

كلية الهندسة	الكلية
الكهرباء	القسم
Electrical Drives	المادة باللغة الانجليزية
المساقات	المادة باللغة العربية
الرابعة	المرحلة الدراسية
م.م. زياد طارق محمد	اسم التدريسي
Induction Motor Drives	عنوان المحاضرة باللغة الانجليزية
محررات البحث	عنوان المحاضرة باللغة العربية
7	رقم المحاضرة
1) Mohummed Rashid" Power electronics circuits, Devices application" 4th edition, 2014 and	المصادر والمراجع
2) Gopal K. Dubey " power semiconductor controlled Drives" 1st edition, 1989	

# Induction Motor Drives

Three-phase induction motors are commonly used in adjustable-speed drives [1] and they have three-phase stator and rotor windings. The stator windings are supplied with balanced three-phase ac voltages, which produce induced voltages in the rotor windings due to transformer action. It is possible to arrange the distribution of stator windings so that there is an effect of multiple poles, producing several cycles of magnetomotive force (mmf) (or field) around the air gap. This field establishes a spatially distributed sinusoidal flux density in the air gap. The speed of rotation of the field is called the *synchronous speed*, which is defined by

$$\omega_s = \frac{2\omega}{p}$$

where  $p$  is the number of poles and  $\omega$  is the supply frequency in rads per second.

When the stator is supplied by a balanced three-phase ac source of frequency  $\omega$  radians per second (or  $f$  Hz), a rotating field moving at a synchronous speed  $\omega_m$  radians per sec is produced

If a stator phase voltage,  $v_s = \sqrt{2}V_s \sin \omega t$ , produces a flux linkage (in the rotor) given by

$$\phi(t) = \phi_m \cos(\omega_m t + \delta - \omega_s t)$$

the induced voltage per phase in the rotor winding is

$$\begin{aligned} e_r &= N_r \frac{d\phi}{dt} = N_r \frac{d}{dt}[\phi_m \cos(\omega_m t + \delta - \omega_s t)] \\ &= -N_r \phi_m (\omega_s - \omega_m) \sin[(\omega_s - \omega_m)t - \delta] \\ &= -s E_m \sin(s\omega_s t - \delta) \\ &= -s \sqrt{2} E_r \sin(s\omega_s t - \delta) \end{aligned}$$

where  $N_r$  = number of turns on each rotor phase;

$\omega_m$  = angular rotor speed or frequency, Hz;

$\delta$  = relative position of the rotor;

$E_r$  = rms value of the induced voltage in the rotor per phase, V;

$E_m$  = peak induced voltage in the rotor per phase, V.

and  $s$  is the slip, defined as

If the rotor speed is  $\omega_m$  rad/ sec then the relative speed between the stator rotating field and the rotor is given by  $\omega_{ml} = \omega_s - \omega_m$ . where  $\omega_{ml}$  is called the slip speed. The parameter  $s$  is known as slip and is given by

$$s = \frac{\omega_s - \omega_m}{\omega_s}$$

$$\omega_m = \omega_s(1 - s)$$

It is also possible to convert a mechanical speed  $\omega_m$  to the rotor electrical speed  $\omega_{re}$  of the rotating field as given by  $\omega_{re} = (p/2) \omega_m$

In that case,  $\omega$  is the synchronous electrical speed,  $\omega_{re}$ . The slip speed becomes  $\omega_{sl} = \omega - \omega_{re} = \omega - \omega_r$ . Thus, the slip can also be defined as

$$s = \frac{\omega - \omega_r}{\omega} = 1 - \frac{\omega_r}{\omega}$$

Which gives the rotor electrical speed as

$$\omega_r = \omega(1 - s)$$

The equivalent circuit for one phase of the rotor is shown in Fig.1.

Where  $R_r$  is the resistance per phase of the rotor windings;  $X_r$  is the leakage reactance per phase of the rotor at the supply frequency;  $E_r$  represents the induced rms phase voltage when the speed is zero or  $s = 1$ . where  $R_r$  and  $X_r$  are referred to the rotor winding

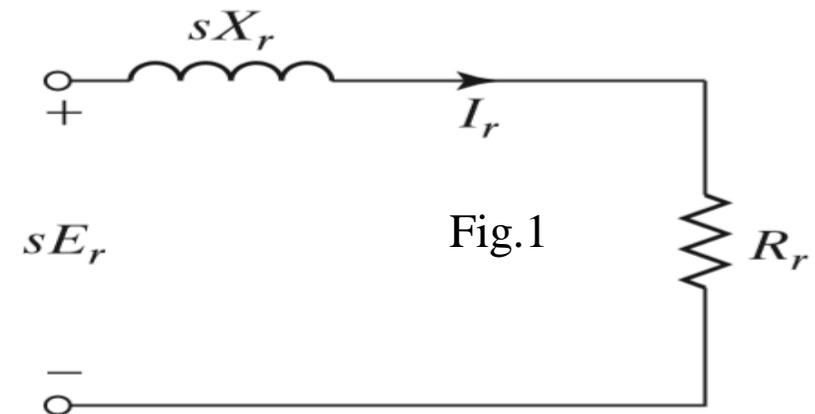
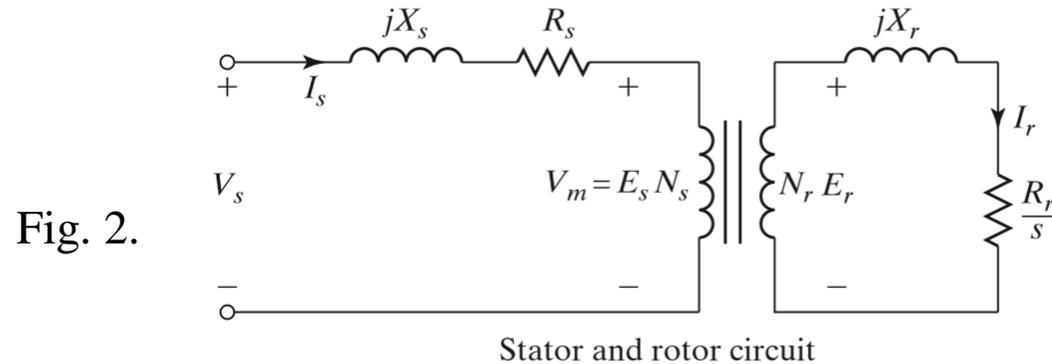


Fig.1  
Rotor circuit

The per-phase circuit model of induction motors is shown in Fig. 2. where  $R_s$  and  $X_s$  are the per-phase resistance and leakage reactance of the stator winding



The complete circuit model with all parameters referred to the stator is shown in Fig.3. where  $R_m$  represents the resistance for excitation (or core) loss and  $X_m$  is the magnetizing reactance.  $R'_r$  and  $X'_r$  are the rotor resistance and reactance referred to the stator.  $I'_r$  is the rotor current referred to the stator. There will be stator core loss, when the supply is connected and the rotor core loss depends on the slip. The friction and windage loss  $P_{no\ load}$  exists when the machine rotates. The core loss  $P_c$  may be included as a part of rotational loss  $P_{no\ load}$ .

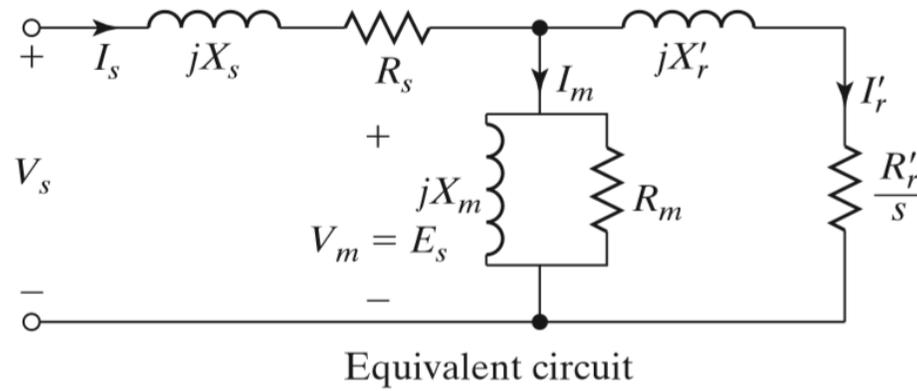


Fig.3

### performance Characteristics

$$P_{su} = 3I_s^2 R_s$$

$$P_{ru} = 3(I'_r)^2 R'_r$$

Core loss  $P_c = \frac{3V_m^2}{R_{...}} \approx \frac{3V_s^2}{R_{...}}$

Gap power (power passing from the stator to the rotor through the air gap)

$$P_g = 3(I'_r)^2 \frac{R'_r}{s}$$

Developed power  $P_d = P_g - P_{ru} = 3(I'_r)^2 \frac{R'_r}{s} (1 - s)$   
 $= P_g (1 - s)$

Developed torque  $T_d = \frac{P_d}{\omega_m} = \frac{P_g(1-s)}{\omega_s(1-s)} = \frac{P_g}{\omega_s}$

Input power  $P_i = 3 V_s I_s \cos \theta_m = P_c + P_{su} + P_g$

where  $\theta_m$  is the angle between  $I_s$  and  $V_s$ .

Output power  $P_o = P_d - P_{no load}$

Efficiency  $\eta = \frac{P_o}{P_i} = \frac{P_d - P_{no load}}{P_c + P_{su} + P_g}$

The value of  $X_m$  is normally large and  $R_m$ , which is much larger, can be removed from the circuit model to simplify the calculations. If  $X_m^2 \gg (R_s^2 + X_s^2)$ , then  $V_s \approx V_m$ , and the magnetizing reactance  $X_m$  may be moved to the stator winding to simplify further; this is shown in Fig 4

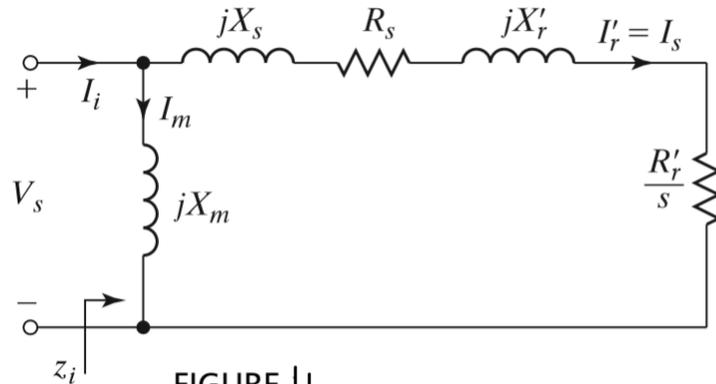


FIGURE 4

Approximate per-phase equivalent circuit.

$$I'_r = \frac{V_s}{[(R_s + R'_r/s)^2 + (X_s + X'_r)^2]^{1/2}}$$

$$T_d = \frac{3 R'_r V_s^2}{s \omega_s [(R_s + R'_r/s)^2 + (X_s + X'_r)^2]}$$

# Steady-State Stability

Let us examine the equilibrium points A and B, which are obtained when an induction motor drives the load TLI, as shown in figure 5. Let us first examine point A for the steady-state stability. A small increase in speed makes the load torque greater than the motor torque. Deceleration occurs and the operation is restored to point A. Similarly, a small decrease in speed causes the motor torque to exceed the load torque. Acceleration occurs and the operation is restored to point A. Thus, A is a stable equilibrium point. Let us next examine the stability of the equilibrium point B. A small increase in speed causes the motor torque to exceed the load torque. Acceleration takes place and the operating point moves away from B. Similarly, a small decrease in speed makes the load torque greater than the motor torque, causing deceleration and the operating point to drift away from B. Thus B is an unstable equilibrium point.

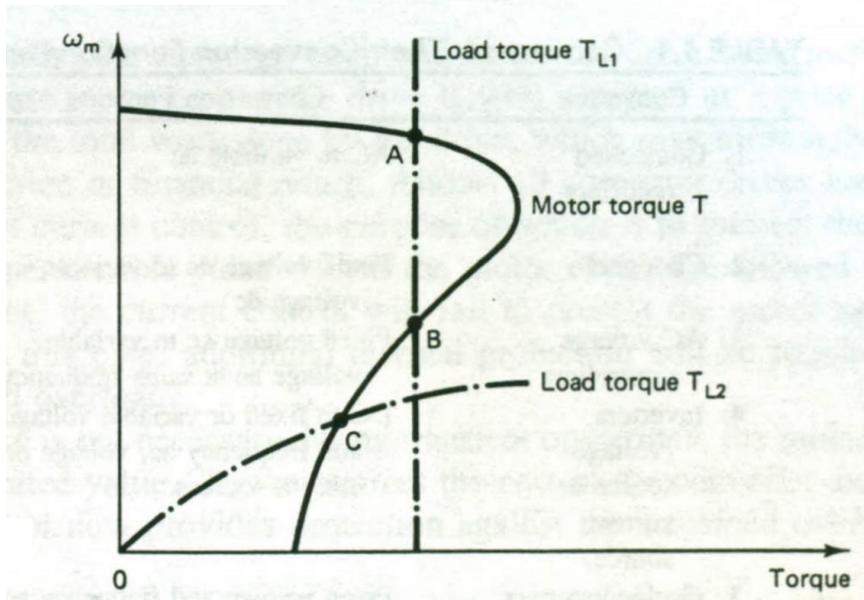


figure 5

## torque–speed Characteristics

If the motor is supplied from a fixed voltage at a constant frequency, the developed torque is a function of the slip and the torque–speed characteristics can be determined. A typical plot of developed torque as a function of slip or speed is shown in Figure 5. The slip is used as the variable instead of the rotor speed because it is nondimensional, and it is applicable to any motor frequency. Near the synchronous speed, that is, at low slips, the torque is linear and is proportional to slip. Beyond the maximum torque (also known as *breakdown torque*), the torque is inversely proportional to slip as shown in Figure 5. At standstill, the slip equals unity, and the torque produced is known as *standstill torque*. To accelerate a load, this standstill torque has to be greater than the load torque. It is desirable that the motor operate close to the low-slip range for higher efficiency. This is due to the fact that the rotor copper losses are directly proportional to slip and are equal to the slip power. Thus, at low slips, the rotor copper losses are small. The operation in the reverse motoring and regenerative braking is obtained by the reversal of the phase sequence of the motor terminals. The reverse speed–torque characteristics are shown by dashed lines. There are three regions of operation: (1) motoring or powering,  $0 \leq s \leq 1$ ; (2) regeneration,  $s < 0$ ; and (3) plugging,  $1 \leq s \leq 2$ .

