9

BJT and JFET Frequency Response

CHAPTER OBJECTIVES

- Develop confidence in the use of logarithms, understand the concept of decibels, and be able to accurately read a logarithmic plot.
- Become acquainted with the frequency response of a BJT and FET amplifier.
- Be able to normalize a frequency plot, establish the dB plot, and find the cutoff frequencies and bandwidth.
- Understand how straight-line segments and cutoff frequencies can result in a Bode plot that will define the frequency response of an amplifier.
- Be able to find the Miller effect capacitance at the input and output of an amplifier due to a feedback capacitor.
- Become familiar with square-wave testing to determine the frequency response of an amplifier.

9.1 INTRODUCTION

The analysis thus far has been limited to a particular frequency. For the amplifier, it was a frequency that normally permitted ignoring the effects of the capacitive elements, reducing the analysis to one that included only resistive elements and sources of the independent and controlled variety. We will now investigate the frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at high frequencies. Because the analysis will extend through a wide frequency range, the logarithmic scale will be defined and used throughout the analysis. In addition, because industry typically uses a decibel scale on its frequency plots, the concept of the decibel is introduced in some detail. The similarities between the frequency response analyses of both BJTs and FETs permit the coverage of both in the same chapter.

9.2 LOGARITHMS

In this field, there is no escaping the need to become comfortable with the logarithmic function. The plotting of a variable between wide limits, comparing levels without having to deal with unwieldy numbers, and identifying levels of particular importance in the design, review, and analysis procedures are all positive features of using the logarithmic function.

As a first step in clarifying the relationship between the variables of a logarithmic function, consider the following mathematical equations:

$$a = b^x, \qquad x = \log_b a \tag{9.1}$$

The variables a, b, and x are the same in each equation. If a is determined by taking the base b to the x power, the same x will result if the log of a is taken to the base b. For instance, if $b = 10$ and $x = 2$,

$$a = b^x = (10)^2 = 100$$

but

$$x = \log_b a = \log_{10} 100 = 2$$

In other words, if you were asked to find the power of a number that would result in a particular level such as

$$10{,}000 = 10^x$$

you could determine the level of x using logarithms. That is,

$$x = \log_{10} 10{,}000 = 4$$

For the electrical/electronics industry and in fact for the vast majority of scientific research, the base in the logarithmic equation is chosen as either 10 or the number $e = 2.71828\ldots$.

Logarithms taken to the base 10 are referred to as *common logarithms,* whereas logarithms taken to the base e are referred to as *natural logarithms.* In summary:

$$\text{Common logarithm:} \quad x = \log_{10} a \tag{9.2}$$

$$\text{Natural logarithm:} \quad y = \log_e a \tag{9.3}$$

The two are related by

$$\log_e a = 2.3 \log_{10} a \tag{9.4}$$

On scientific calculators, the common logarithm is typically denoted by the **log** key and the natural logarithm by the **ln** key.

EXAMPLE 9.1 Using the calculator, determine the logarithm of the following numbers to the base indicated:

a. $\log_{10} 10^6$.
b. $\log_e e^3$.
c. $\log_{10} 10^{-2}$.
d. $\log_e e^{-1}$.

Solution:

a. **6** b. **3** c. **−2** d. **−1**

The results in Example 9.1 clearly reveal that

the logarithm of a number taken to a power is simply the power of the number if the number matches the base of the logarithm.

In the next example, the base and the variable x are not related by an integer power of the base.

EXAMPLE 9.2 Using the calculator, determine the logarithm of the following numbers:

a. $\log_{10} 64$.
b. $\log_e 64$.
c. $\log_{10} 1600$.
d. $\log_{10} 8000$.

Solution:

a. **1.806** b. **4.159** c. **3.204** d. **3.903**

Note in parts (a) and (b) of Example 9.2 that the logarithms $\log_{10} a$ and $\log_e a$ are indeed related as defined by Eq. (9.4). In addition, note that the logarithm of a number does not increase in the same linear fashion as the number. That is, 8000 is 125 times larger than 64, but the logarithm of 8000 is only about 2.16 times larger than the magnitude of the logarithm of 64, revealing a very nonlinear relationship. In fact, Table 9.1 clearly shows how the logarithm of a number increases only as the exponent of the number. If the antilogarithm of a number is desired, the 10^x or e^x calculator function is employed.

TABLE 9.1

$\log_{10} 10^0$	$= 0$
$\log_{10} 10$	$= 1$
$\log_{10} 100$	$= 2$
$\log_{10} 1{,}000$	$= 3$
$\log_{10} 10{,}000$	$= 4$
$\log_{10} 100{,}000$	$= 5$
$\log_{10} 1{,}000{,}000$	$= 6$
$\log_{10} 10{,}000{,}000$	$= 7$
$\log_{10} 100{,}000{,}000$	$= 8$
etc.	

EXAMPLE 9.3 Using a calculator, determine the antilogarithm of the following expressions:

a. $1.6 = \log_{10} a$.
b. $0.04 = \log_e a$.

Solution:

a. $a = 10^{1.6}$

Using the 10^x key: $a = \mathbf{39.81}$

b. $a = e^{0.04}$

Using the e^x key: $a = \mathbf{1.0408}$

Because the remaining analysis of this chapter employs the common logarithm, we review a few properties of logarithms using solely the common logarithm. In general, however, the same relationships hold true for logarithms to any base. First, note that

$$\boxed{\log_{10} 1 = 0} \tag{9.5}$$

as clearly revealed by Table 9.1, because $10^0 = 1$. Next,

$$\boxed{\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b} \tag{9.6}$$

which for the special case of $a = 1$ becomes

$$\boxed{\log_{10} \frac{1}{b} = -\log_{10} b} \tag{9.7}$$

which shows that for any b greater than 1, the logarithm of a number less than 1 is always negative. Finally,

$$\boxed{\log_{10} ab = \log_{10} a + \log_{10} b} \tag{9.8}$$

In each case, the equations employing natural logarithms have the same format.

EXAMPLE 9.4 Using a calculator, determine the logarithm of the following numbers:

a. $\log_{10} 0.5$.

b. $\log_{10} \dfrac{4000}{250}$.

c. $\log_{10} (0.6 \times 30)$.

Solution:

a. $\mathbf{-0.3}$

b. $\log_{10} 4000 - \log_{10} 250 = 3.602 - 2.398 = \mathbf{1.204}$

$$\text{Check: } \log_{10} \frac{4000}{250} = \log_{10} 16 = \mathbf{1.204}$$

c. $\log_{10} 0.6 + \log_{10} 30 = -0.2218 + 1.477 = \mathbf{1.255}$

$$\text{Check: } \log_{10} (0.6 \times 30) = \log_{10} 18 = \mathbf{1.255}$$

The use of log scales can significantly expand the range of variation of a particular variable on a graph. Most graph paper available is of the semilog or double-log (log-log) variety. The term *semi* (meaning one-half) indicates that only one of the two scales is a log scale, whereas double-log indicates that both scales are log scales. A semilog scale appears in Fig. 9.1. Note that the vertical scale is a linear scale with equal divisions. The spacing between the lines of the log plot is shown on the graph. The log of 2 to the base 10 is approximately 0.3. The distance from 1 ($\log_{10} 1 = 0$) to 2 is therefore 30% of the span. The log of 3 to the base 10 is 0.4771 or almost 48% of the span (very close to one-half

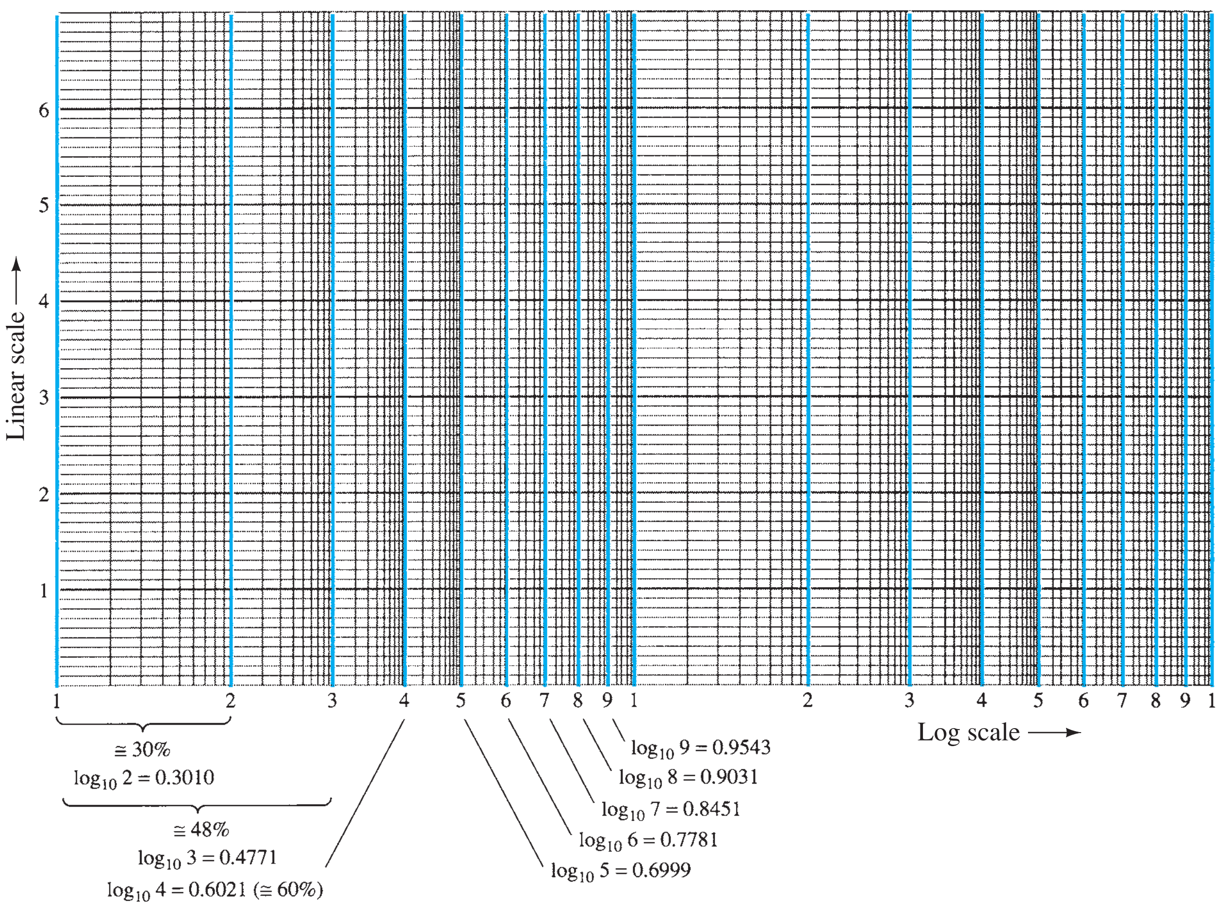

FIG. 9.1
Semilog graph paper.

