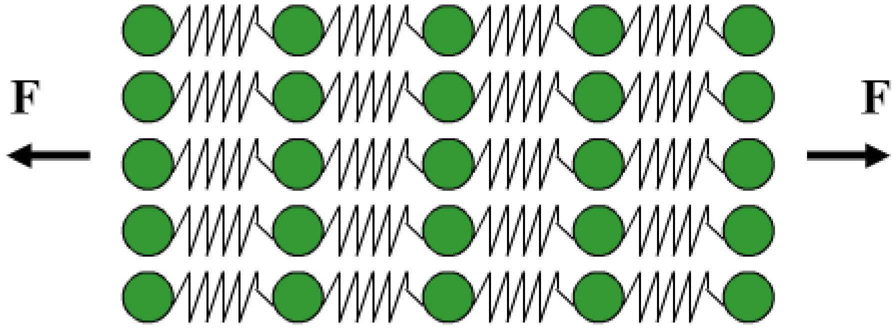
  - **Tangent** modulus is taken as the slope of the stress–strain curve at some specified level of stress.
  - **secant** modulus represents the slope of a secant drawn from the origin to some given point of the curve



- Slope of stress-strain plot)which is proportional to the elastic modulus) depends on **bond strength (bond stiffness)** of metal



$$E \propto \left(\frac{dF}{dr}\right)_{r_o}$$



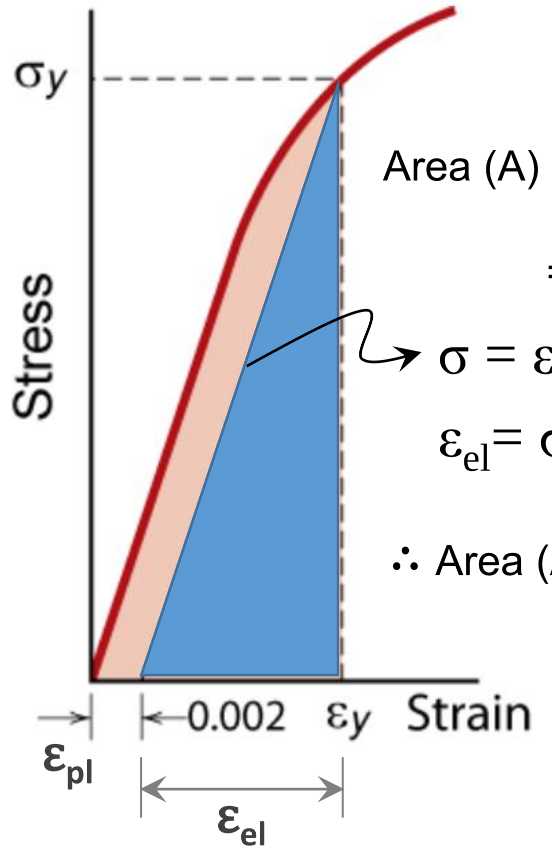
Resilience

- In materials science, **resilience** is the ability and the capacity of a material to **absorb energy** when it is deformed **elastically** and then, upon unloading, to **recover** this amount of energy.

- The maximum energy that can be absorbed up to the elastic limit, without creating a permanent deformation is known as **proof resilience**.

- In the stress-strain curve, it is given by the area under the portion of a stress–strain curve (up to yield point).

- Under assumption of linear elasticity or up to proportional limit, **resilience** can be calculated by integrating the stress–strain curve from zero to the **proportional limit**.