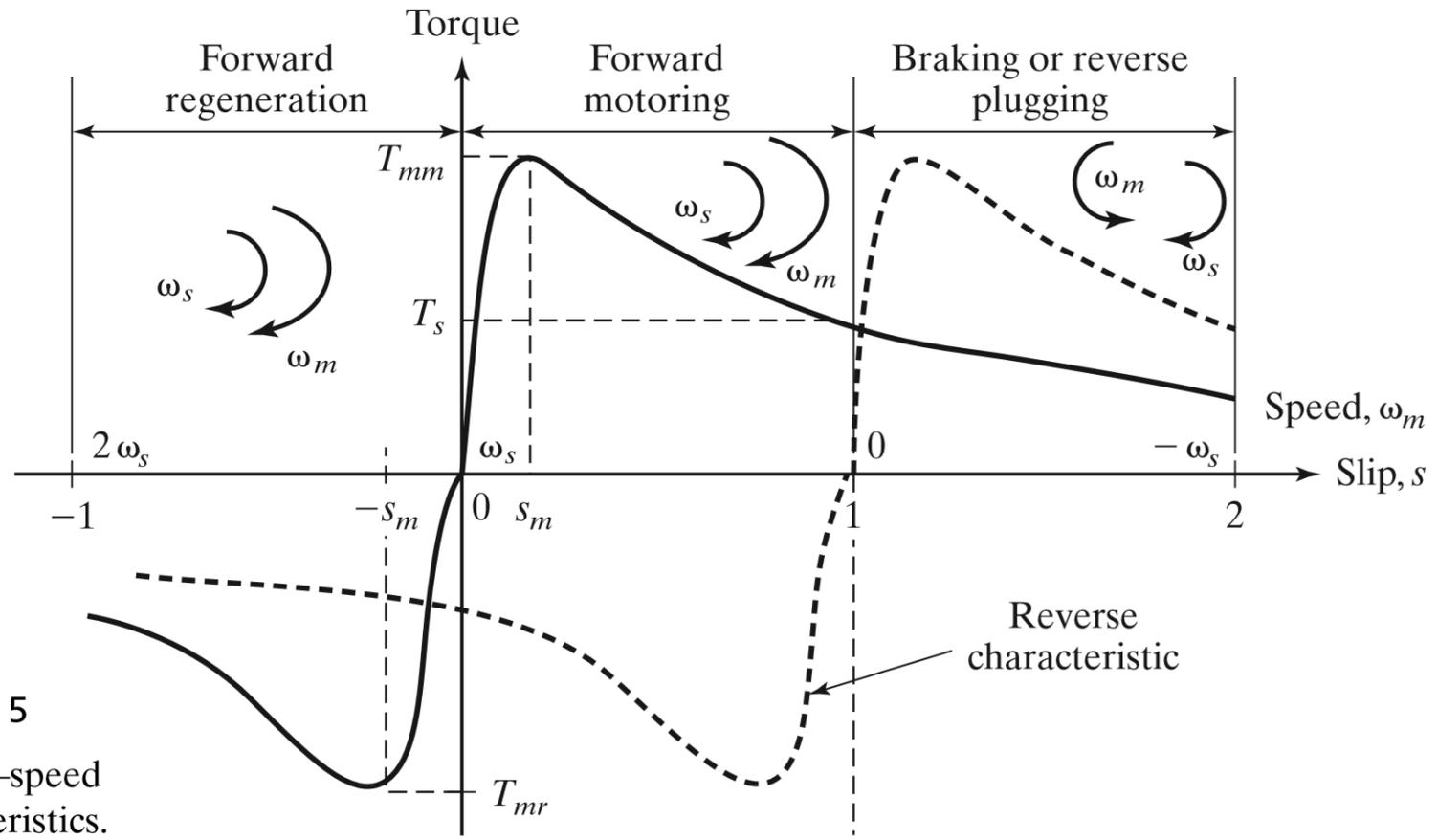


FIGURE 5  
Torque-speed characteristics.

$$T_s = \frac{3 R_r' V_s^2}{\omega_s [(R_s + R_r')^2 + (X_s + X_r')^2]}$$

$$s_m = \pm \frac{R_r'}{[R_s^2 + (X_s + X_r')^2]^{1/2}}$$

$$T_{mm} = \frac{3 V_s^2}{2\omega_s [R_s + \sqrt{R_s^2 + (X_s + X_r')^2}]}$$

$$T_{mr} = \frac{3 V_s^2}{2\omega_s [-R_s + \sqrt{R_s^2 + (X_s + X_r')^2}]}$$

$$T_d = \frac{3 R_r' V_s^2}{s\omega_s [(R_r'/s)^2 + (X_s + X_r')^2]}$$

$$T_s = \frac{3 R_r' V_s^2}{\omega_s [(R_r')^2 + (X_s + X_r')^2]}$$

$$s_m = \pm \frac{R_r'}{X_s + X_r'}$$

$R_s=0$

$$T_{mm} = -T_{mr} = \frac{3 V_s^2}{2\omega_s (X_s + X_r')}$$

$$\frac{T_s}{T_{mm}} = \frac{2R'_r(X_s + X'_r)}{(R'_r)^2 + (X_s + X'_r)^2} = \frac{2s_m}{s_m^2 + 1}$$

If  $s < 1$ ,  $s^2 < s_m^2$

$$\frac{T_d}{T_{mm}} = \frac{2s}{s_m} = \frac{2(\omega_s - \omega_m)}{s_m \omega_s}$$

which gives the speed as a function of torque.

$$\omega_m = \omega_s \left( 1 - \frac{s_m}{2T_{mm}} T_d \right)$$

The speed and torque of induction motors can be varied by one of the following

1. Stator voltage control
2. Rotor voltage control
3. Frequency control
4. Stator voltage and frequency control
5. Stator current control
6. Voltage, current, and frequency control

note. read example 15.1 pp.773

## 1. stator voltage Control

Torque indicates that the torque is proportional to the square of the stator supply voltage and a reduction in stator voltage can produce a reduction in speed. If the terminal voltage is reduced to  $bV$ . Since one cannot allow the terminal voltage to be more than the rated value, this method allows speed control only below the normal rated speed.

$$T_d = \frac{3R'_r(bV_s)^2}{s\omega_s[(R_s + R'_r/s)^2 + (X_s + X'_r)^2]}$$

where  $b < 1$ . Figure 6 shows the typical torque–speed characteristics for various values of  $b$ . The points of intersection with the load line define the stable operating points. In any magnetic circuit, the induced voltage is proportional to flux and frequency, and the rms air-gap flux can be expressed as  $V_a = bV_s = K_m\omega\phi$

$$\phi = \frac{V_a}{K_m \omega} = \frac{b V_s}{K_m \omega}$$

where  $K_m$  is a constant and depends on the number of turns of the stator winding. As the stator voltage is reduced, the air-gap flux and the torque are also reduced. At a

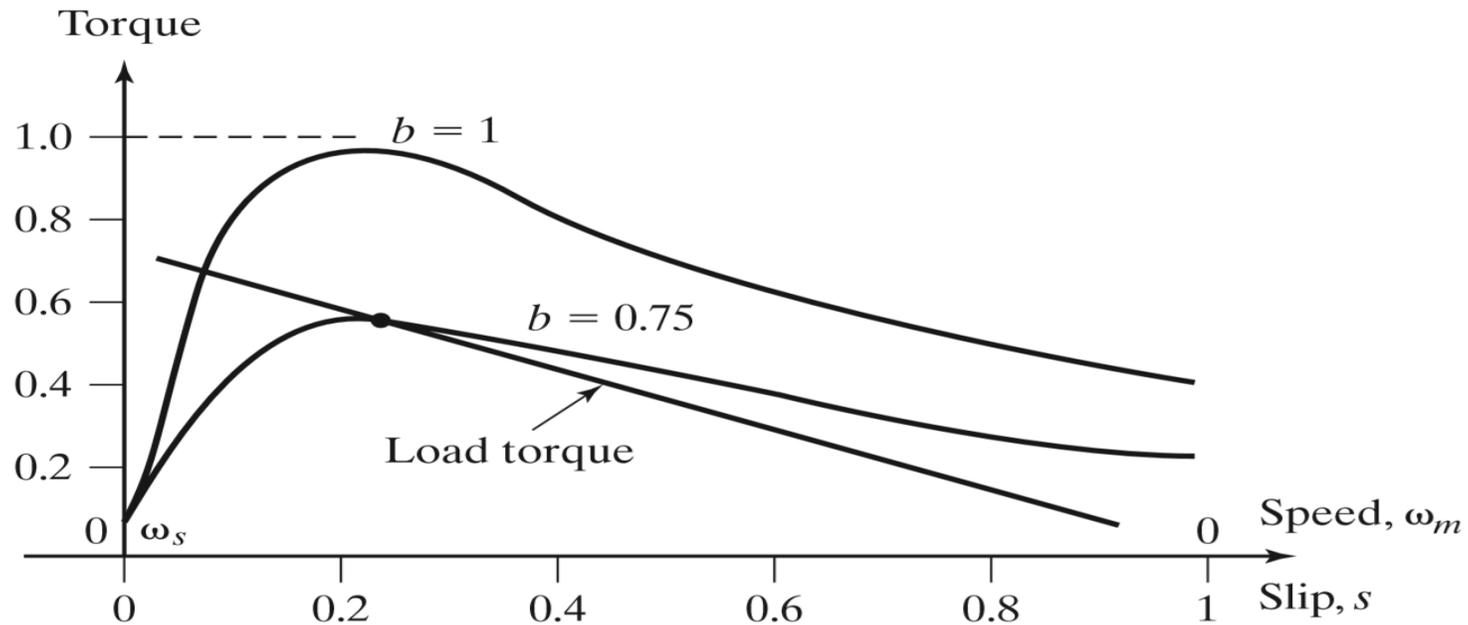


Figure 6 Torque–speed characteristics with variable stator voltage.

lower voltage, the current can be peaking at a slip of  $s_a = \frac{1}{3}$ . The range of speed control depends on the slip for maximum torque  $s_m$ . For a low-slip motor, the speed range is very narrow. This type of voltage control is not suitable for a constant-torque load and is normally applied to applications requiring low-starting torque and a narrow range of speed at a relatively low slip.

The stator voltage can be varied by three-phase (1) ac voltage controllers, (2) voltage-fed variable dc-link inverters, or (3) pulse-width modulation (PWM) inverters. However, due to limited speed range requirements, the ac voltage controllers are normally used to provide the voltage control. The ac voltage controllers are very simple. However, the harmonic contents are high and the input PF of the controllers is low. They are used mainly in low-power applications, such as fans, blowers, and centrifugal pumps, where the starting torque is low. They are also used for starting high-power induction motors to limit the in-rush current.