the distance between power-of-10 increments on the log scale). Because $\log_{10} 5 \cong 0.7$, it is marked off at a point 70% of the distance. Note that between any two digits the same compression of the lines appears as you progress from the left to the right. It is important to note the resulting numerical value and the spacing, because plots will typically only have the tic marks indicated in Fig. 9.2 due to a lack of space. The longer bars for this figure have the numerical values of 0.3, 3, and 30 associated with them, whereas the next-shorter bars have values of 0.5, 5, and 50 and the shortest bars 0.7, 7, and 70.

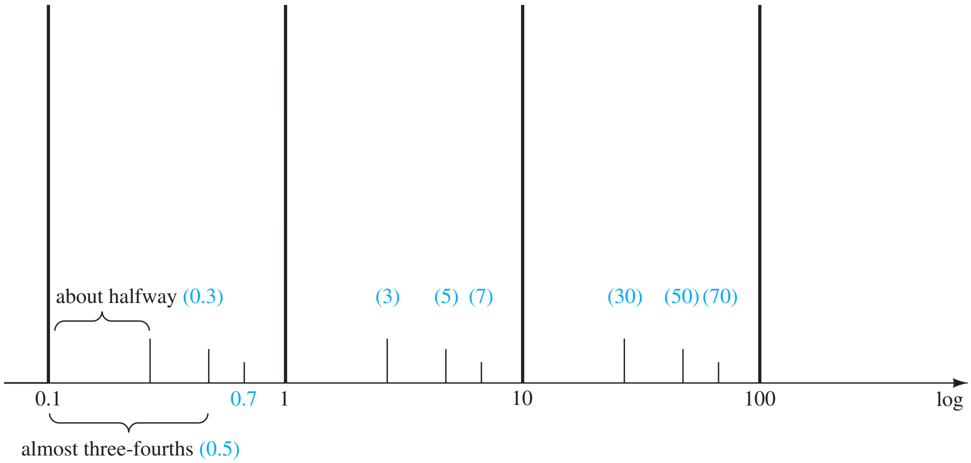

FIG. 9.2
Identifying the numerical values of the tic marks on a log scale.

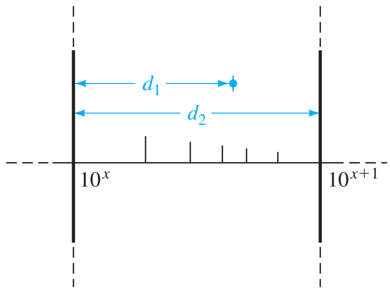

FIG. 9.3
Finding a value on a log plot.

On many log plots, the tick marks for most of the intermediate levels are left off because of space constraints. The following equation can be used to determine the logarithmic level at a particular point between known levels using a ruler or simply estimating the distances. The parameters are defined by Fig. 9.3.

$$\boxed{\text{Value} = 10^x \times 10^{d_1/d_2}} \tag{9.9}$$

The derivation of Eq. (9.9) is simply an extension of the details regarding distance appearing in Fig. 9.1.

EXAMPLE 9.5 Determine the value of the point appearing on the logarithmic plot in Fig. 9.4 using the measurements made by a ruler (linear).

Solution:

$$\frac{d_1}{d_2} = \frac{7/16''}{3/4''} = \frac{0.438''}{0.750''} = 0.584$$

Using a calculator:

$$10^{d_1/d_2} = 10^{0.584} = 3.837$$

Applying Eq. (9.9):

$$\begin{aligned}\text{Value} &= 10^x \times 10^{d_1/d_2} = 10^2 \times 3.837\\ &= \mathbf{383.7}\end{aligned}$$

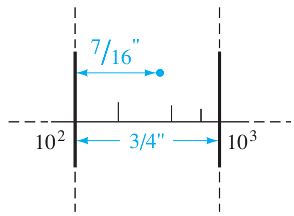

FIG. 9.4
Example 9.5.

Plotting a function on a log scale can change the general appearance of the waveform as compared to a plot on a linear scale. A straight-line plot on a linear scale can develop a curve on a log scale, and a nonlinear plot on a linear scale can take on the appearance of a straight line on a log plot. The important point is that the results extracted at each level should be correctly labeled by developing a familiarity with the spacing of Figs. 9.1 and 9.2. This is particularly true for some of the log-log plots that appear later in this book.

9.3 DECIBELS

Power Levels

The concept of the decibel (dB) and the associated calculations will become increasingly important in the remaining sections of this chapter. The term *decibel* has its origin in the fact that power and audio levels are related on a logarithmic basis. That is, an increase in power level from, say, 4 W to 16 W does not result in an audio level increase by a factor of $16/4 = 4$, but by a factor of 2, as derived from the power of 4 in the following manner: $(4)^2 = 16$. For a change of 4 W to 64 W, the audio level will increase by a factor of 3 because $(4)^3 = 64$. In logarithmic form, the relationship can be written as $\log_4 64 = 3$.

The term *bel* is derived from the surname of Alexander Graham Bell. For standardization, the bel (B) is defined by the following equation relating two power levels, P_1 and P_2:

$$G = \log_{10}\frac{P_2}{P_1} \quad \text{bel} \tag{9.10}$$

It was found, however, that the bel was too large a unit of measurement for practical purposes, so the decibel (dB) is defined such that 10 decibels = 1 bel. Therefore,

$$G_{\text{dB}} = 10\log_{10}\frac{P_2}{P_1} \quad \text{dB} \tag{9.11}$$

The terminal rating of electronic communication equipment (amplifiers, microphones, etc.) is commonly in decibels. Equation (9.11) indicates clearly, however, that the decibel rating is a measure of the difference in magnitude between *two* power levels. For a specified terminal (output) power (P_2) there must be a reference power level (P_1). The reference level is generally accepted to be 1 mW, although on occasion, the 6-mW standard of earlier years is applied. The resistance associated with the 1-mW power level is 600 Ω, chosen because it is the characteristic impedance of audio transmission lines. When the 1-mW level is employed as the reference level, the decibel symbol frequently appears as dBm. In equation form,

$$G_{\text{dBm}} = 10\log_{10}\left.\frac{P_2}{1\text{ mW}}\right|_{600\,\Omega} \quad \text{dBm} \tag{9.12}$$

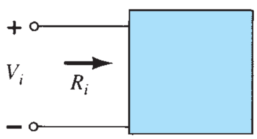

FIG. 9.5
Configuration employed in the discussion of Eq. (9.13).

There exists a second equation for decibels that is applied frequently. It can be best described through the system of Fig. 9.5. For V_i equal to some value V_1, $P_1 = V_1^2/R_i$, where R_i is the input resistance of the system of Fig. 9.5. If V_i should be increased (or decreased) to some other level, V_2 then $P_2 = V_2^2/R_i$. If we substitute into Eq. (9.11) to determine the resulting difference in decibels between the power levels, we obtain

$$G_{\text{dB}} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_i}{V_1^2/R_i} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2$$

and

$$G_{\text{dB}} = 20\log_{10}\frac{V_2}{V_1} \quad \text{dB} \tag{9.13}$$

Frequently, the effect of different impedances ($R_1 \neq R_2$) is ignored and Eq. (9.13) applied simply to establish a basis of comparison between levels—voltage or current. For situations of this type, the decibel gain should more correctly be referred to as the voltage or current gain in decibels to differentiate it from the common usage of decibel as applied to power levels.

In particular note the multiplying factor of 20 rather than the 10 of earlier equations.

Cascaded Stages

One of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages. For example, the magnitude of the overall voltage gain of a cascaded system is given by

$$|A_{v_T}| = |A_{v_1}| \cdot |A_{v_2}| \cdot |A_{v_3}| \cdots |A_{v_n}| \tag{9.14}$$

Applying the proper logarithmic relationship results in

$$G_v = 20\log_{10}|A_{v_T}| = 20\log_{10}|A_{v_1}| + 20\log_{10}|A_{v_2}| + 20\log_{10}|A_{v_3}| + \cdots + 20\log_{10}|A_{v_n}| \quad \text{(db)} \qquad \textbf{(9.15)}$$

In words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage, that is,

$$\boxed{G_{\text{dB}_T} = G_{\text{dB}_1} + G_{\text{dB}_2} + G_{\text{dB}_3} + \cdots + G_{\text{dB}_n}} \quad \text{dB} \qquad \textbf{(9.16)}$$

Voltage Gains versus dB Levels

Table 9.2 shows the association between dB levels and voltage gains. First note that a gain of 2 results in a dB level of +6 dB, whereas a drop to $\frac{1}{2}$ results in a −6-dB level. A change in V_o/V_i from 1 to 10, 10 to 100, or 100 to 1000 results in the same 20-dB change in level. When $V_o = V_i$, $V_o/V_i = 1$, and the dB level is 0. At a very high gain of 1000, the dB level is 60, whereas at the much higher gain of 10,000, the dB level is 80 dB, an increase of only 20 dB—a result of the logarithmic relationship. Table 9.2 clearly reveals that voltage gains of 50 dB or higher should immediately be recognized as being quite high.