$$\text{Area (A)} = \frac{1}{2}\,b.h$$

$$= \frac{1}{2}\,\varepsilon_{el}.\sigma$$

$$\sigma = \varepsilon_{el}.E$$

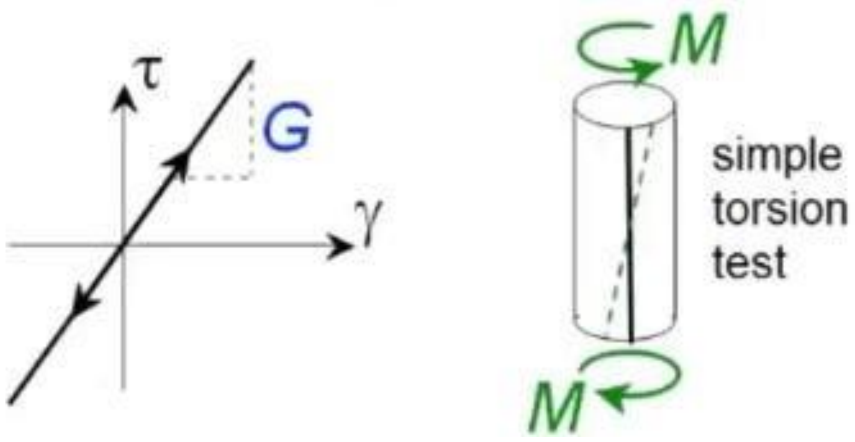
$$\varepsilon_{el} = \sigma/E$$

$$\therefore \text{Area (A)} = \frac{\sigma_y^2}{E}$$

Linear Elastic Properties

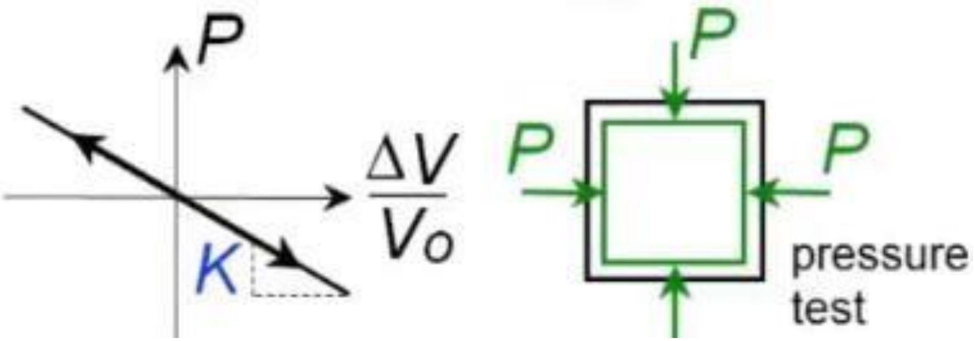
- **Elastic shear modulus (G):**

$$\tau = G\gamma$$



- **Elastic bulk modulus (*k*):**

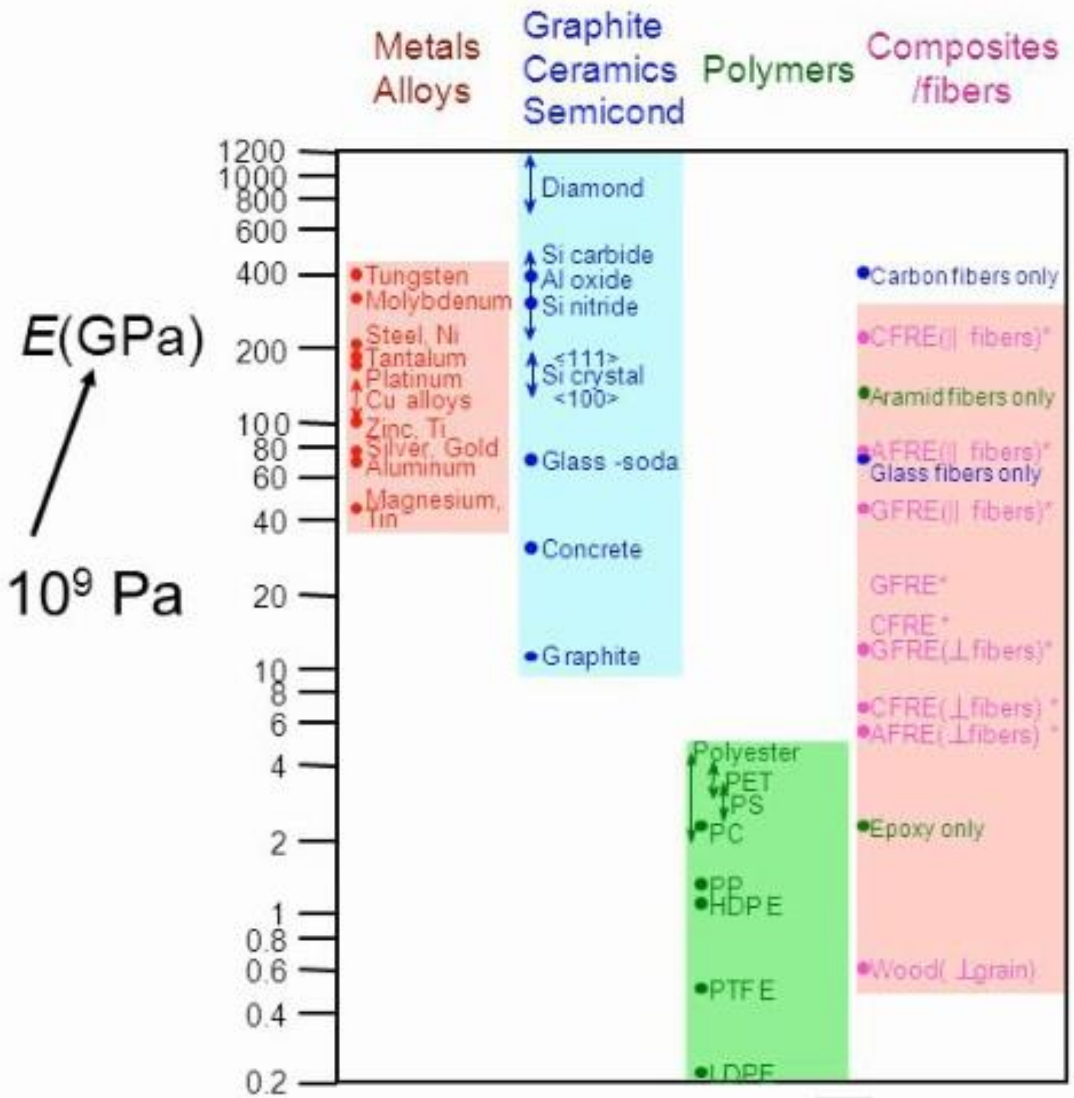
$$P = k\,(\Delta V / V_o)$$



- **Special relations for isotropic materials**

$$G = \frac{E}{2(1+v)} \qquad k = \frac{E}{3(1-2v)}$$

Modulus of elasticity (comparison)



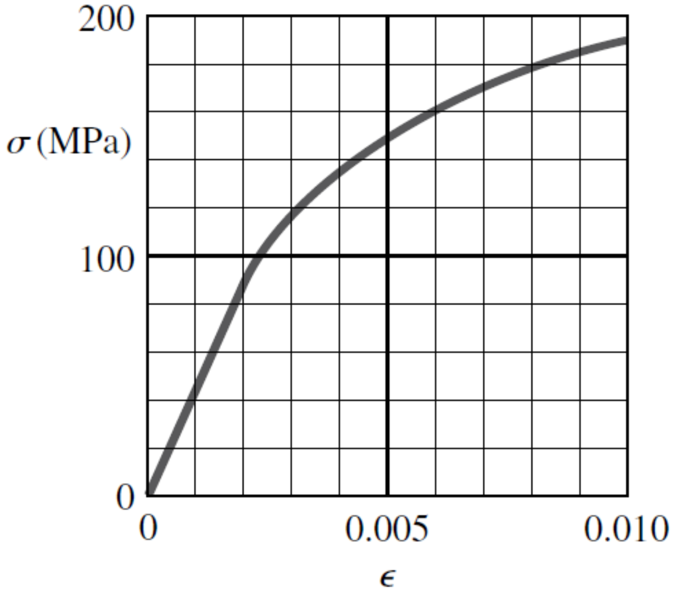
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Example:

A circular bar of magnesium alloy is 750 mm long. The stress strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?

(b) If the bar is reloaded, what is the proportional limit?