## Control of Induction Motors by AC Voltage Controllers

Fig.7 shows two commonly used symmetrical 3-phase ac voltage controller circuits for wye- and delta-connected stators, respectively. For small size motors, each antiparallel thyristor pair can be replaced by a triac. Connection  $C_1$  can also be used with a delta-connected stator. With a delta connection, a third harmonic current may circulate, increasing the motor losses. Each thyristor pair of circuit  $C_1$  carries the line current, whereas each thyristor pair of circuit  $C_2$  carries only the phase current. Thus, in  $C_2$  the thyristor current rating

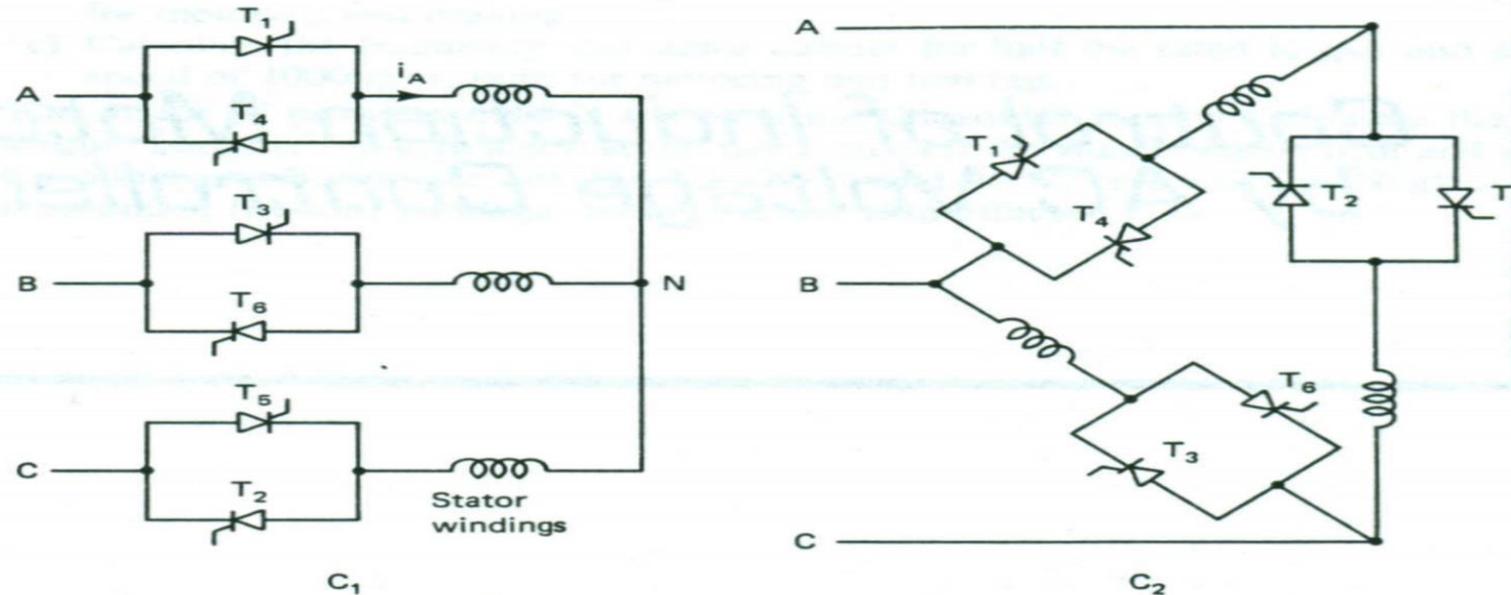


Figure 7. 3-phase ac voltage controller circuits.

Under normal operation, the maximum voltage to which the thyristors of circuit C<sub>1</sub> are subjected is  $(\sqrt{3}/2)$  times smaller than that of the thyristors of circuit C<sub>2</sub>

The thyristors of the controllers of fig.7. are fired in the sequence of their numbers with a phase difference of 60 degree. For circuit C<sub>1</sub> the firing angle  $\alpha$  is measured from the instant the phase voltage  $V_{AN}$  has a zero value. For circuit C<sub>2</sub>, firing angle  $\alpha$  is measured from line voltage  $V_{AB}$ . Let us define an angle  $\phi = \tan^{-1}(X_{in}/R_{in})$  where  $(R_{in} + jX_{in})$  is the input impedance of the induction motor. For the firing angle  $\alpha \leq \phi$ . the motor terminal voltage remains constant and nearly equal to the supply voltage. Both motor voltage and current are sinusoidal. For higher values of  $\alpha$ , the current flows discontinuously and the motor voltage decreases with an increase in  $\alpha$ . The zero motor voltage and current are reached at  $\alpha = 150$  and  $\alpha = 180$  for the circuits C<sub>1</sub> and C<sub>2</sub>, respectively.

## FAN OR PUMP AND CRANE HOIST DRIVES

Two major applications of induction motors fed by ac voltage controllers are fan or pump drives and the crane hoist drive

In fan and pump drives, the torque varies as the square of the speed and the power as the cube of the speed. The pump and fan drives require speed control only in a narrow range. Because torque reduces as the square of the speed and the speed control is required only in a narrow range, the ac voltage controller fed squirrel-cage induction motor with a is found suitable for these applications.

$$T = \frac{3}{\omega_{ms}} I_r'^2 \frac{R_r'}{s}$$

$$T_L = C\omega_m^2 = C(1 - s)^2\omega_{ms}^2$$

where  $C$  is a constant.

If the friction, windage, and core loss torques are neglected,

$$T = T_L$$

$$I_r' = K \left[ \frac{(1-s)\sqrt{s}}{\sqrt{R_r'}} \right]$$

$$\text{where } K = \sqrt{(C\omega_{ms}^3/3)}$$

In approximation circuit  $I_s = I_r'$

$$I_s = K \left[ \frac{(1-s)\sqrt{s}}{\sqrt{R_r'}} \right]$$

the motor current is inversely proportional to the square root of the rotor resistance,  $R_r'$ . If the full-load stator current and the motor slip are denoted by  $I_{\text{rated}}$  and  $s_{\text{rated}}$ , respectively,

$$I_{\text{rated}} = K \left[ \frac{(1-s_{\text{rated}})\sqrt{s_{\text{rated}}}}{\sqrt{R_r'}} \right]$$

$$\frac{I_s}{I_{\text{rated}}} = \frac{(1-s)\sqrt{s}}{(1-s_{\text{rated}})\sqrt{s_{\text{rated}}}}$$

The slip at which the maximum value of  $I_s$ , occurs is obtained by equating  $(d I_s / ds)$  to zero, giving  $s_m = 1/3$

$$\frac{I_{\text{max}}}{I_{\text{rated}}} = \frac{2}{3\sqrt{3}(1-s_{\text{rated}})\sqrt{s_{\text{rated}}}}$$

example 1 .Finding the performance parameters of a three-phase Induction Motor with stator voltage Control

A three-phase, 460-V, 60-Hz, four-pole Y-connected induction motor has the following parameters:  $R_s = 1.01 \Omega$ ,  $R'_r = 0.69 \Omega$ ,  $X_s = 1.3 \Omega$ ,  $X'_r = 1.94 \Omega$ , and  $X_m = 43.5 \Omega$ . The no-load loss,  $P_{\text{no load}}$ , is negligible. The load torque, which is proportional to the speed squared, is  $41 \text{ N} \cdot \text{m}$  at 1740 rpm. If the motor speed is 1550 rpm, determine (a) the load torque  $T_L$ , (b) the rotor current  $I_r$ , (c) the stator supply voltage  $V_a$ , (d) the motor input current  $I_i$ , (e) the motor input power  $P_i$ , (f) the slip for maximum current  $s_a$ , (g) the maximum rotor current  $I_{r(\text{max})}$ , (h) the speed at maximum rotor current  $\omega_a$ , and (i) the torque at the maximum current  $T_a$ .

### ***Solution***

$p = 4$ ,  $f = 60 \text{ Hz}$ ,  $V_s = 460/\sqrt{3} = 265.58 \text{ V}$ ,  $R_s = 1.01 \Omega$ ,  $R'_r = 0.69 \Omega$ ,  $X_s = 1.3 \Omega$ ,  $X'_r = 1.94 \Omega$ ,  $X_m = 43.5 \Omega$ ,  $\omega = 2\pi \times 60 = 377 \text{ rad/s}$ , and  $\omega_s = 377 \times 2/4 = 188.5 \text{ rad/s}$ . Because torque is proportional to speed squared,

$$T_L = K_m \omega_m^2$$

At  $\omega_m = 1740 \pi/30 = 182.2 \text{ rad/s}$ ,  $T_L = 41 \text{ N} \cdot \text{m}$ , yields  $K_m = 41/182.2^2 = 1.235 \times 10^{-3}$  and  $\omega_m = 1550 \pi/30 = 162.3 \text{ rad/s}$ .  $s = (188.5 - 162.3)/ 188.500 = 0.139$ .

**a.**  $T_L = 1.235 \times 10^{-3} \times 162.3^2 = 32.5 \text{ N} \cdot \text{m}$ .

**b.**  $P_d = 3(I'_r)^2 \frac{R'_r}{s} (1 - s) = T_L \omega_m + P_{\text{no load}}$

For negligible no-load loss,

$$I_r = \left[ \frac{sT_L\omega_m}{3R'_r(1-s)} \right]^{1/2}$$

$$= \left[ \frac{0.139 \times 32.5 \times 162.3}{3 \times 0.69(1-0.139)} \right]^{1/2} = 20.28 \text{ A}$$

c. The stator supply voltage

$$V_a = I'_r \left[ \left( R_s + \frac{R'_r}{s} \right)^2 + (X_s + X'_r)^2 \right]^{1/2}$$

$$= 20.28 \times \left[ \left( 1.01 + \frac{0.69}{0.139} \right)^2 + (1.3 + 1.94)^2 \right]^{1/2} = 137.82$$

d.  $\mathbf{Z}_i = \frac{-43.5 \times (1.3 + 1.94) + j43.5 \times (1.01 + 0.69/0.139)}{1.01 + 0.69/0.139 + j(43.5 + 1.3 + 1.94)} = 6.27 \angle 35.82^\circ$

$$\mathbf{I}_i = \frac{V_a}{\mathbf{Z}_i} = \frac{137.82}{6.27} \angle -144.26^\circ = 22 \angle -35.82^\circ \text{ A}$$

e.  $\text{PF}_m = \cos(-35.82^\circ) = 0.812$  (lagging).

$$P_i = 3 \times 137.82 \times 22.0 \times 0.812 = 7386 \text{ W}$$

f. Substituting  $\omega_m = \omega_s(1 - s)$  and  $T_L = K_m\omega_m^2$

$$I'_r = \left[ \frac{sT_L\omega_m}{3R'_r(1-s)} \right]^{1/2} = (1-s)\omega_s \left( \frac{sK_m\omega_s}{3R'_r} \right)^{1/2}$$

The slip at which  $I'_r$  becomes maximum can be obtained by setting  $dI'_r/ds = 0$ , and this yields

$$s_a = \frac{1}{3}$$

$$\begin{aligned} I'_{r(\max)} &= \omega_s \left( \frac{4K_m\omega_s}{81R'_r} \right)^{1/2} \\ &= 188.5 \times \left( \frac{4 \times 1.235 \times 10^{-3} \times 188.5}{81 \times 0.69} \right)^{1/2} = 24.3 \text{ A} \end{aligned}$$

h. The speed at the maximum current

$$\begin{aligned} \omega_a &= \omega_s(1-s_a) = (2/3)\omega_s = 0.6667\omega_s \\ &= 188.5 \times 2/3 = 125.27 \text{ rad/s} \quad \text{or} \quad 1200 \text{ rpm} \end{aligned}$$

i.  $T_a = 9I_{r(\max)}^2 \frac{R_r}{\omega_s} = 9 \times 24.3^2 \times \frac{0.69}{188.5} = 19.45 \text{ N}\cdot\text{m}$

### 3- Frequency Control

The synchronous speed is directly proportional to the supply frequency. Hence, the synchronous speed and the motor speed can be controlled below and above the normal full-load speed by changing the supply frequency. If the synchronous speed corresponding to the rated frequency is called the base speed  $\omega_b$ , the synchronous speed at any other frequency  $\omega_s = \beta \omega_b$  becomes where  $\beta > 1$

$$\omega_s = \beta \omega_b$$

$$s = \frac{\beta \omega_b - \omega_m}{\beta \omega_b} = 1 - \frac{\omega_m}{\beta \omega_b}$$

The torque expression :

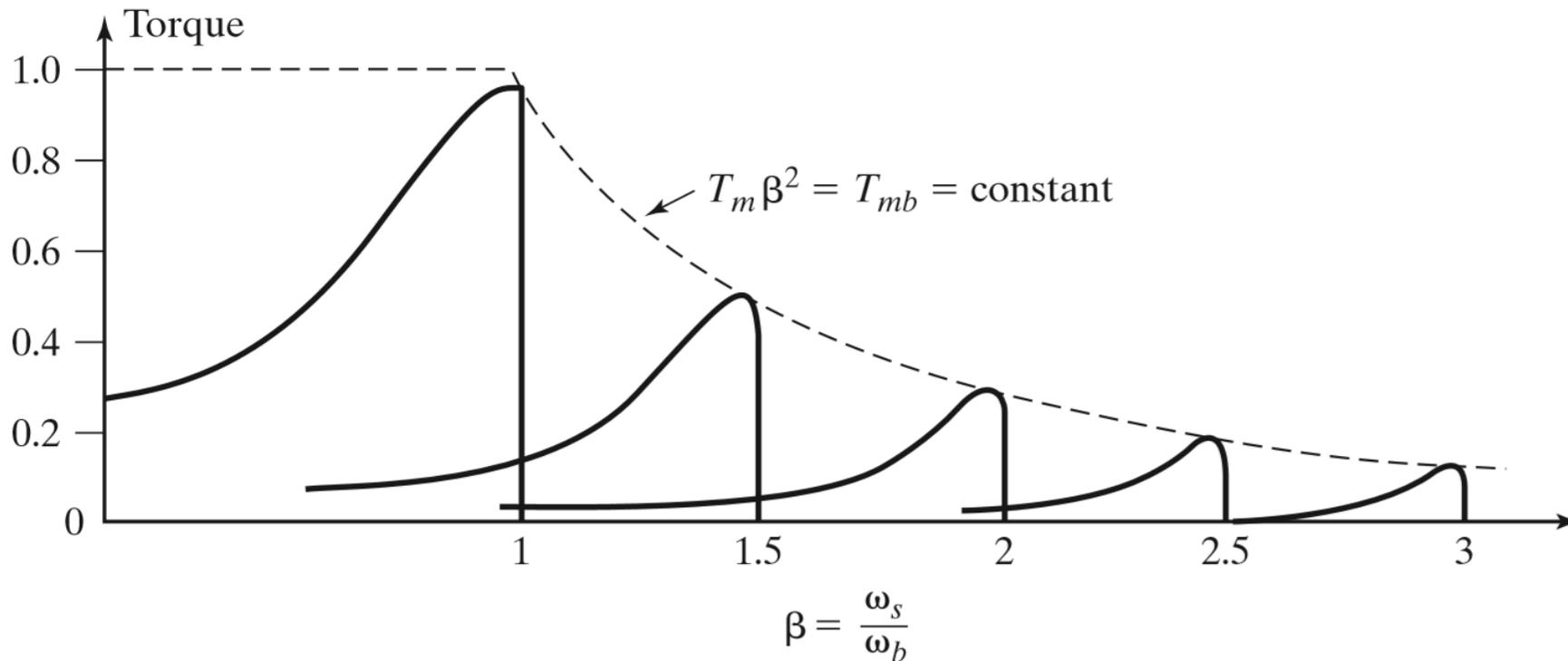
$$T_d = \frac{3 R_r' V_a^2}{s \beta \omega_b [(R_s + R_r'/s)^2 + (\beta X_s + \beta X_r')^2]}$$