TABLE 9.2

Comparing $A_v = \dfrac{V_o}{V_i}$ *to dB*

Voltage Gain, V_o/V_i	dB Level
0.5	–6
0.707	–3
1	0
2	6
10	20
40	32
100	40
1000	60
10,000	80
etc.	

EXAMPLE 9.6 Find the magnitude gain corresponding to a voltage gain of 100 dB.

Solution: By Eq. (9.13),

$$G_{\text{dB}} = 20\log_{10}\frac{V_2}{V_1} = 100 \text{ dB} \Rightarrow \log_{10}\frac{V_2}{V_1} = 5$$

so that

$$\frac{V_2}{V_1} = \mathbf{10^5 = 100{,}000}$$

EXAMPLE 9.7 The input power to a device is 10,000 W at a voltage of 1000 V. The output power is 500 W and the output impedance is 20 Ω.

a. Find the power gain in decibels.
b. Find the voltage gain in decibels.
c. Explain why parts (a) and (b) agree or disagree.

Solution:

a. $$G_{\text{dB}} = 10\log_{10}\frac{P_o}{P_i} = 10\log_{10}\frac{500\text{ W}}{10\text{ kW}} = 10\log_{10}\frac{1}{20} = -10\log_{10}20$$
$$= -10(1.301) = \mathbf{-13.01\ dB}$$

b. $$G_v = 20\log_{10}\frac{V_o}{V_i} = 20\log_{10}\frac{\sqrt{PR}}{1000} = 20\log_{10}\frac{\sqrt{(500\text{ W})(20\ \Omega)}}{1000\text{ V}}$$
$$= 20\log_{10}\frac{100}{1000} = 20\log_{10}\frac{1}{10} = -20\log_{10}10 = \mathbf{-20\ dB}$$

c. $$R_i = \frac{V_i^2}{P_i} = \frac{(1\text{ kV})^2}{10\text{ kW}} = \frac{10^6}{10^4} = \mathbf{100\ \Omega \neq R_o = 20\ \Omega}$$

EXAMPLE 9.8 An amplifier rated at 40-W output is connected to a 10-Ω speaker.

a. Calculate the input power required for full power output if the power gain is 25 dB.
b. Calculate the input voltage for rated output if the amplifier voltage gain is 40 dB.

Solution:

a. Eq. (9.11): $$25 = 10\log_{10}\frac{40\text{ W}}{P_i} \Rightarrow P_i = \frac{40\text{ W}}{\text{antilog}\,(2.5)} = \frac{40\text{ W}}{3.16 \times 10^2}$$
$$= \frac{40\text{ W}}{316} \cong \mathbf{126.5\ mW}$$

b. $G_v = 20 \log_{10} \dfrac{V_o}{V_i} \Rightarrow 40 = 20 \log_{10} \dfrac{V_o}{V_i}$

$$\frac{V_o}{V_i} = \text{antilog } 2 = 100$$

$$V_o = \sqrt{PR} = \sqrt{(40 \text{ W})(10 \text{ V})} = 20 \text{ V}$$

$$V_i = \frac{V_o}{100} = \frac{20 \text{ V}}{100} = 0.2 \text{ V} = \mathbf{200\ mV}$$

Human Auditory Response

One of the most frequent applications of the decibel scale is in the communication and entertainment industries. The human ear does not respond in a linear fashion to changes in source power level, that is, a doubling of the audio power level from 1/2 W to 1 W does not result in a doubling of the loudness level for the human ear. In addition, a change from 5 W to 10 W is received by the ear as the same change in sound intensity as experienced from 1/2 W to 1 W. In other words, the ratio between levels is the same in each case (1 W/0.5 W = 10 W/5 W = 2), resulting in the same decibel or logarithmic change defined by Eq. (9.11). The ear, therefore, responds in a logarithmic fashion to changes in audio power levels.

To establish a basis for comparison between audio levels, a reference level of 0.0002 **microbar** (μbar) was chosen, where 1 μbar is equal to the sound pressure of 1 dyne per square centimeter, or about 1 millionth of the normal atmospheric pressure at sea level. The 0.0002 μbar level is the threshold level of hearing. Using this reference level, the sound pressure level in decibels is defined by the following equation:

$$\text{dB}_s = 20 \log_{10} \frac{P}{0.0002\ \mu\text{bar}} \qquad \textbf{(9.17)}$$

where P is the sound pressure in microbars.

The decibel levels in Table 9.3 are defined by Eq. (9.17). Meters designed to measure audio levels are calibrated to the levels defined by Eq. (9.17) and shown in Table 9.3.

In particular take note of the sound level for iPods and MP3 players, for which it is suggested, based on research, that they should not be used for more than 1 hour a day at 60% volume to prevent permanent hearing damage. Always remember that hearing damage is usually not reversible, so that any loss is for the long term.

A common question regarding audio levels is how much the power level of an acoustical source must be increased to double the sound level received by the human ear. The question is not as simple as it first seems due to considerations such as the frequency content of the sound, the acoustical conditions of the surrounding area, the physical characteristics of the surrounding medium, and—of course—the unique characteristics of the human ear. However, a general conclusion can be formulated that has practical value if we note the power levels of an acoustical source appearing to the left in Table 9.3. Each power level is associated with a particular decibel level, and a change of 10 dB in the scale corresponds to an increase or a decrease in power by a factor of 10. For instance, a change from 90 dB to 100 dB is associated with a change in wattage from 3 W to 30 W. Through experimentation, it has been found that on an average basis the loudness level doubles for every 10 dB change in audio level—a conclusion somewhat verified by the examples to the right in Table 9.3.

To double the sound level received by the human ear, the power rating of the acoustical source (in watts) must be increased by a factor of 10.

In other words, doubling the sound level available from a 1 W acoustical source requires moving up to a 10 W source.

Further:

At normal hearing levels, it would take a change of about 3 dB (twice the power level) for the change to be noticeable to the human ear.

At low levels of sound, a change of 2 dB may be noticeable, but it may take a 6 dB (four times the power level) change for much higher levels of sound.

TABLE 9.3

Typical sound levels and their decibel levels.

Output Power. Average value in watts.	dB_s		
	160	Jet engine	
	150		
	140	Community siren	
	130	Jackhammer	
Threshold of pain	120	Live music concert, iPods and MP3 players at full volume	
300	110	Health club, movie theater	
100		Chain saw	
30	100	Very loud music, motorcycle	
10			
3	90	Loud music, heavy truck, subway train	
1			
0.3	80	Orchestra, highway traffic, alarm clock	
0.1			
0.03	70		
0.01		Average conversation	
0.003	60	Quiet music	Dynamic range $\cong$ 120 dB_s
0.001			
0.0003	50	Average residence, computer system	
	40	Background music	
	30	Quiet office, computer hard drive	
	20	Whispering	
	10	Faint sounds, paper rustling	
0.0002 μbar of pressure	0	Threshold of hearing	

One final example of the use of dB as a unit of measurement is the LRAD (Long Range Acoustic Device) appearing in Fig. 9.6. It emits a tone between 2100 Hz and 3100 Hz at 145 dB that is effective at up to 500 m, or almost two football fields. The sound at its peak is thousands of times louder than a smoke alarm. It can be used to transmit critical information and instructions and is capable of strong deterrent tones against intruders.

FIG. 9.6

LRAD (Long Range Acoustic Device) 1000X. (Courtesy of LRAD Corporation.)

Instrumentation

A number of modern VOMs and DMMs have a dB scale designed to provide an indication of power ratios referenced to a standard level of 1 mW at 600 Ω. Since the reading is accurate only if the load has a characteristic impedance of 600 Ω, the 1 mW, 600 Ω reference level is normally printed somewhere on the face of the meter, as shown in Fig. 9.7. The dB scale is usually calibrated to the lowest ac scale of the meter. In other words, when making the dB measurement, choose the lowest ac voltage scale, but read the dB scale. If a higher voltage scale is chosen, a correction factor must be used, which is sometimes printed on the face of the meter but is always available in the meter manual. If the impedance is other than 600 Ω or not purely resistive, other correction factors must be used that are normally included in the meter manual. Using the basic power equation $P = V^2/R$ reveals that 1 mW across a 600 Ω load is the same as applying 0.775 V rms across a 600 Ω load; that is, $V = \sqrt{PR} = \sqrt{(1\text{ mW})(600\ \Omega)} = 0.775$ V. The result is that an analog display will have 0 dB [defining the reference point of 1 mW, $\text{dB} = 10 \log_{10} P_2/P_1 = 10 \log_{10}$ (1mW/1 mW(ref) = 0 dB] and 0.775 V rms on the same pointer projection, as shown in Fig. 9.7. A voltage of 2.5 V across a 600 Ω load results in a dB level of $\text{dB} = 20 \log_{10} V_2/V_1 = 20 \log_{10} 25\text{ V}/0.775 = 10.17$ dB, resulting in 2.5 V and 10.17 dB appearing along the same pointer projection. A voltage of less than 0.775 V, such as 0.5 V, results in a dB level of $\text{dB} = 20 \log_{10} V_2/V_1 = 20 \log_{10} 0.5\text{ V}/0.775\text{ V} = -3.8$ dB, also shown on the scale in Fig. 9.7. Although a reading of 10 dB reveals that the power level is 10 times the reference, don't assume that a reading of 5 dB means that the output level is 5 mW. The 10 : 1 ratio is a special one in logarithmic use. For the 5 dB level, the power level must be found using the antilogarithm (3.126), which reveals that the power level associated with 5 dB is about 3.1 times the reference or 3.1 mW. A conversion table is usually provided in the manual for such conversions.