Solution:

Stress and strain at point B:

$$\varepsilon_B = \frac{\delta}{L} \qquad \varepsilon_B = 8 \times 10^{-3} \ \& \ \sigma_B = 180\text{MPa}$$

Elastic recovery

$$Slope = \frac{178}{0.004} \qquad Slope = 4.45 \times 10^4$$

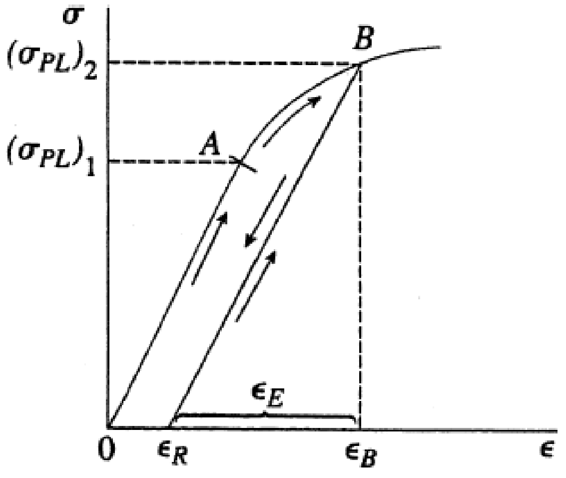
$$\varepsilon_E = \frac{\sigma_B}{slope} = 4.045 \times 10^{-3}$$

Residual strain:

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 3.95 \times 10^{-3}$$

Permanent set :

$$\varepsilon_R L = 2.97mm$$



H.W.

A bar made of structural steel having the stress strain diagram shown in the figure has a length of 48 in. The yield stress of the steel is 42 ksi and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 30 103 ksi. The bar is loaded axially until it elongates 0.20 in., and then the load is removed. How does the final length of the bar compare with its original length?