The typical torque–speed characteristics are shown in Figure 1. for various values of  $\beta$ . The dc-ac converter( inverter) in can vary the frequency at a fixed voltage. If  $R_s$  is negligible, the torque equation will be:-

$$T_{mb} = \frac{3 V_a^2}{2\omega_b (X_s + X_r')}$$

The maximum torque at any other frequency is

$$T_m = \frac{3}{2\omega_b (X_s + X_r')} \left( \frac{V_a}{\beta} \right)^2 \quad \text{the corresponding slip is} \quad s_m = \frac{R_r'}{\beta (X_s + X_r')}$$



Torque characteristics with frequency control.

$$\frac{T_m}{T_{mb}} = \frac{1}{\beta^2} \quad \text{and} \quad T_m \beta^2 = T_{mb}$$

Thus, it can be concluded that the maximum torque is inversely proportional to frequency squared, and  $T_m \beta^2$  remains constant. In this type of control, the motor is said to be operated in a field-weakening mode. For  $\beta > 1$ , the motor is operated at a constant terminal voltage and the flux is reduced, thereby limiting the torque capability of the motor. For  $1 < \beta < 1.5$ , the relation between  $T_m$  and  $\beta$  can be considered approximately linear. For  $\beta < 1$ , the motor is normally operated at a constant flux by reducing the terminal voltage  $V_a$  along with the frequency so that the flux remains constant.

## Example 1 Finding the Performance Parameters of a Three-Phase Induction Motor with Frequency Control

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole Y-connected induction motor has the following parameters:  $R_s = 0$ ,  $R_r' = 0.38 \Omega$ ,  $X_s = 1.14 \Omega$ ,  $X_r' = 1.71 \Omega$ , and  $X_m = 33.2 \Omega$ . The motor is controlled by varying the supply frequency. If the breakdown torque requirement is  $35 \text{ N}\cdot\text{m}$ , calculate (a) the supply frequency and (b) the speed  $\omega_m$  at the maximum torque.

### Solution

$V_a = V_s = 460/\sqrt{3} = 258 \times 58 \text{ V}$ ,  $\omega_b = 2\pi \times 60 = 377 \text{ rad/s}$ ,  $p = 4$ ,  $P_0 = 11,200 \text{ W}$ ,  $T_{mb} \times 1750 \pi/30 = 11,200$ ,  $T_{mb} = 61.11 \text{ N}\cdot\text{m}$ , and  $T_m = 35 \text{ N}\cdot\text{m}$ .

$$\mathbf{a.} \quad \beta = \sqrt{\frac{T_{mb}}{T_m}} = \sqrt{\frac{61.11}{35}} = 1.321 \quad T_m \beta^2 = T_{mb}$$
$$\omega_s = \beta \omega_b = 1.321 \times 377 = 498.01 \text{ rad/s}$$

the supply frequency is  $\omega = \frac{4 \times 498.01}{2} = 996 \text{ rad/s}$  or  $158.51 \text{ Hz}$

**b.** the slip for maximum torque is

$$s_m = \frac{R_r'/\beta}{X_s + X_r'} = \frac{0.38/1.321}{1.14 + 1.71} = 0.101$$
$$\omega_m = 498.01 \times (1 - 0.101) = 447.711 \text{ rad/s} \quad \text{or} \quad 4275 \text{ rpm}$$

## 4-voltage and Frequency Control

If the ratio of voltage to frequency is kept constant, the flux in  $\phi = \frac{V_a}{K_m \omega}$

Equation:- $T_m \beta^2 = T_{mb}$ , indicates that the maximum torque, which is independent of frequency, can be maintained approximately constant. However, at a high frequency, the air-gap flux is reduced due to the drop in the stator impedance and the voltage has to be increased to maintain the torque level. This type of control is usually known as volts/hertz control. If  $\omega_s = \beta \omega_b$ , and the voltage-to-frequency ratio is constant so that

$$\frac{V_a}{\omega_s} = d$$

The ratio  $d$ , which is determined from the rated terminal voltage  $V_s$  and the base speed  $\omega_b$ , is given by

$$d = \frac{V_s}{\omega_b} \longrightarrow V_a = d \omega_s = \frac{V_s}{\omega_b} \beta \omega_b = V_s \omega_b$$

$$s_m = \frac{R'_r}{[R_s^2 + \beta^2 (X_s + X'_r)^2]^{1/2}}$$

The typical torque–speed characteristics are shown in Figure 2. As the frequency is reduced,  $\beta$  decreases and the slip for maximum torque increases. For a given torque demand, the speed can be controlled by changing the frequency. Therefore, by varying both the voltage and frequency, the torque and speed can be controlled. The torque is normally maintained constant while the speed is varied. The voltage at variable frequency can be obtained from three-phase inverters or cycloconverters. The cycloconverters are used in very large power applications (e.g., locomotives and cement mills), where the frequency requirement is one-half or one-third of the line frequency.

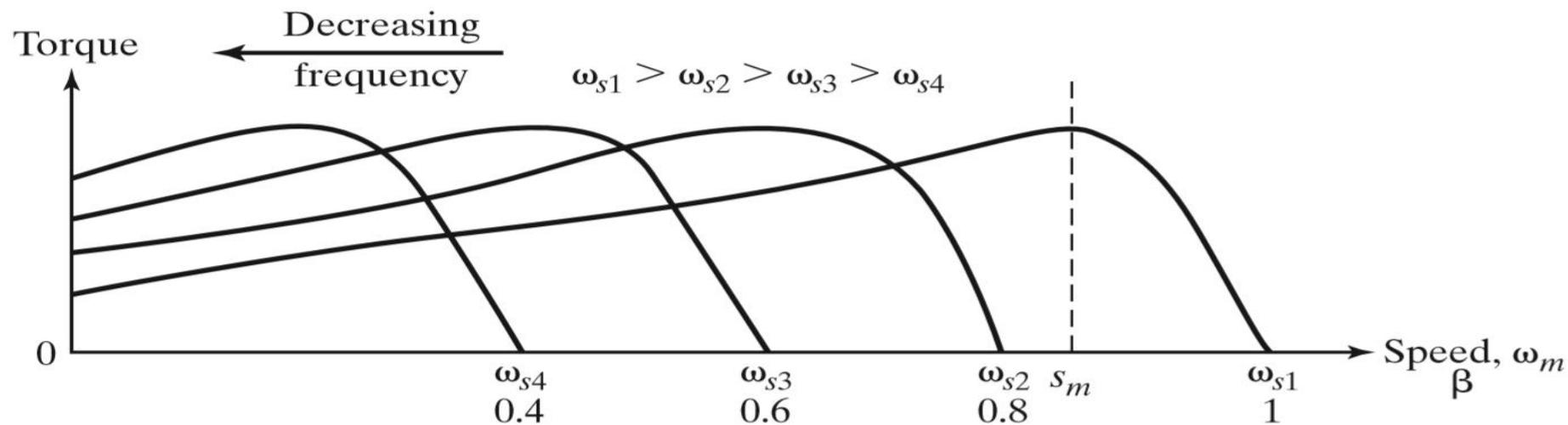
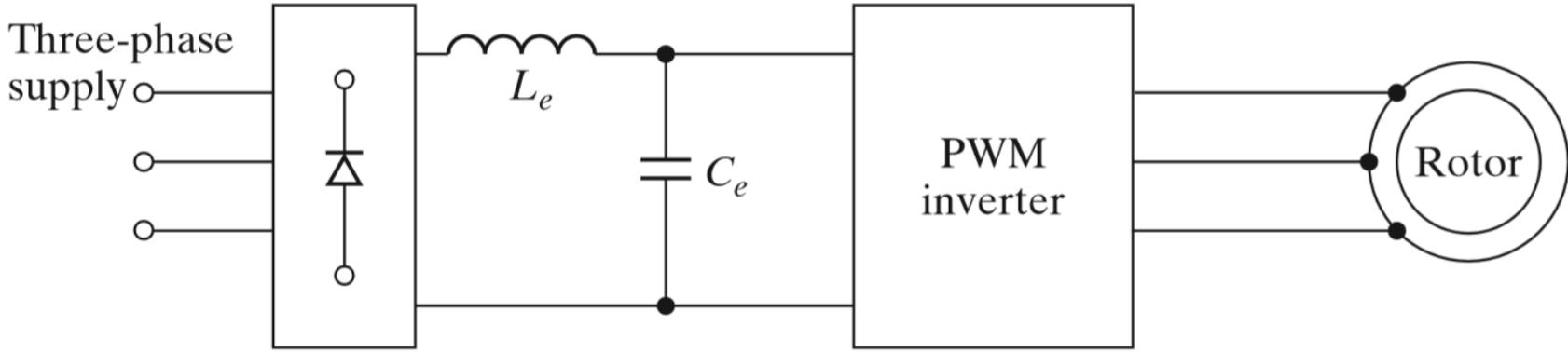


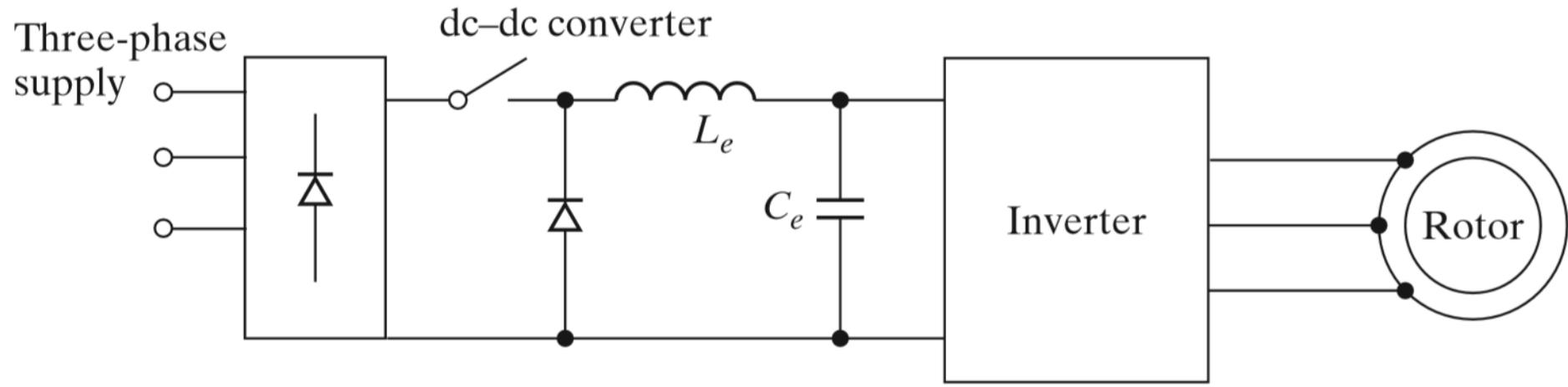
FIGURE 2

Torque–speed characteristics with volts/hertz control.

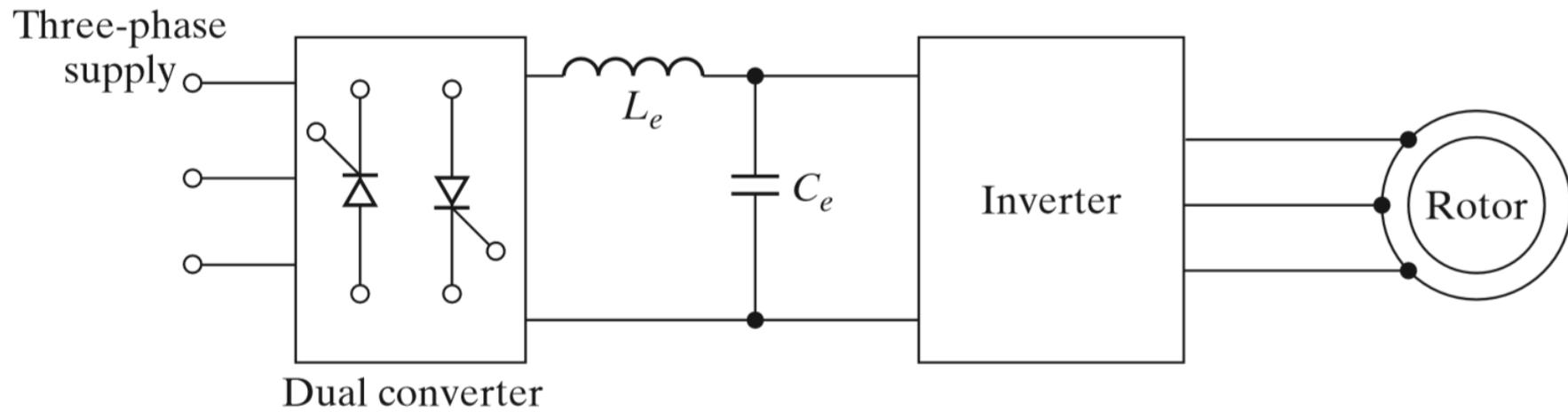
Three possible circuit arrangements for obtaining variable voltage and frequency are shown in Figure 3. In Figure 3 a, the dc voltage remains constant and the PWM techniques are applied to vary both the voltage and frequency within the inverter. Due to diode rectifier, regeneration is not possible and the inverter would generate harmonics into the ac supply. In Figure 3 b, the dc-dc converter varies the dc voltage to the inverter and the inverter controls the frequency. Due to the dc converter, the harmonic injection into the ac supply is reduced. In Figure 3 c, the dc voltage is varied by the dual converter and frequency is controlled within the inverter. This arrangement permits regeneration; however, the input PF of the converter is low, especially at a high delay angle.