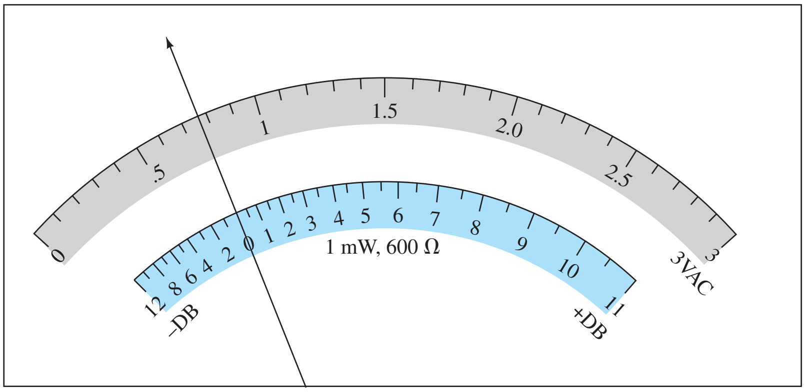

FIG. 9.7
Defining the relationship between a dB scale referenced to 1 mW, 600 Ω and a 3 V rms voltage scale.

9.4 GENERAL FREQUENCY CONSIDERATIONS

The frequency of the applied signal can have a pronounced effect on the response of a single-stage or multistage network. The analysis thus far has been for the midfrequency spectrum. At low frequencies, we shall find that the coupling and bypass capacitors can no longer be replaced by the short-circuit approximation because of the increase in reactance of these elements. The frequency-dependent parameters of the small-signal equivalent circuits and the stray capacitive elements associated with the active device and the network will limit the high-frequency response of the system. An increase in the number of stages of a cascaded system will also limit both the high- and low-frequency responses.

Low-Frequency Range

To demonstrate how the larger coupling and bypass capacitors of a network will affect the frequency response of a system, the reactance of a 1-μF (typical value for such applications) capacitor is tabulated in Table 9.4 for a wide range of frequencies.

TABLE 9.4

Variation in $X_C = \frac{1}{2\pi f_C}$ *with frequency for a 1-μF capacitor*

f	X_C	
10 Hz	15.91 kΩ	Range of possible effect
100 Hz	1.59 kΩ	
1 kHz	159 Ω	
10 kHz	15.9 Ω	
100 kHz	1.59 Ω	Range of lesser concern (≅ short-circuit equivalence)
1 MHz	0.159 Ω	
10 MHz	15.9 mΩ	
100 MHz	1.59 mΩ	

Two regions have been defined in Table 9.4. For the range of 10 Hz to 10 kHz the magnitude of the reactance is sufficiently large that it may have an impact on the response of the system. However, for much higher frequencies it appears as though the capacitor is behaving much like the short-circuit equivalent it is designed to match.

Clearly, therefore,

the larger capacitors of a system will have an important impact on the response of a system in the low-frequency range and can be ignored for the high-frequency region.

High-Frequency Range

For the smaller capacitors that come into play due to the parasitic capacitances of the device or network, the frequency range of concern will be the higher frequencies. Consider a 5-pF capacitor, typical of a parasitic capacitance of a transistor or the level of capacitance introduced simply by the wiring of the network, and the level of reactance that results for the same frequency range appearing in Table 9.4. The results appear in Table 9.5 and clearly reveal that at low frequencies they have a very large impedance matching the desired open-circuit equivalence. However, at higher frequencies they are approaching a short-circuit equivalence that can severely affect the response of a network.

TABLE 9.5

Variation in $X_C = \frac{1}{2\pi fC}$ *with frequency for a 5 pF capacitor*

f	X_C	
10 Hz	3,183 MΩ	Range of lesser concern (≅ open-circuit equivalent)
100 Hz	318.3 MΩ	
1 kHz	31.83 MΩ	
10 kHz	3.183 MΩ	
100 kHz	318.3 kΩ	Range of possible effect
1 MHz	31.83 kΩ	
10 MHz	3.183 kΩ	
100 MHz	318.3 Ω	

Clearly, therefore,

the smaller capacitors of a system will have an important impact on the response of a system in the high-frequency range and can be ignored for the low-frequency region.

Mid-Frequency Range

In the mid-frequency range the effect of the capacitive elements is largely ignored and the amplifier considered ideal and composed simply of resistive elements and controlled sources.

The result is that

the effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determined.

Typical Frequency Response

The magnitudes of the gain response curves of an *RC*-coupled, direct-coupled, and transformer-coupled amplifier system are provided in Fig. 9.8. Note that the horizontal scale is a logarithmic scale to permit a plot extending from the low- to the high-frequency regions. For each plot, a low-, a high-, and a mid-frequency region has been defined. In addition, the primary reasons for the drop in gain at low and high frequencies have also been indicated within the parentheses. For the *RC*-coupled amplifier, the drop at low frequencies is due to the increasing reactance of C_C, C_s, or C_E, whereas its upper frequency limit is determined by either the parasitic capacitive elements of the network or the frequency dependence of the gain of the active device. An explanation of the drop in gain for the transformer-coupled system requires a basic understanding of "transformer action" and the transformer equivalent circuit. For the moment, let us say that it is simply due to the "shorting effect" (across the input terminals of the transformer) of the magnetizing inductive reactance at low frequencies ($X_L = 2\pi fL$). The gain must obviously be zero at $f = 0$ because at this point there is no longer a changing flux established through the core to induce a secondary or output voltage. As indicated in Fig. 9.8, the high-frequency response is controlled primarily by the stray capacitance between the turns of the primary and secondary windings. For the direct-coupled amplifier, there are no coupling or bypass capacitors to cause a drop in gain at low frequencies. As the figure indicates, it is a flat response to the upper cutoff

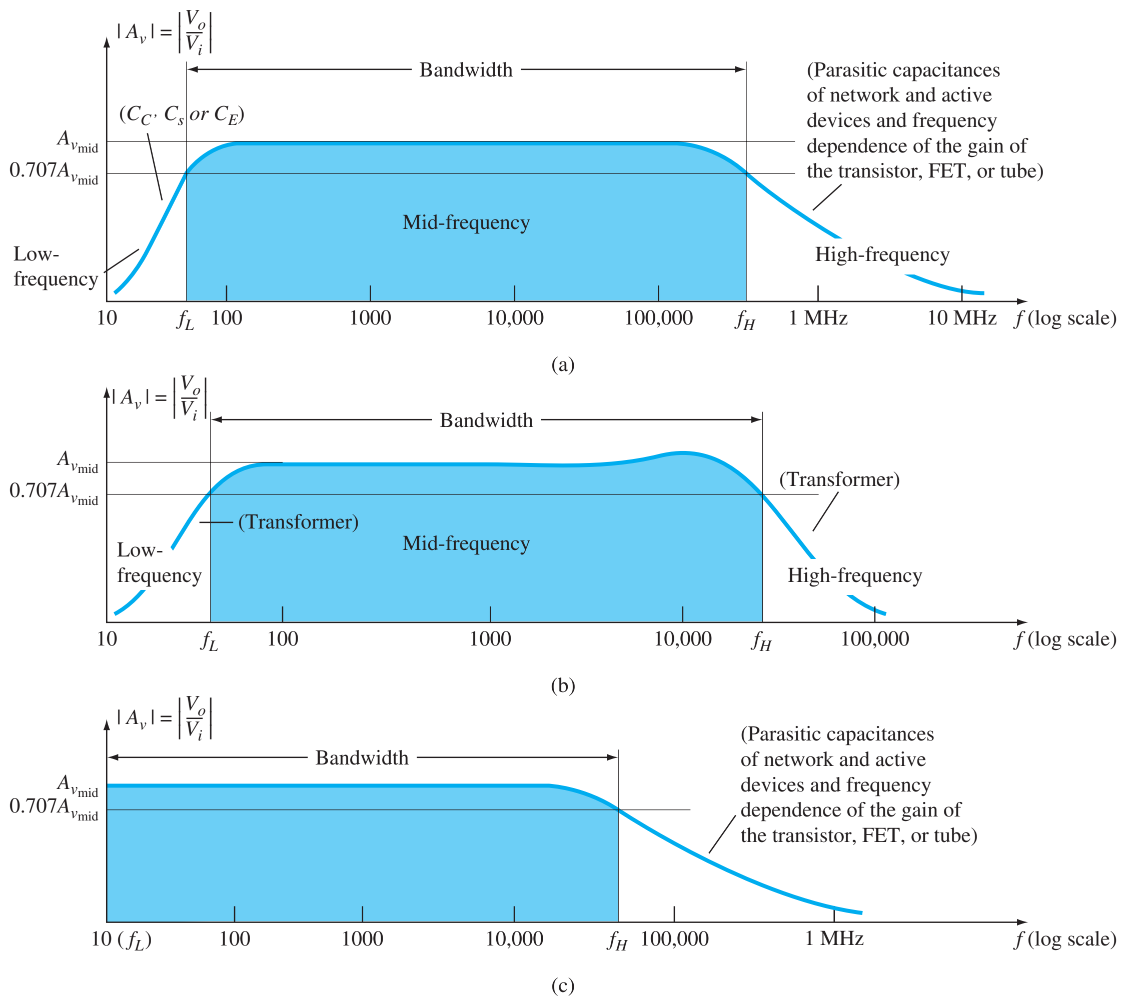

FIG. 9.8

Gain versus frequency: (a) RC-coupled amplifiers; (b) transformer-coupled amplifiers; (c) direct-coupled amplifiers.

frequency, which is determined by either the parasitic capacitances of the circuit or the frequency dependence of the gain of the active device.