(a) Fixed dc and PWM inverter drive



(b) Variable dc and inverter



(c) Variable dc from dual converter and inverter

FIGURE 3

Voltage-source induction motor drives.

## Example 2. Finding the performance parameters of a three-phase Induction Motor with voltage and Frequency Control

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole, Y-connected induction motor has the following parameters:  $R_s = 0.66 \Omega$ ,  $R'_r = 0.38 \Omega$ ,  $X_s = 1.14 \Omega$ ,  $X'_r = 1.71 \Omega$ , and  $X_m = 33.2 \Omega$ . The motor is controlled by varying both the voltage and frequency. The volts/hertz ratio, which corresponds to the rated voltage and rated frequency, is maintained constant.

(a) Calculate the maximum torque  $T_m$  and the corresponding speed  $\omega_m$  for 60 and 30 Hz.

(b) Repeat (a) if  $R_s$  is negligible.

### ***Solution***

$$p = 4, V_a = V_s = 460/\sqrt{3} = 265.58 \text{ V}, \omega = 2\pi \times 60 = 377 \text{ rad/s}, \omega_b = 2 \times 377/4 = 188.5 \text{ rad/s}, d = 265.58/188.5 = 1.409.$$

**a.** At 60 Hz,  $\omega_b = \omega_s = 188.5 \text{ rad/s}$ ,  $\beta = 1$ , and  $V_a = d\omega_s = 1.409 \times 188.5 = 265.58 \text{ V}$ .

$$s_m = \frac{R'_r}{[R_s^2 + \beta^2(X_s + X'_r)^2]^{1/2}}$$

$$s_m = \frac{0.38}{[0.66^2 + (1.14 + 1.71)^2]^{1/2}} = 0.1299$$

$$\omega_m = 188.5 \times (1 - 0.1299) = 164.01 \text{ rad/s} \quad \text{or} \quad 1566 \text{ rpm}$$

$$T_{mm} = \frac{3V_s^2}{2\omega_s[R_s + \sqrt{R_s^2 + (X_s + X'_r)^2}]}$$

the maximum torque is

$$T_m = \frac{3 \times 265.58^2}{2 \times 188.5 \times [0.66 + \sqrt{0.66^2 + (1.14 + 1.71)^2}]} = 156.55 \text{ N} \cdot \text{m}$$

## Rotor Voltage Control

In a wound-rotor motor, an external three-phase resistor may be connected to its slip rings, as shown in Figure 15.6a. The developed torque may be varied by varying the resistance  $R_x$ . If  $R_x$  is referred to the stator winding and added to  $R_r$ , Eq. (15.18) may be applied to determine the developed torque. The typical torque–speed characteristics for variations in rotor resistance are shown in Figure 15.6b. This method increases the starting torque while limiting the starting current. However, this is an inefficient

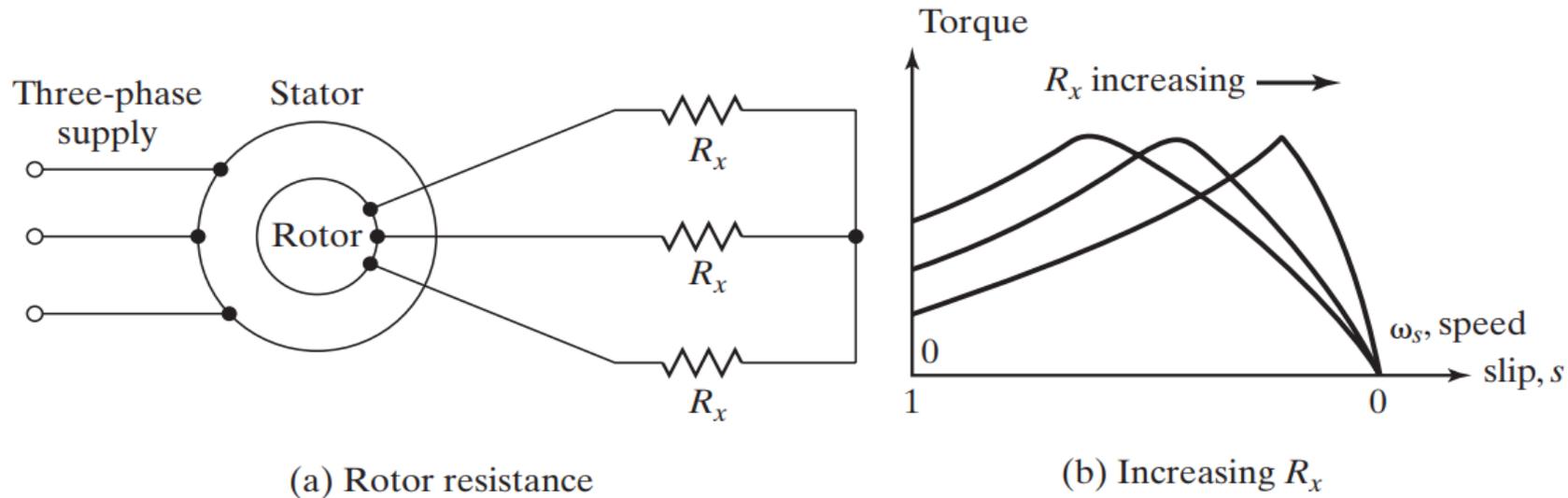


FIGURE 15.6

Speed control by motor resistance.

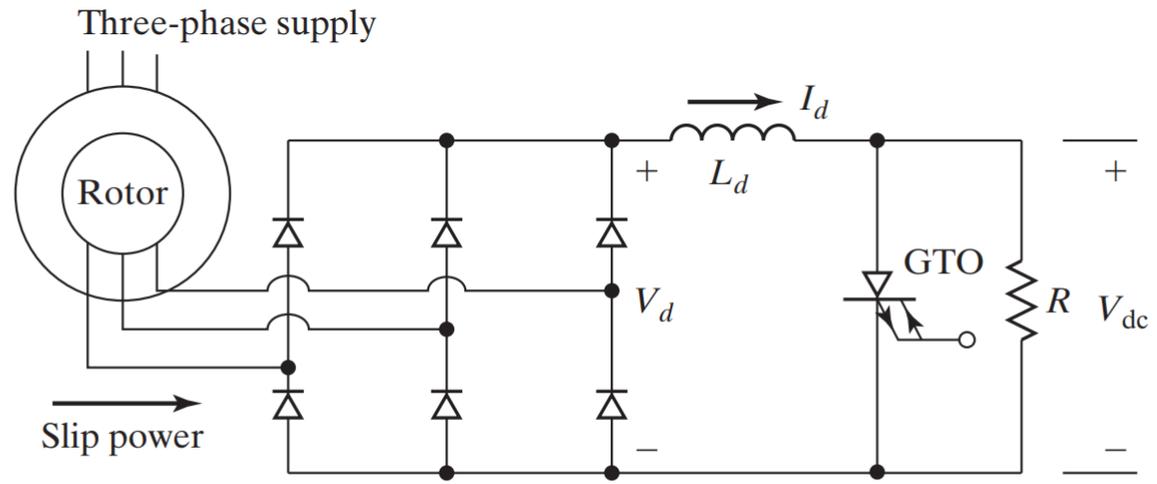
method and there would be imbalances in voltages and currents if the resistances in the rotor circuit are not equal. A wound-rotor induction motor is designed to have a low-rotor resistance so that the running efficiency is high and the full-load slip is low. The increase in the rotor resistance does not affect the value of maximum torque but increases the slip at maximum torque. The wound-rotor motors are widely used in applications requiring frequent starting and braking with large motor torques (e.g., crane hoists). Because of the availability of rotor windings for changing the rotor resistance, the wound rotor offers greater flexibility for control. However, it increases the cost and needs maintenance due to slip rings and brushes. The wound-rotor motor is less widely used as compared with the squirrel-case motor.

The three-phase resistor may be replaced by a three-phase diode rectifier and a dc converter, as shown in Figure 15.7a, where the gate-turn-off thyristor (GTO) or an insulated-gate bipolar transistor (IGBT) operates as a dc converter switch. The inductor  $L_d$  acts as a current source  $I_d$  and the dc converter varies the effective resistance, which can be found from Eq. (14.40):

$$R_e = R(1 - k) \quad (15.49)$$

where  $k$  is the duty cycle of the dc converter. The speed can be controlled by varying the duty cycle. The portion of the air-gap power, which is not converted into mechanical power, is called *slip power*. The slip power is dissipated in the resistance  $R$ .

The slip power in the rotor circuit may be returned to the supply by replacing the dc converter and resistance  $R$ , with a three-phase full converter, as shown in Figure 15.7b. The converter is operated in the inversion mode with delay range of  $\pi/2 \leq \alpha \leq \pi$ , thereby returning energy to the source. The variation of the delay angle permits PF and speed control. This type of drive is known as a *static Kramer* drive. Again, by replacing the bridge rectifiers by three three-phase dual converters (or cycloconverters), as shown in Figure 15.7c, the slip PF in either direction is possible and this arrangement is called a *static Scherbius* drive. The static Kramer and Scherbius drives are used in large power pump and blower applications where limited range of speed control is required. Because the motor is connected directly to the source, the PF of these drives is generally high.



(a) Slip control by dc converter

### Example 15.3 Finding the Performance Parameters of a Three-Phase Induction Motor with Rotor Voltage Control

A three-phase, 460-V, 60-Hz, six-pole Y-connected wound-rotor induction motor whose speed is controlled by slip power, as shown in Figure 15.7a, has the following parameters:  $R_s = 0.041 \Omega$ ,  $R_r' = 0.044 \Omega$ ,  $X_s = 0.29 \Omega$ ,  $X_r' = 0.44 \Omega$ , and  $X_m = 6.1 \Omega$ . The turns ratio of the rotor to stator windings is  $n_m = N_r/N_s = 0.9$ . The inductance  $L_d$  is very large and its current  $I_d$  has

negligible ripple. The values of  $R_s$ ,  $R_r$ ,  $X_s$ , and  $X_r$  for the equivalent circuit in Figure 15.2 can be considered negligible compared with the effective impedance of  $L_d$ . The no-load loss of the motor is negligible. The losses in the rectifier, inductor  $L_d$ , and the GTO dc converter are also negligible.

The load torque, which is proportional to speed squared, is  $750 \text{ N} \cdot \text{m}$  at 1175 rpm. (a) If the motor has to operate with a minimum speed of 800 rpm, determine the resistance  $R$ . With this value of  $R$ , if the desired speed is 1050 rpm, calculate (b) the inductor current  $I_d$ , (c) the duty cycle of the dc converter  $k$ , (d) the dc voltage  $V_d$ , (e) the efficiency, and (f) the input  $\text{PF}_s$  of the drive.

#### **Solution**

$V_a = V_s = 460/\sqrt{3} = 265.58 \text{ V}$ ,  $p = 6$ ,  $\omega = 2\pi \times 60 = 377 \text{ rad/s}$ , and  $\omega_s = 2 \times 377/6 = 125.66 \text{ rad/s}$ . The equivalent circuit of the drive is shown in Figure 15.8a, which is reduced to Figure 15.8b provided the motor parameters are neglected. From Eq. (15.49), the dc voltage at the rectifier output is

$$V_d = I_d R_e = I_d R(1 - k) \quad (15.59)$$

and

$$E_r = sV_s \frac{N_r}{N_s} = sV_s n_m \quad (15.60)$$

For a three-phase rectifier, Eq. (3.33) relates  $E_r$  and  $V_d$  as

$$V_d = 1.654 \times \sqrt{2} E_r = 2.3394 E_r$$

Using Eq. (15.60),

$$V_d = 2.3394 s V_s n_m \quad (15.61)$$

$$V_d = I_d R_e = I_d R(1 - k) \quad (15.59)$$

and

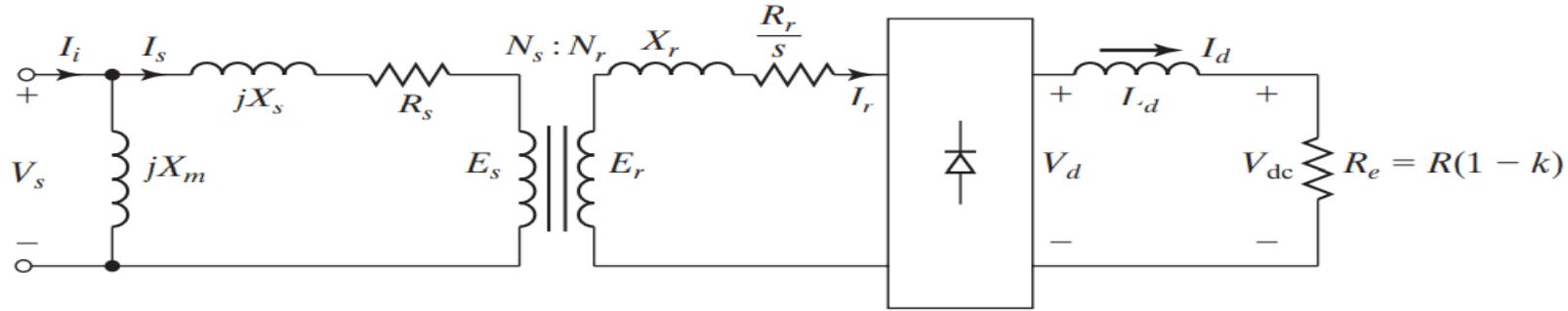
$$E_r = sV_s \frac{N_r}{N_s} = sV_s n_m \quad (15.60)$$

For a three-phase rectifier, Eq. (3.33) relates  $E_r$  and  $V_d$  as

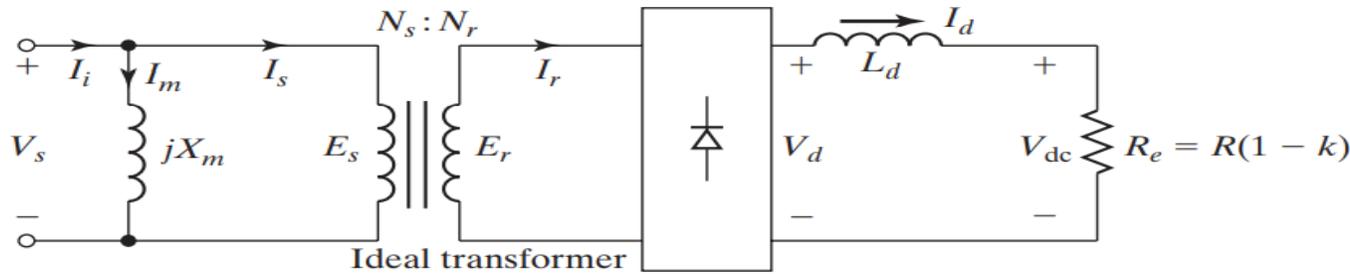
$$V_d = 1.654 \times \sqrt{2} E_r = 2.3394 E_r$$

Using Eq. (15.60),

$$V_d = 2.3394 s V_s n_m \quad (15.61)$$



(a) Equivalent circuit



(b) Approximate equivalent circuit

FIGURE 15.8

Equivalent circuits for Example 15.3.

If  $P_r$  is the slip power, Eq. (15.9) gives the gap power

$$P_g = \frac{P_r}{s}$$

and Eq. (15.10) gives the developed power as

$$P_d = 3(P_g - P_r) = 3\left(\frac{P_r}{s} - P_r\right) = \frac{3P_r(1-s)}{s} \quad (15.62)$$

Because the total slip power is  $3P_r = V_d I_d$  and  $P_d = T_L \omega_m$ , Eq. (15.62) becomes

$$P_d = \frac{(1-s)V_d I_d}{s} = T_L \omega_m = T_L \omega_s (1-s) \quad (15.63)$$

Substituting  $V_d$  from Eq. (15.61) in Eq. (15.63) and solving for  $I_d$  gives

$$I_d = \frac{T_L \omega_s}{2.3394 V_s n_m} \quad (15.64)$$

which indicates that the inductor current is independent of the speed. Equating Eq. (15.59) to Eq. (15.61) gives

$$2.3394sV_s n_m = I_d R(1-k)$$

which gives

$$s = \frac{I_d R(1-k)}{2.3394 V_s n_m} \quad (15.65)$$

The speed can be found from Eq. (15.65) as

$$\omega_m = \omega_s(1-s) = \omega_s \left[ 1 - \frac{I_d R(1-k)}{2.3394 V_s n_m} \right] \quad (15.66)$$

$$= \omega_s \left[ 1 - \frac{T_L \omega_s R(1-k)}{(2.3394 V_s n_m)^2} \right] \quad (15.67)$$

which shows that for a fixed duty cycle, the speed decreases with load torque. By varying  $k$  from 0 to 1, the speed can be varied from a minimum value to  $\omega_s$ .

**a.**  $\omega_m = 800 \pi/30 = 83.77$  rad/s. From Eq. (15.40) the torque at 900 rpm is

$$T_L = 750 \times \left( \frac{800}{1175} \right)^2 = 347.67 \text{ N} \cdot \text{m}$$

From Eq. (15.64), the corresponding inductor current is

$$I_d = \frac{347.67 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 78.13 \text{ A}$$

The speed is minimum when the duty cycle  $k$  is zero and Eq. (15.66) gives the minimum speed,

$$83.77 = 125.66 \left( 1 - \frac{78.13R}{2.3394 \times 265.58 \times 0.9} \right)$$

and this yields  $R = 2.3856 \Omega$ .

**b.** At 1050 rpm

$$T_L = 750 \times \left( \frac{1050}{1175} \right)^2 = 598.91 \text{ N} \cdot \text{m}$$

$$I_d = \frac{598.91 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 134.6 \text{ A}$$

**c.**  $\omega_m = 1050 \pi/30 = 109.96 \text{ rad/s}$  and Eq. (15.66) gives

$$109.96 = 125.66 \left[ 1 - \frac{134.6 \times 2.3856(1 - k)}{2.3394 \times 265.58 \times 0.9} \right]$$

which gives  $k = 0.782$ .

**d.** Using Eq. (15.4), the slip is

$$s = \frac{125.66 - 109.96}{125.66} = 0.125$$

From Eq. (15.61),

$$V_d = 2.3394 \times 0.125 \times 265.58 \times 0.9 = 69.9 \text{ V}$$

**e.** The power loss,

$$P_1 = V_d I_d = 69.9 \times 134.6 = 9409 \text{ W}$$

The output power,

$$P_o = T_L \omega_m = 598.91 \times 109.96 = 65,856 \text{ W}$$

The rms rotor current referred to the stator is

$$I'_r = \sqrt{\frac{2}{3}} I_d n_m = \sqrt{\frac{2}{3}} \times 134.6 \times 0.9 = 98.9 \text{ A}$$

The rotor copper loss is  $P_{ru} = 3 \times 0.044 \times 98.9^2 = 1291 \text{ W}$ , and the stator copper loss is  $P_{su} = 3 \times 0.041 \times 98.9^2 = 1203 \text{ W}$ . The input power is

$$P_i = 65,856 + 9409 + 1291 + 1203 = 77,759 \text{ W}$$

The efficiency is  $65,856/77,759 = 85\%$ .

- f.** From Eq. (10.19) for  $n = 1$ , the fundamental component of the rotor current referred to the stator is

$$\begin{aligned} I'_{r1} &= 0.7797 I_d \frac{N_r}{N_s} = 0.7797 I_d n_m \\ &= 0.7797 \times 134.6 \times 0.9 = 94.45 \text{ A} \end{aligned}$$

and the rms current through the magnetizing branch is

$$I_m = \frac{V_a}{X_m} = \frac{265.58}{6.1} = 43.54 \text{ A}$$

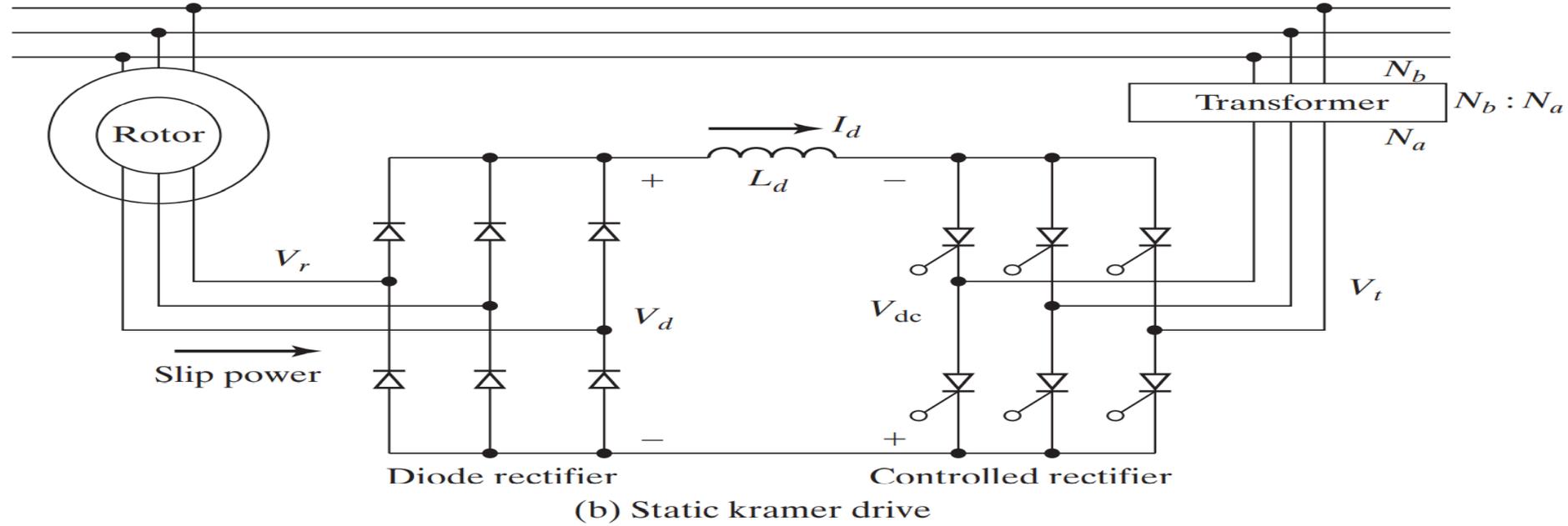
The rms fundamental component of the input current is

$$\begin{aligned} I_{i1} &= \left[ (0.7797 I_d n_m)^2 + \left( \frac{V_a}{X_m} \right)^2 \right]^{1/2} \\ &= (94.45^2 + 43.54^2)^{1/2} = 104 \text{ A} \end{aligned} \tag{15.68}$$

The PF angle is given approximately by

$$\begin{aligned} \theta_m &= -\tan^{-1} \frac{V_a/X_m}{0.7797 I_d n_m} \\ &= -\tan^{-1} \frac{43.54}{94.45} = \underline{\underline{-24.74^\circ}} \end{aligned} \tag{15.69}$$

The input PF is  $\text{PFs} = \cos -24.74^\circ = 0.908$  lagging



Assuming  $n_r$  is the effective turns ratio of the stator and the rotor windings, the rotor voltage is related to the stator (and line voltage  $V_L$ ) by

$$V_r = \frac{sV_L}{n_r} \quad (15.50)$$

The dc output voltage of the three-phase rectifier is

$$V_d = 1.35V_r = \frac{11.35sV_L}{n_r} \quad (15.51)$$

Neglecting the resistive voltage in the series inductor  $L_d$ ,

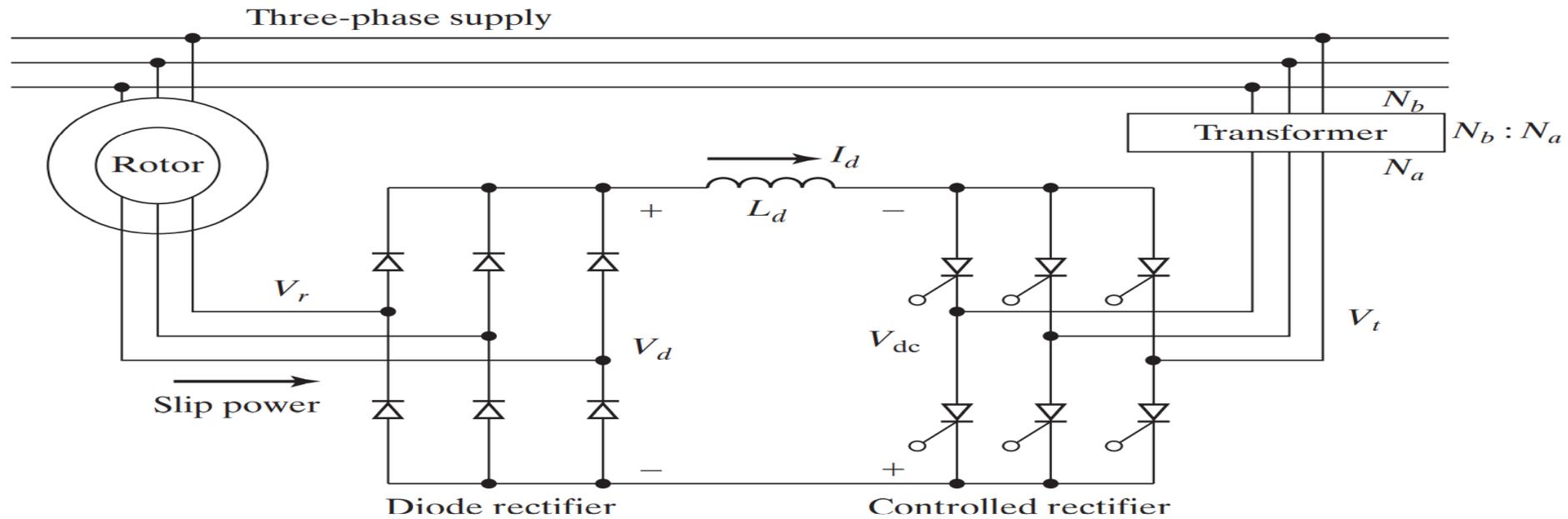
$$V_d = -V_{dc} \quad (15.52)$$

$V_{dc}$ , which is the output voltage of a phase-controlled converter, is given by