For each system of Fig. 9.8, there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the midband value. To fix the frequency boundaries of relatively high gain, $0.707A_{v_{mid}}$ was chosen to be the gain at the cutoff levels. The corresponding frequencies f_1 and f_2 are generally called the *corner, cutoff, band, break,* or *half-power frequencies.* The multiplier 0.707 was chosen because at this level the output power is half the midband power output, that is, at midfrequencies,

$$P_{o_{mid}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

and at the half-power frequencies,

$$P_{o_{HPF}} = \frac{|0.707\, A_{v_{mid}} V_i|^2}{R_o} = 0.5\frac{|A_{v_{mid}} V_i|^2}{R_o}$$

and

$$P_{o_{HPF}} = 0.5\, P_{o_{mid}} \quad \textbf{(9.18)}$$

The bandwidth (or passband) of each system is determined by f_H and f_L, that is,

$$\text{bandwidth (BW)} = f_H - f_L \quad \textbf{(9.19)}$$

with f_H and f_L defined in each curve of Fig. 9.8.

9.5 NORMALIZATION PROCESS

For applications of a communication nature (audio, video) a decibel plot versus frequency is normally provided rather than the gain versus frequency plot of Fig. 9.8. In other words, when you pick up a specification sheet on a particular amplifier or system, the plot will typically be of dB versus frequency rather than gain versus frequency.

To obtain such a dB plot the curve is first *normalized*—a process whereby the vertical parameter is divided by a specific level or quantity sensitive to a combination or variables of the system. For this area of investigation, it is usually the midband or maximum gain for the frequency range of interest.

For example, in Fig. 9.9 the curve of Fig. 9.8a is normalized by dividing the output voltage gain at each frequency by the midband level. Note that the curve has the same shape, but the band frequencies are now defined by simply the 0.707 level and not linked to the actual midband level. It clearly reveals that

The band frequencies define a level where the gain or quantity of interest will be 70.7% or its maximum value.

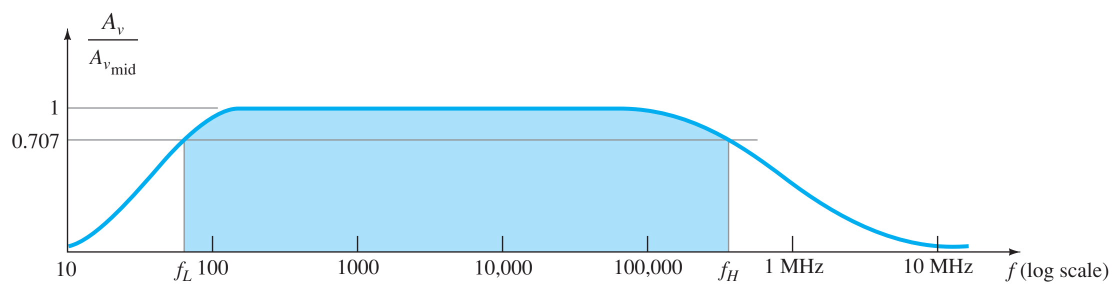

FIG. 9.9
Normalized gain versus frequency plot.

Consider also that the plot of Fig. 9.9 is not sensitive to the actual level of the midband gain. The midband gain could be 50, 100, or even 200, and the resulting plot of Fig. 9.9 would be the same. The plot of Fig. 9.9 is now defining frequencies where the relative gain is defined rather than concerning itself with the "actual gain."

The next example will demonstrate the normalization process for a typical amplifier response.

EXAMPLE 9.9 Given the frequency response of Fig. 9.10:

a. Find the cutoff frequencies f_L and f_H using the measurements provided.
b. Find the bandwidth of the response.
c. Sketch the normalized response.

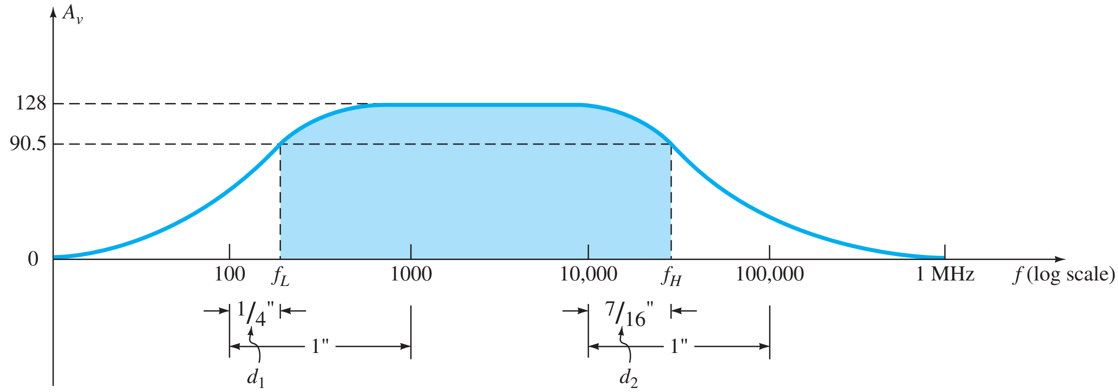

FIG. 9.10
Gain plot for Example 9.8.

Solution:

a. For f_L: $\dfrac{d_1}{d_2} = \dfrac{1/4''}{1''} = 0.25$

$10^{d_1/d_2} = 10^{0.25} = 1.7783$

$\text{Value} = 10^x \times 10^{d_1/d_2} = 10^2 \times 1.7783 = \mathbf{177.83\ Hz}$

For f_H: $\dfrac{d_1}{d_2} = \dfrac{7/16''}{1''} = 0.438$

$10^{d_1/d_2} = 10^{0.438} = 2.7416$

$\text{Value} = 10^x \times 10^{d_1/d_2} = 10^4 \times 2.7416 = \mathbf{27{,}416\ Hz}$

b. The bandwidth:

$\text{BW} = f_H - f_L = 27{,}416\text{ Hz} - 177.83\text{ Hz} \cong \mathbf{27.24\ KHz}$

c. The normalized response is determined by simply dividing each level of Fig. 9.10 by the midband level of 128, as shown in Fig. 9.11. The result is a maximum value of 1 and cutoff levels of 0.707.

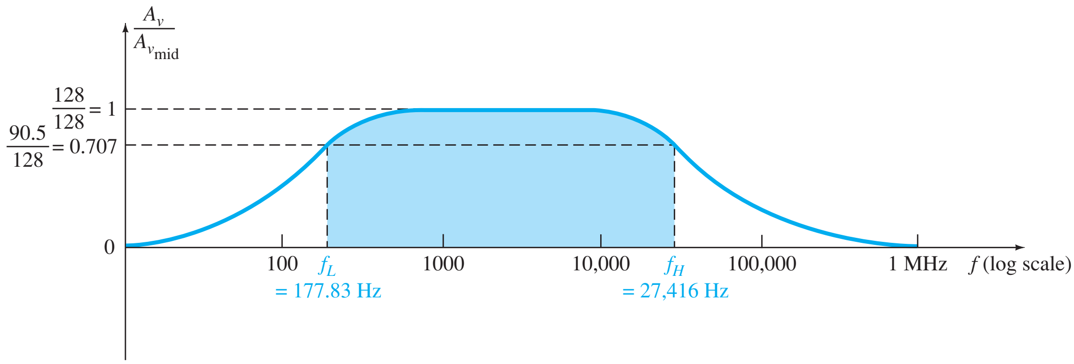

FIG. 9.11
Narmalized plot of Fig. 9.10.

A decibel plot of Fig. 9.11 can be obtained by applying Eq. (9.13) in the following manner:

$$\left.\frac{A_v}{A_{v_{mid}}}\right|_{dB} = 20 \log_{10} \frac{A_v}{A_{v_{mid}}} \tag{9.20}$$

At midband frequencies, $20 \log_{10} 1 = 0$, and at the cutoff frequencies, $20 \log_{10} 1/\sqrt{2} = -3$ dB. Both values are clearly indicated in the resulting decibel plot of Fig. 9.12. The smaller the fraction ratio, the more negative is the decibel level.

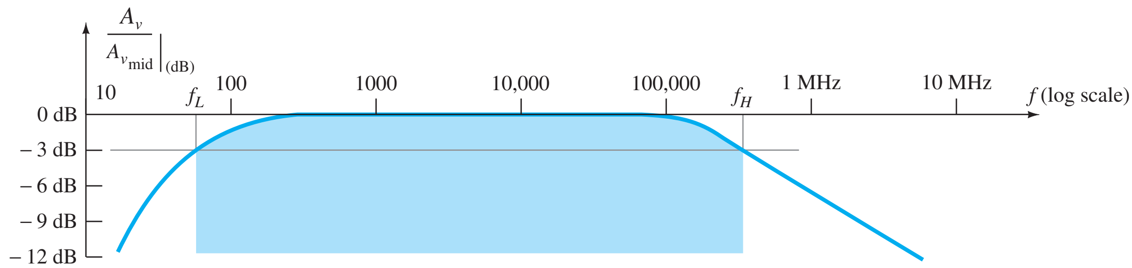

FIG. 9.12
Decibel plot of the normalized gain versus frequency plot of Fig. 9.9.

For the greater part of the discussion to follow, a decibel plot will be made only for the low- and high-frequency regions. Keep Fig. 9.12 in mind, therefore, to permit a visualization of the broad system response.

Most amplifiers introduce a 180° phase shift between input and output signals. This fact must now be expanded to indicate that this is the case only in the midband region. At low frequencies, there is a phase shift such that V_o lags V_i by an increased angle. At high frequencies, the phase shift drops below 180°. Figure 9.13 is a standard phase plot for an *RC*-coupled amplifier.

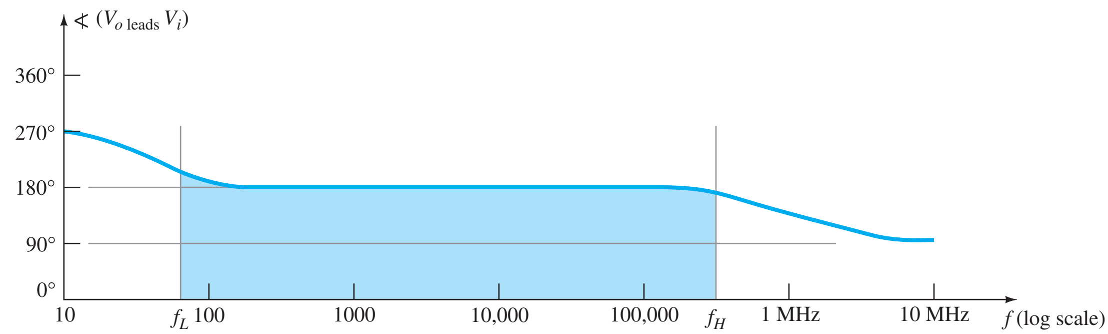

FIG. 9.13
Phase plot for an RC-coupled amplifier system.

9.6 LOW-FREQUENCY ANALYSIS—BODE PLOT

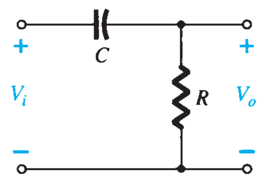

FIG. 9.14
RC combination that will define a low-cutoff frequency.

In the low-frequency region of the single-stage BJT or FET amplifier, it is the *RC* combinations formed by the network capacitors C_C, C_E, and C_s and the network resistive parameters that determine the cutoff frequencies. In fact, an *RC* network similar to Fig. 9.14 can be established for each capacitive element, and the frequency at which the output voltage drops to 0.707 of its maximum value can be determined. Once the cutoff frequencies due to each capacitor are determined, they can be compared to establish which will determine the low-cutoff frequency for the system.

Consider, for example, the voltage-divider BJT network of Fig. 9.15 that was analyzed in detail in Section 5.6. The analysis of that section resulted in an input impedance of

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$

and an equivalent circuit at the input as shown in Fig. 9.16.

For the mid-frequency range the capacitor C_s is assumed to be an equivalent short-circuit state, and $V_b = V_i$. The result is a high midband gain for the amplifier that is not affected by the coupling or bypass capacitors. However, as we lower the applied frequency the reactance of the capacitor will increase and take an increasing share of the applied voltage V_i. Neglecting the effects of the coupling capacitor C_C and bypass capacitor C_E for the moment, if the voltage V_b should decrease, it will result in the same decrease in overall gain V_o/V_i.

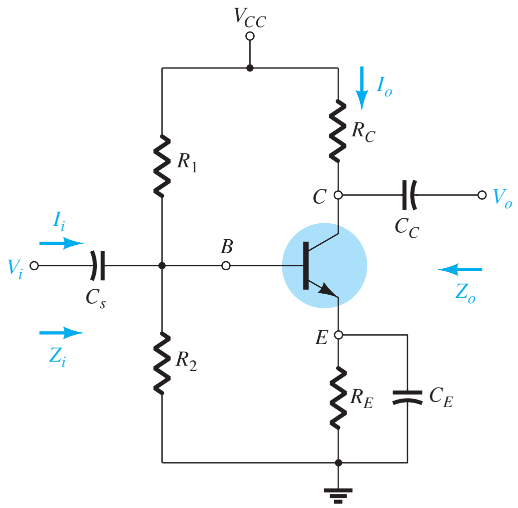

FIG. 9.15
Voltage-divider bias configuration.

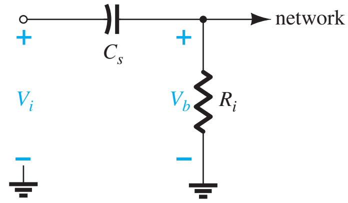

FIG. 9.16
Equivalent input circuit for the network of Fig. 9.15.

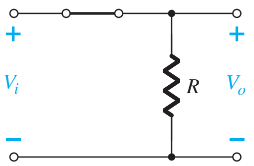

FIG. 9.17
RC circuit of Fig. 9.14 at very high frequencies.

Less of the applied voltage is reaching the base of the transistor reducing the output voltage V_o. In fact if the V_b should drop to 0.707 of the peak possible value of V_i the overall gain will drop the same amount. In total, therefore, if we find the frequency that will result in V_b being only 0.707 V_i, we will have the low-cutoff frequency for the full amplifier response.

Finding this frequency will now be examined by analyzing the generic *RC* network of Fig. 9.14 introduced above. Once the results are obtained it can be applied to any *RC* combination that may develop due to the other coupling capacitors or bypass capacitors. At high frequencies, the reactance of the capacitor of Fig. 9.14 is

$$X_C = \frac{1}{2\pi f C} \cong 0\ \Omega$$

and the short-circuit equivalent can be substituted for the capacitor as shown in Fig. 9.17. The result is that $V_o \cong V_i$ at high frequencies. At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(0)C} = \infty\ \Omega$$

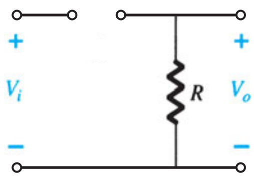

FIG. 9.18
RC circuit of Fig. 9.14 at f = 0 Hz.

and the open-circuit approximation can be applied as shown in Fig. 9.18, with the result that $V_o = 0$ V.

Between the two extremes, the ratio $A_v = V_o/V_i$ will vary as shown in Fig. 9.19. As the frequency increases, the capacitive reactance decreases, and more of the input voltage appears across the output terminals.

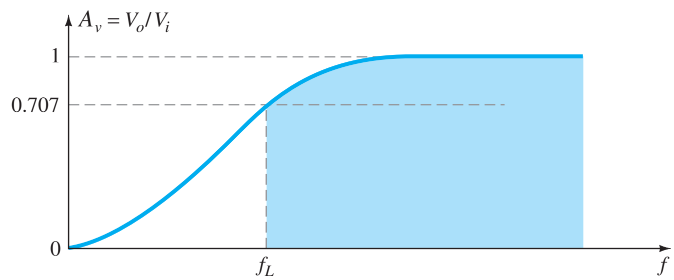

FIG. 9.19
Low-frequency response for the RC circuit of Fig. 9.14.

The output and input voltages are related by the voltage-divider rule in the following manner:

$$\mathbf{V_o = \frac{RV_i}{R + X_C}}$$

where the boldface roman characters represent magnitude and angle of each quantity.

The magnitude of V_o is determined as follows:

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

For the special case where $X_C = R$,

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}}V_i$$

and

$$|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707|_{X_C=R} \tag{9.21}$$

the level of which is indicated on Fig. 9.19. In other words, at the frequency for which $X_C = R$, the output will be 70.7% of the input for the network of Fig. 9.14.

The frequency at which this occurs is determined from

$$X_C = \frac{1}{2\pi f_L C} = R$$

and

$$f_L = \frac{1}{2\pi RC} \tag{9.22}$$

In terms of logs,

$$G_v = 20 \log_{10} A_v = 20 \log_{10} \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

whereas at $A_v = V_o/V_i = 1$ or $V_o = V_i$ (the maximum value),

$$G_v = 20 \log_{10} 1 = 20(0) = 0 \text{ dB}$$

In Fig. 9.8, we recognize that there is a 3-dB drop in gain from the midband level when $f = f_L$. In a moment, we will find that an RC network will determine the low-frequency cutoff for a BJT transistor and f_L will be determined by Eq. (9.22).

If the gain equation is written as

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

we obtain, using the frequency defined above,

$$A_v = \frac{1}{1 - j(f_L/f)} \tag{9.23}$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_L/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_L/f)}_{\text{phase} \angle \text{ by which } V_o \text{ leads } V_i} \tag{9.24}$$

For the magnitude when $f = f_L$,

$$|A_v| = \frac{1}{\sqrt{1 + (1)^2}} = \frac{1}{\sqrt{2}} = 0.707 \Rightarrow -3 \text{ dB}$$

In the logarithmic form, the gain in dB is

$$A_{v(\text{dB})} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_L/f)^2}} \tag{9.25}$$

Expanding Eq. (9.25):

$$A_{v(\text{dB})} = -20 \log_{10}\left[1 + \left(\frac{f_L}{f}\right)^2\right]^{1/2}$$
$$= -\left(\tfrac{1}{2}\right)(20) \log_{10}\left[1 + \left(\frac{f_L}{f}\right)^2\right]$$
$$= -10 \log_{10}\left[1 + \left(\frac{f_L}{f}\right)^2\right]$$

For frequencies where $f \ll f_L$ or $(f_L/f)^2 \gg 1$, the equation above can be approximated by

$$A_{v(\text{dB})} = -10 \log_{10}\left(\frac{f_L}{f}\right)^2$$

and finally,

$$A_{v(\text{dB})} = -20 \log_{10}\frac{f_L}{f}\Bigg|_{f \ll f_L} \qquad \textbf{(9.26)}$$

Ignoring the condition $f \ll f_L$ for a moment, we find that a plot of Eq. (9.26) on a frequency log scale yields a result very useful for future decibel plots.

$$\text{At } f = f_L\text{:} \quad \frac{f_L}{f} = 1 \text{ and } -20 \log_{10} 1 = 0 \text{ dB}$$
$$\text{At } f = \tfrac{1}{2}f_L\text{:} \quad \frac{f_L}{f} = 2 \text{ and } -20 \log_{10} 2 \cong -6 \text{ dB}$$
$$\text{At } f = \tfrac{1}{4}f_L\text{:} \quad \frac{f_L}{f} = 4 \text{ and } -20 \log_{10} 4 \cong -12 \text{ dB}$$
$$\text{At } f = \tfrac{1}{10}f_L\text{:} \quad \frac{f_L}{f} = 10 \text{ and } -20 \log_{10} 10 = -20 \text{ dB}$$

A plot of these points is indicated in Fig. 9.20 from $0.1f_L$ to f_L with a dark blue straight line. In the same figure, a straight line is also drawn for the condition of 0 dB for $f \gg f_L$. As stated earlier, the straight-line segments (asymptotes) are only accurate for 0 dB when $f \gg f_L$ and the sloped line when $f_L \gg f$. We know, however, that when $f = f_L$, there is a 3-dB drop from the midband level. Employing this information in association with the straight-line segments permits a fairly accurate plot of the frequency response as indicated in the same figure.