$$V_{dc} = 1.35V_t \cos \alpha \quad (15.53)$$

where

$$V_t = \frac{N_a}{N_b}V_L = n_t V_L \quad (15.54)$$



(b) Static kramer drive

where  $n_t$  is the turns ratio of the transformer in the converter side. Using Eq. (15.51) to (15.54), the slip can be found from

$$s = -n_r n_t \cos \alpha \quad (15.55)$$

This gives the delay angle as

$$\alpha = \cos^{-1} \left( \frac{-s}{n_r n_t} \right) \quad (15.56)$$

The delay angle can be varied in the inversion mode from  $90^\circ$  to  $180^\circ$ . But the power switching devices limit the upper range to  $155^\circ$ , and thus the practical range of the delay angle is

$$90^\circ \leq \alpha \leq 155^\circ \quad (15.57)$$

which gives the slip range as

$$0 \leq s \leq (0.906 \times n_r n_t) \quad (15.58)$$

### Example 15.4 Finding the Performance Parameters of a Static Kramer Drive

The induction motor in Example 15.3 is controlled by a static Kramer drive, as shown in Figure 15.7b. The turns ratio of the converter ac voltage to supply voltage is  $n_c = N_a/N_b = 0.40$ . The load torque is  $750 \text{ N} \cdot \text{m}$  at  $1175 \text{ rpm}$ . If the motor is required to operate at a speed of  $1050 \text{ rpm}$ , calculate (a) the inductor current  $I_d$ ; (b) the dc voltage  $V_d$ ; (c) the delay angle of the converter  $\alpha$ ; (d) the efficiency; and (e) the input PF of the drive,  $\text{PF}_s$ . The losses in the diode rectifier, converter, transformer, and inductor  $L_d$  are negligible.

#### **Solution**

$V_a = V_s = 460/\sqrt{3} = 265.58 \text{ V}$ ,  $p = 6$ ,  $\omega = 2\pi \times 60 = 377 \text{ rad/s}$ ,  $\omega_s = 2 \times 377/6 = 125.66 \text{ rad/s}$ , and  $\omega_m = 1050 \pi/30 = 109.96 \text{ rad/s}$ . Then

$$s = \frac{125.66 - 109.96}{125.66} = 0.125$$

$$T_L = 750 \times \left(\frac{1050}{1175}\right)^2 = 598.91 \text{ N} \cdot \text{m}$$

- a.** The equivalent circuit of the drive is shown in Figure 15.9, where the motor parameters are neglected. From Eq. (15.64), the inductor current is

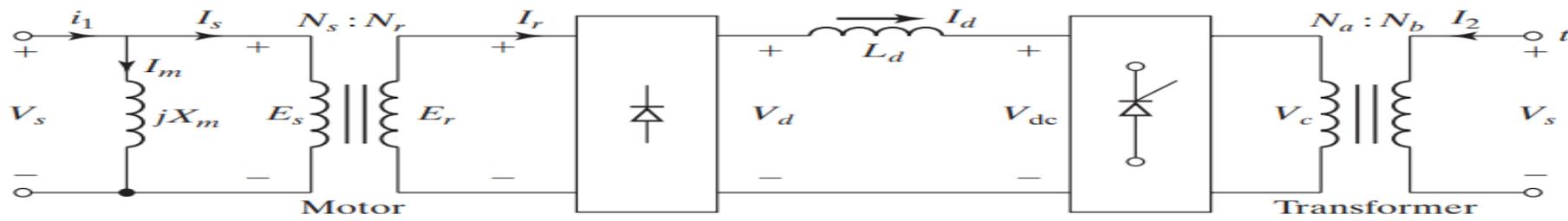
$$I_d = \frac{598.91 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 134.6 \text{ A}$$

- b.** From Eq. (15.61),

$$V_d = 2.3394 \times 0.125 \times 265.58 \times 0.9 = 69.9 \text{ V}$$

- c.** Because the ac input voltage to the converter is  $V_c = n_c V_s$ , Eq. (10.15) gives the average voltage at the dc side of the converter as

$$V_{dc} = -\frac{3\sqrt{3}\sqrt{2} n_c V_s}{\pi} \cos \alpha = -2.3394 n_c V_s \cos \alpha \quad (15.70)$$



**FIGURE 15.9**  
Equivalent circuit for static Kramer drive.

Because  $V_d = V_{dc}$ , Eqs. (15.61) and (15.70) give

$$2.3394sV_s n_m = -2.3394n_c V_s \cos \alpha$$

which gives

$$s = \frac{-n_c \cos \alpha}{n_m} \quad (15.71)$$

The speed, which is independent of torque, becomes

$$\omega_m = \omega_s(1 - s) = \omega_s \left( 1 + \frac{n_c \cos \alpha}{n_m} \right) \quad (15.72)$$

$$109.96 = 125.66 \times \left( 1 + \frac{0.4 \cos \alpha}{0.9} \right)$$

which gives the delay angle,  $\alpha = 106.3^\circ$ .

**d.** The power fed back

$$P_1 = V_d I_d = 69.9 \times 134.6 = 9409 \text{ W}$$

The output power

$$P_o = T_L \omega_m = 598.91 \times 109.96 = 65,856 \text{ W}$$

The rms rotor current referred to the stator is

$$I'_r = \sqrt{\frac{2}{3}} I_d n_m = \sqrt{\frac{2}{3}} \times 134.6 \times 0.9 = 98.9 \text{ A}$$

$$P_{ru} = 3 \times 0.044 \times 98.9^2 = 1291 \text{ W}$$

$$P_{su} = 3 \times 0.041 \times 98.9^2 = 1203 \text{ W}$$

$$P_i = 65,856 + 1291 + 1203 = 68,350 \text{ W}$$

The efficiency is  $65,856/68,350 = 96\%$ .

- e. From (f) in Example 15.3,  $I'_{r1} = 0.7797I_d n_m = 94.45 \text{ A}$ ,  $I_m = 265.58/6.1 = 43.54 \text{ A}$ , and  $\mathbf{I}_{i1} = 104 \angle -24.74^\circ$ . From Example 10.5, the rms current fed back to the supply is

$$\mathbf{I}_{i2} = \sqrt{\frac{2}{3}} I_d n_c \angle -\alpha = \sqrt{\frac{2}{3}} \times 134.6 \times 0.4 \angle -\alpha = 41.98 \angle -106.3^\circ$$

The effective input current of the drive is

$$\mathbf{I}_i = \mathbf{I}_{i1} + \mathbf{I}_{i2} = 104 \angle -24.74^\circ + 41.98 \angle -106.3^\circ = 117.7 \angle -45.4^\circ \text{ A}$$

The input PF is  $\text{PF}_s = \cos(-45.4^\circ) = 0.702$  (lagging).

---



At 30 Hz,  $\omega_s = 2 \times 2 \times \pi 30/4 = 94.25$  rad/s,  $\beta = 30/60 = 0.5$ , and  $V_a = d\omega_s = 1.409 \times 94.25 = 132.79$  V. the slip for maximum torque is

$$s_m = \frac{0.38}{[0.66^2 + 0.5^2 \times (1.14 + 1.71)^2]^{1/2}} = 0.242$$

$$\omega_m = 94.25 \times (1 - 0.242) = 71.44 \text{ rad/s} \quad \text{or} \quad 682 \text{ rpm}$$

$$T_m = \frac{3 \times 132.79^2}{2 \times 94.25 \times [0.66 + \sqrt{0.66^2 + 0.5^2 \times (1.14 + 1.71)^2}]} = 125.82 \text{ N} \cdot \text{m}$$

**b.** At 60 Hz,  $\omega_b = \omega_s = 188.5$  rad/s and  $V_a = 265.58$  V.

$$s_m = \frac{0.38}{1.14 + 1.71} = 0.1333$$

$$\omega_m = 188.5 \times (1 - 0.1333) = 163.36 \text{ rad/s} \quad \text{or} \quad 1560 \text{ rpm}$$

the maximum torque is  $T_m = 196.94$  N·m.

At 30 Hz,  $\omega_s = 94.25$  rad/s,  $\beta = 0.5$ , and  $V_a = 132.79$  V. From Eq. (15.77)

$$s_m = \frac{0.38/0.5}{1.14 + 1.71} = 0.2666$$

$$\omega_m = 94.25 \times (1 - 0.2666) = 69.11 \text{ rad/s} \quad \text{or} \quad 660 \text{ rpm}$$

the maximum torque is  $T_m = 196.94$  N·m.

## 5-Current Control

The torque of induction motors can be controlled by varying the rotor current. The input current, which is readily accessible, is varied instead of the rotor current. For a fixed input current, the rotor current depends on the relative values of the magnetizing and rotor circuit impedances. From Figure 1, the rotor current can be found as:

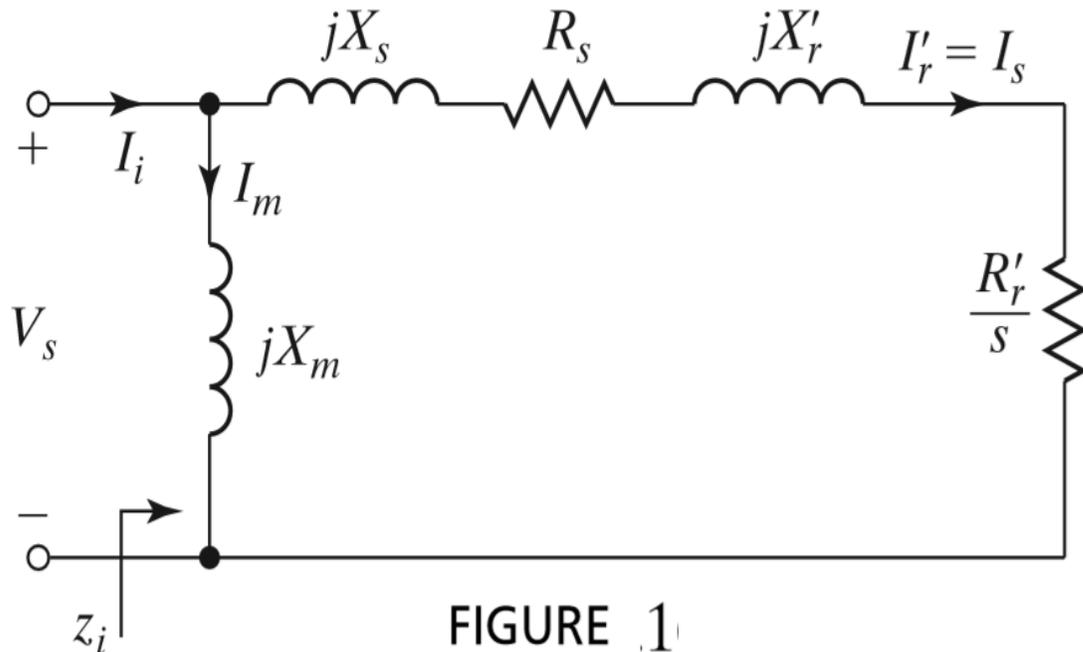


FIGURE 1

Approximate per-phase equivalent circuit.

$$\bar{\mathbf{I}}'_r = \frac{jX_m I_i}{R_s + R'_r/s + j(X_m + X_s + X'_r)} = I'_r \angle \theta_1$$

$$P_g = 3(I_r')^2 \frac{R_r'}{s} \text{ and } T_d = \frac{P_g}{\omega_s}$$

the developed torque is 
$$T_d = \frac{3 R_r' (X_m I_i)^2}{s \omega_s [(R_s + R_r'/s)^2 + (X_m + X_s + X_r')^2]}$$

and the starting torque at  $s = 1$  is

$$T_s = \frac{3 R_r' (X_m I_i)^2}{\omega_s [(R_s + R_r')^2 + (X_m + X_s + X_r')^2]}$$

The slip for maximum torque is

$$s_m = \pm \frac{R_r'}{[R_s^2 + (X_m + X_s + X_r')^2]^{1/2}} \quad \text{Home work (derive this Formula)}$$

Generally,  $X_m$  is much greater than  $X_s$  and  $R_s$ , which can be neglected for most applications. Neglecting the values of  $R_s$  and  $X_s$ .

$$s_m = \pm \frac{R'_r}{X_m + X'_r}$$

Home work (derive this Formula)

and at  $s = s_m$ , The maximum torque,

$$T_m = \frac{3 X_m^2}{2\omega_s (X_m + X'_r)} I_i^2 = \frac{3 L_m^2}{2(L_m + L'_r)} I_i^2 \quad (1)$$