The piecewise linear plot of the asymptotes and associated breakpoints is called a Bode plot of the magnitude versus frequency.

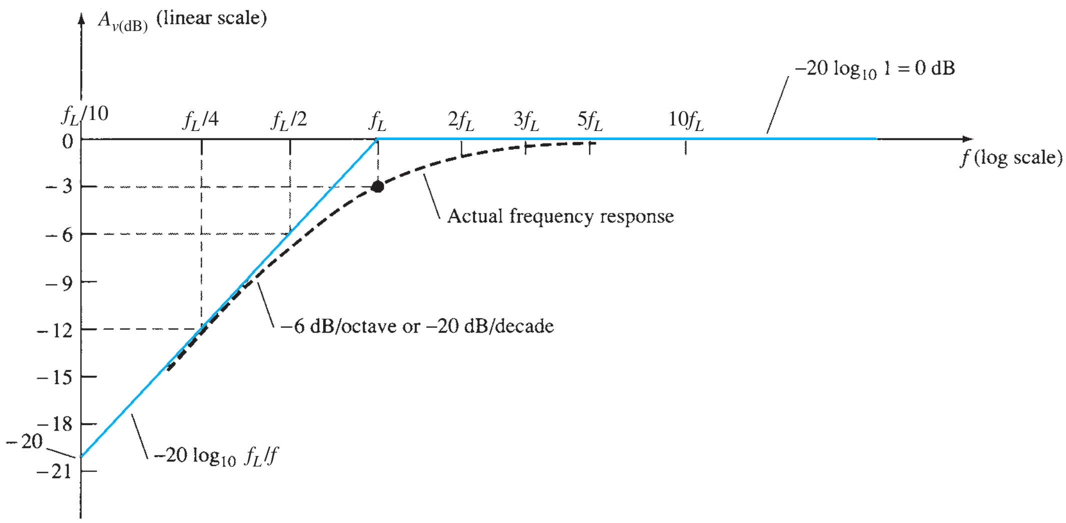

FIG. 9.20
Bode plot for the low-frequency region.

The approach was developed by Professor Hendrik Bode in the 1940s (Fig. 9.21).

The calculations above and the curve itself demonstrate clearly that:

A change in frequency by a factor of two, equivalent to one octave, results in a 6-dB change in the ratio, as shown by the change in gain from $f_L/2$ to f_L.

As noted by the change in gain from $f_L/2$ to f_L:

For a 10:1 change in frequency, equivalent to one decade, there is a 20-dB change in the ratio, as demonstrated between the frequencies of $f_L/10$ and f_L.

Therefore, a decibel plot can easily be obtained for a function having the format of Eq. (9.26). First, simply find f_L from the circuit parameters and then sketch two asymptotes—one along the 0-dB line and the other drawn through f_L sloped at 6 dB/octave or 20 dB/decade. Then, find the 3-dB point corresponding to f_L and sketch the curve.

The gain at any frequency can be determined from the frequency plot in the following manner:

$$A_{v(\text{dB})} = 20 \log_{10} \frac{V_o}{V_i}$$

but

$$\frac{A_{v(\text{dB})}}{20} = \log_{10} \frac{V_o}{V_i}$$

and

$$A_v = \frac{V_o}{V_i} = 10^{A_{v(\text{dB})}/20} \tag{9.27}$$

For example, if $A_{v(\text{dB})} = -3$ dB,

$$A_v = \frac{V_o}{V_i} = 10^{(-3/20)} = 10^{(-0.15)} \cong 0.707 \qquad \text{as expected}$$

The quantity $10^{-0.15}$ is determined using the 10^x function found on most scientific calculators.

The phase angle of θ is determined from

$$\theta = \tan^{-1}\frac{f_L}{f} \tag{9.28}$$

from Eq. (9.24).

For frequencies $f \ll f_L$,

$$\theta = \tan^{-1}\frac{f_L}{f} \rightarrow 90°$$

For instance, if $f_L = 100f$,

$$\theta = \tan^{-1}\frac{f_L}{f} = \tan^{-1}(100) = 89.4°$$

For $f = f_L$,

$$\theta = \tan^{-1}\frac{f_L}{f} = \tan^{-1}1 = 45°$$

For $f \gg f_L$,

$$\theta = \tan^{-1}\frac{f_L}{f} \rightarrow 0°$$

For instance, if $f = 100f_L$,

$$\theta = \tan^{-1}\frac{f_L}{f} = \tan^{-1}0.01 = 0.573°$$

A plot of $\theta = \tan^{-1}(f_L/f)$ is provided in Fig. 9.22. If we add the additional 180° phase shift introduced by an amplifier, the phase plot of Fig. 9.13 is obtained. The magnitude and phase response for an *RC* combination have now been established. In Section 9.7, each capacitor of importance in the low-frequency region will be redrawn in an *RC* format and the cutoff frequency for each determined to establish the low-frequency response for the BJT amplifier.

American (Madison, WI; Summit, NJ; Cambridge, MA)
(1905–1981)
V.P. at Bell Laboratories
Professor of Systems Engineering, Harvard University

In his early years at Bell Laboratories, Hendrik Bode was involved with *electric filter* and *equalizer design*. He then transferred to the Mathematics Research Group, where he specialized in research pertaining to electrical networks theory and its application to long distance communication facilities. In 1948 he was awarded the Presidential Certificate of Merit for his work in electronic fire control devices. In addition to the publication of the book *Network Analysis and Feedback Amplifier Design* in 1945, which is considered a classic in its field, he has been granted 25 patents in electrical engineering and systems design. Upon retirement, Bode was elected Gordon McKay Professor of Systems Engineering at Harvard University. He was a fellow of the IEEE and American Academy of Arts and Sciences.

FIG. 9.21
Hendrik Wade Bode.
(Courtesy of AT&T Archives and History Center.)

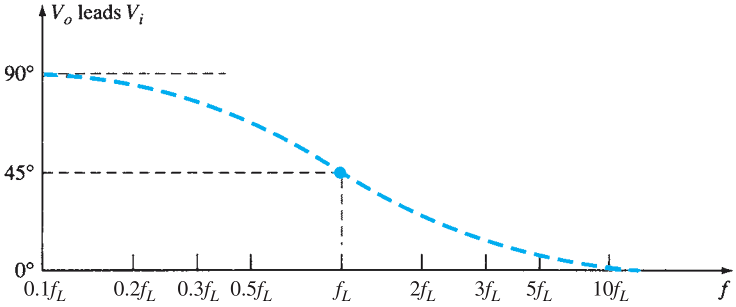

FIG. 9.22
Phase response for the RC circuit of Fig. 9.14.

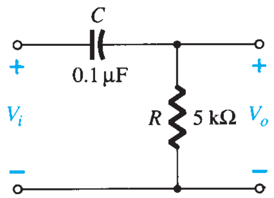

FIG. 9.23
Example 9.10.

EXAMPLE 9.10 For the network of Fig. 9.23:

a. Determine the break frequency.
b. Sketch the asymptotes and locate the −3-dB point.
c. Sketch the frequency response curve.
d. Find the gain at $A_{v(\text{dB})} = -6$ dB.

Solution:

a. $$f_L = \frac{1}{2\pi RC} = \frac{1}{(6.28)(5 \times 10^3\ \Omega)(0.1 \times 10^{-6}\ \text{F})}$$
$$\cong \mathbf{318.5\ Hz}$$

b. and c. See Fig. 9.24.

d. Eq. (9.27): $$A_v = \frac{V_o}{V_i} = 10^{A_{v(\text{dB})}/20}$$
$$= 10^{(-6/20)} = 10^{-0.3} = 0.501$$

and $V_o = 0.501\ V_i$ or approximately 50% of V_i.

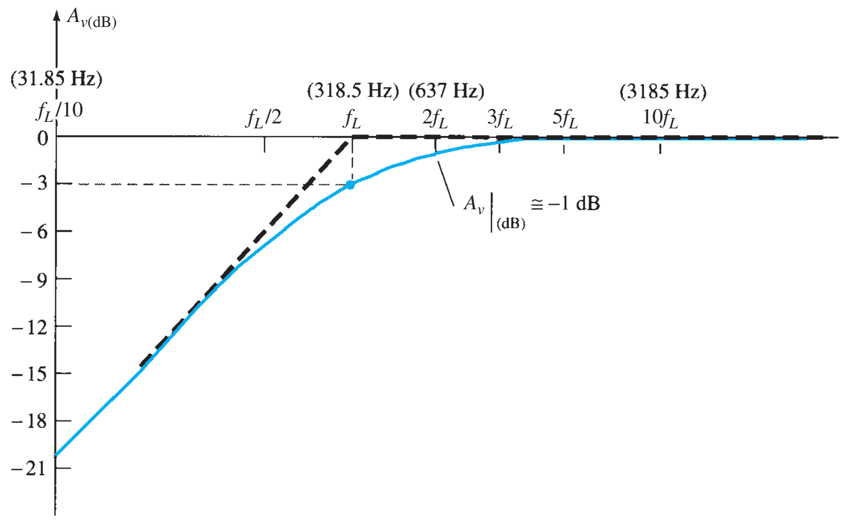

FIG. 9.24
Frequency response for the RC circuit of Fig. 9.23.