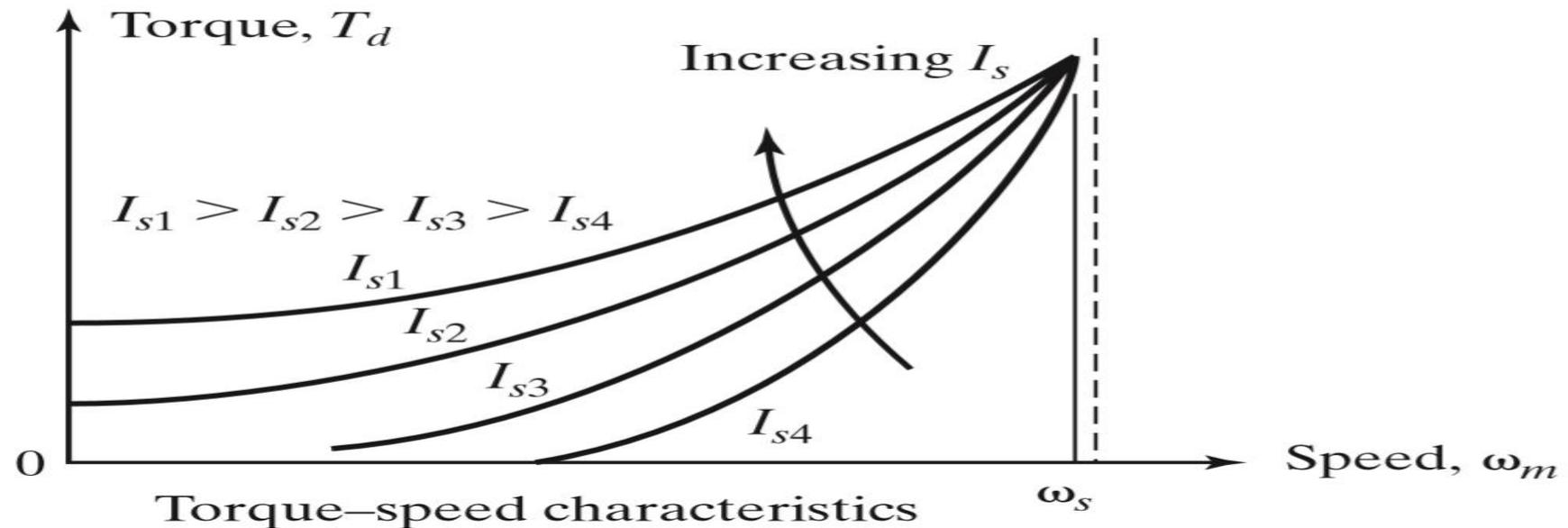
It can be noticed from Eq.1. that the maximum torque depends on the square of the current and is approximately independent of the frequency

The input current  $I_i$  is supplied from a dc current source  $I_d$  consisting of a large inductor. The fundamental stator rms phase current of the three-phase current-source inverter is related to  $I_d$  by

$$I_i = I_s = \frac{\sqrt{2}\sqrt{3}}{\pi} I_d = 0.779 I_d$$

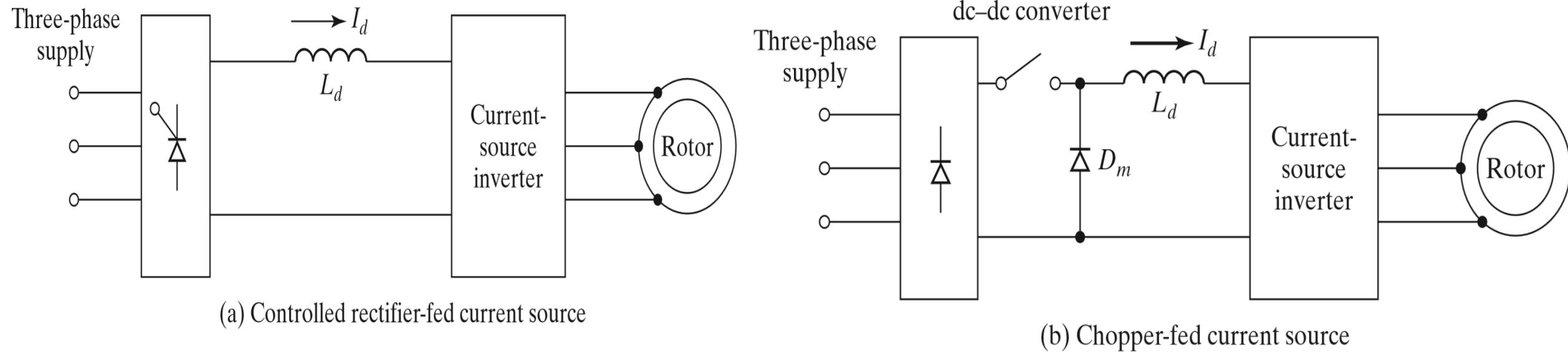
$$T_m = \frac{3L_m^2}{2(L_m + L'_r)} I_i^2 \quad \text{--- 1}$$

The typical torque–speed characteristics are shown in Fig.2. for increasing values of stator current. Because  $X_m$  is large as compared with  $X_s$  and  $X'_r$ , the starting torque is low. As the speed increases (or slip decreases), the stator voltage rises and the torque increases. The starting current is low due to the low values of flux (as  $I_m$  is low and  $X_m$  is large) and rotor current compared with their rated values. The torque increases with the speed due to the increase in flux.



A further increase in speed toward the positive slope of the characteristics increases the terminal voltage beyond the rated value. The flux and the magnetizing current are also increased, thereby saturating the flux. The torque can be controlled by the stator current and slip. To keep the air-gap flux constant and to avoid saturation due to high voltage, the motor is normally operated on the negative slope of the equivalent torque–speed characteristics with voltage control. The negative slope is in the unstable region and the motor must be operated in closed-loop control. At a low slip, the terminal voltage could be excessive and the flux would saturate. Due to saturation, the torque peaking, as shown in Figure 2, is less than that as shown.

The constant current can be supplied by three-phase current-source inverters. The current-fed inverter has the advantages of fault current control and the current is less sensitive to the motor parameter variations. However, they generate harmonics and torque pulsation. Two possible configurations of current-fed inverter drives are shown in Fig. 3. In Fig.3a, the inductor acts as a current source and the controlled rectifier controls the current source. The input PF of this arrangement is very low. In Fig.3b, the dc–dc converter controls the current source and the input PF is higher.



**FIGURE 3** Current-source inductor motor drive.

## Example 1

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole, Y-connected induction motor has the following parameters:  $R_s = 0.66 \Omega$ ,  $R_r' = 0.38 \Omega$ ,  $X_s = 1.14 \Omega$ ,  $X_r' = 1.71 \Omega$ , and  $X_m = 33.2 \Omega$ . The no-load loss is negligible. The motor is controlled by a current-source inverter and the input current is maintained constant at 20 A. If the frequency is 40 Hz and the developed torque is 55 N·m, determine (a) the slip for maximum torque  $s_m$  and maximum torque  $T_m$ , (b) the slip  $s$ , (c) the rotor speed  $\omega_m$ , (d) the terminal voltage per phase  $V_a$ , and (e) the  $\text{PF}_m$ .

### ***Solution***

$V_{a(\text{rated})} = 460/\sqrt{3} = 265.58 \text{ V}$ ,  $I_i = 20 \text{ A}$ ,  $T_L = T_d = 55 \text{ N}\cdot\text{m}$ , and  $p = 4$ . At 40 Hz,  $\omega = 2\pi \times 40 = 251.33 \text{ rad/s}$ ,  $\omega_s = 2 \times 251.33/4 = 125.66 \text{ rad/s}$ ,  $R_s = 0.66 \Omega$ ,  $R_r' = 0.38 \Omega$ ,  $X_s = 1.14 \times 40/60 = 0.76 \Omega$ ,  $X_r' = 1.71 \times 40/60 = 1.14 \Omega$ , and  $X_m = 33.2 \times 40/60 = 22.13 \Omega$ .

$$s_m = \frac{0.38}{[0.66^2 + (22.13 + 0.78 + 1.14)^2]^{1/2}} = 0.0158$$

for  $s = s_m$ ,  $T_m = 94.68 \text{ N}\cdot\text{m}$ .

$$T_d = 55 = \frac{3(R_r/s) (22.13 \times 20)^2}{125.66 \times [(0.66 + R_r/s)^2 + (22.13 + 0.76 + 1.14)^2]}$$

which gives  $(R_r/s)^2 - 83.74(R_r/s) + 578.04 = 0$ , and solving for  $R_r/s$  yields

$$\frac{R_r'}{s} = 76.144 \quad \text{or} \quad 7.581$$

and  $s = 0.00499$  or  $0.0501$ . Because the motor is normally operated with a large slip in the negative slope of the torque–speed characteristic,  $s = 0.0501$

- c.**  $\omega_m = 125.656 \times (1 - 0.0501) = 119.36$  rad/s or 1140 rpm.
- d.** From Figure 15.2, the input impedance can be derived as

$$\bar{\mathbf{Z}}_i = R_i + jX_i = (R_i^2 + X_i^2)^{1/2} \angle \theta_m = Z_i \angle \theta_m$$

$$R_i = \frac{X_m^2 (R_s + R_r/s)}{(R_s + R_r/s)^2 + (X_m + X_s + X_r)^2}$$
$$= 6.26 \Omega$$

$$X_i = \frac{X_m [(R_s + R_r/s)^2 + (X_s + X_r)(X_m + X_s + X_r)]}{(R_s + R_r/s)^2 + (X_m + X_s + X_r)^2} = 3.899 \Omega$$

and

$$\theta_m = \tan^{-1} \frac{X_i}{R_i}$$

$$= 31.9^\circ$$

$$Z_i = (6.26^2 + 3.899^2)^{1/2} = 7.38 \Omega$$

$$V_a = Z_i I_i = 7.38 \times 20 = 147.6 \text{ V}$$

**e.**  $\text{PF}_m = \cos(31.9^\circ) = 0.849$  (lagging).

## 6-voltage, Current, and Frequency Control

The torque–speed characteristics of induction motors depend on the type of control. It may be necessary to vary the voltage, frequency, and current to meet the torque–speed requirements, as shown in Fig.3, where there are three regions. **In the first region, the speed can be varied by voltage(or current) control at constant torque.** **In the second region, the motor is operated at constant current and the slip is varied.** **In the third region, the speed is controlled by frequency at a reduced stator current.**

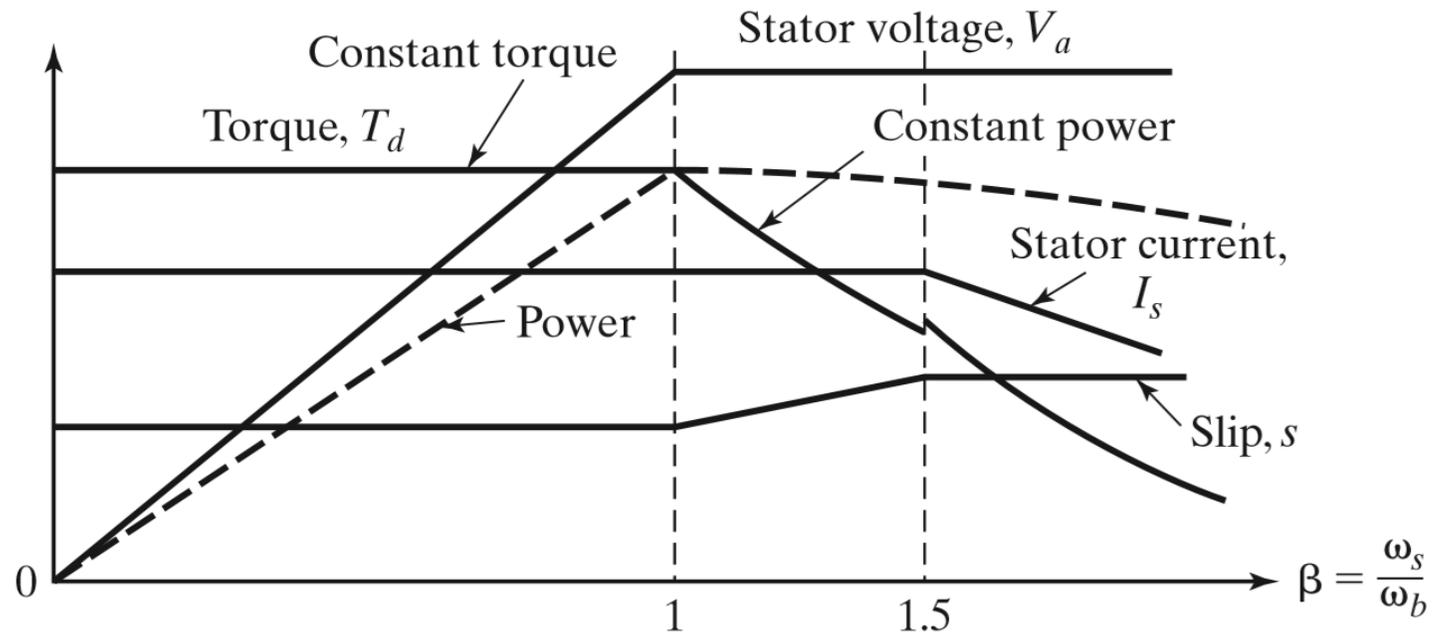


FIGURE 3 Control variables versus frequency.