9.7 LOW-FREQUENCY RESPONSE—BJT AMPLIFIER WITH R_L

The analysis of this section will employ the loaded (R_L) voltage-divider BJT bias configuration introduced earlier in Section 9.6. For the network of Fig. 9.25, the capacitors C_s, C_C, and C_E will determine the low-frequency response. We will now examine the impact of each independently in the order listed.

C_s Because C_s is normally connected between the applied source and the active device, the general form of the RC configuration is established by the network of Fig. 9.26, matching that of Fig. 9.16 with $R_i = R_1 \| R_2 \| \beta r_e$.

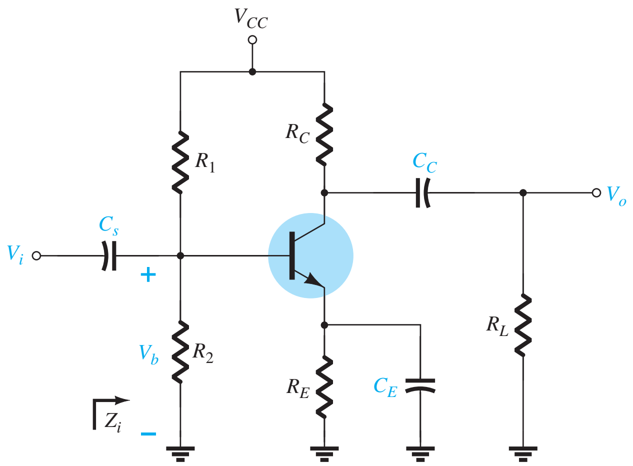

FIG. 9.25
Loaded BJT amplifier with capacitors that affect the low-frequency response.

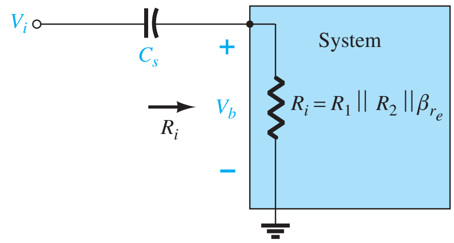

FIG. 9.26
Determining the effect of C_s on the low-frequency response.

Applying the voltage-divider rule:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_i}{R_i - jX_{C_s}} \tag{9.29}$$

The cutoff frequency defined by C_s can be determined by manipulating the above equation into a standard form or simply using the results of Section 9.6. As a verification of the results of Section 9.6 the manipulation process is defined in detail below. For future RC networks, the results of Section 9.6 will simply be applied.

Rewriting Eq. (9.29):

$$\frac{\mathbf{V}_b}{\mathbf{V}_i} = \frac{R_i}{R_i - jX_{C_s}} = \frac{1}{1 - j\dfrac{X_{C_s}}{R_i}}$$

The factor

$$\frac{X_{C_s}}{R_i} = \left(\frac{1}{2\pi f C_s}\right)\left(\frac{1}{R_i}\right) = \frac{1}{2\pi f R_i C_s}$$

Defining

$$f_{L_s} = \frac{1}{2\pi R_i C_s} \tag{9.30}$$

we have

$$\mathbf{A}_v = \frac{\mathbf{V}_b}{\mathbf{V}_i} = \frac{1}{1 - j(f_{L_s}/f)} \tag{9.31}$$

At f_{L_s} the voltage V_b will be 70.7% of the mid band value assuming C_s is the only capacitive element controlling the low-frequency response.

For the network of Fig. 9.25, when we analyze the effects of C_s we must make the assumption that C_E and C_C are performing their designed function or the analysis becomes too unwieldy, that is, that the magnitudes of the reactances of C_E and C_C permit employing a short-circuit equivalent in comparison to the magnitude of the other series impedances.

C_C Because the coupling capacitor is normally connected between the output of the active device and the applied load, the *RC* configuration that determines the low-cutoff frequency due to C_C appears in Fig. 9.27. The total series resistance is now $R_o + R_L$, and the cutoff frequency due to C_C is determined by

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \qquad \textbf{(9.32)}$$

Ignoring the effects of C_s and C_E, we find that the output voltage V_o will be 70.7% of its midband value at f_{L_C}. For the network of Fig. 9.25, the ac equivalent network for the output section with $V_i = 0$ V appears in Fig. 9.28. The resulting value for R_o in Eq. (9.32) is then simply

$$R_o = R_C \| r_o \qquad \textbf{(9.33)}$$

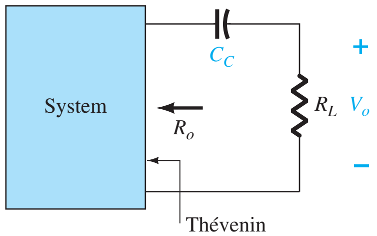

FIG. 9.27
Determining the effect of C_C on the low-frequency response.

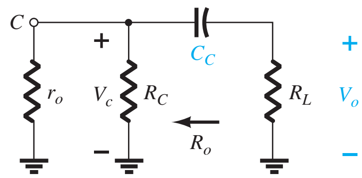

FIG. 9.28
Localized ac equivalent for C_C with $V_i = 0$ V.

C_E To determine f_{L_E}, the network "seen" by C_E must be determined as shown in Fig. 9.29. Once the level of R_e is established, the cutoff frequency due to C_E can be determined using the following equation:

$$f_{L_E} = \frac{1}{2\pi R_e C_E} \qquad \textbf{(9.34)}$$

For the network of Fig. 9.25, the ac equivalent as "seen" by C_E appears in Fig. 9.30 as derived from Fig. 5.38. The value of R_e is therefore determined by

$$R_e = R_E \left\| \left(\frac{R_1 \| R_2}{\beta} + r_e \right) \right. \qquad \textbf{(9.35)}$$

The effect of C_E on the gain is best described in a quantitative manner by recalling that the gain for the configuration of Fig. 9.31 is given by

$$A_v = \frac{-R_C}{r_e + R_E}$$

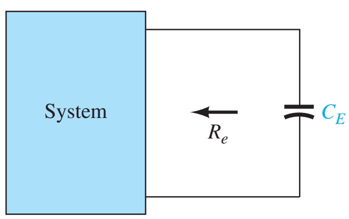

FIG. 9.29
Determining the effect of C_E on the low-frequency response.

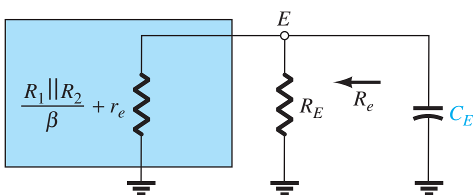

FIG. 9.30
Localized ac equivalent of C_E.

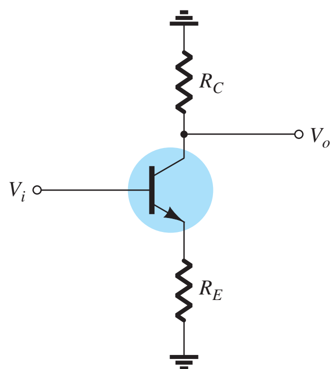

FIG. 9.31
Network employed to describe the effect of C_E on the amplifier gain.

The maximum gain is obviously available where R_E is 0 Ω. At low frequencies, with the bypass capacitor C_E in its "open-circuit" equivalent state, all of R_E appears in the gain equation above, resulting in the minimum gain. As the frequency increases, the reactance of the capacitor C_E will decrease, reducing the parallel impedance of R_E and C_E until the resistor R_E is effectively "shorted out" by C_E. The result is a maximum or midband gain determined by $A_v = -R_C/r_e$. At f_{L_E} the gain will be 3 dB below the midband value determined with R_E "shorted out."

Before continuing, keep in mind that C_s, C_C, and C_E will affect only the low-frequency response. At the midband frequency level, the short-circuit equivalents for the capacitors can be inserted. Although each will affect the gain $A_v = V_o/V_i$ in a similar frequency range, the highest low-frequency cutoff determined by C_s, C_C, or C_E will have the greatest impact because it will be the last encountered before the midband level. If the frequencies are relatively far apart, the highest cutoff frequency will essentially determine the lower cutoff frequency for the entire system. If there are two or more "high" cutoff frequencies, the effect will be to raise the lower cutoff frequency and reduce the resulting bandwidth of the system. In other words, there is an interaction between capacitive elements that can affect the resulting low-cutoff frequency. However, if the cutoff frequencies established by each capacitor are sufficiently separated, the effect of one on the other can be ignored with a high degree of accuracy—a fact that will be demonstrated by the printouts to appear in the following example.

EXAMPLE 9.11 Determine the cutoff frequencies for the network of Fig. 9.25 using the following parameters:

$C_s = 10\ \mu\text{F}, \quad C_E = 20\ \mu\text{F}, \quad C_C = 1\ \mu\text{F}$

$R_1 = 40\ \text{k}\Omega, \quad R_2 = 10\ \text{k}\Omega, \quad R_E = 2\ \text{k}\Omega, \quad R_C = 4\ \text{k}\Omega,$

$R_L = 2.2\ \text{k}\Omega$

$\beta = 100, \quad r_o = \infty\ \Omega, \quad V_{CC} = 20\ \text{V}$

Solution: To determine r_e for dc conditions, we first apply the test equation:

$$\beta R_E = (100)(2\ \text{k}\Omega) = 200\ \text{k}\Omega \gg 10R_2 = 100\ \text{k}\Omega$$

Since satisfied the dc base voltage is determined by

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\ \text{k}\Omega(20\ \text{V})}{10\ \text{k}\Omega + 40\ \text{k}\Omega} = \frac{200\ \text{V}}{50} = 4\ \text{V}$$

with
$$I_E = \frac{V_E}{R_E} = \frac{4\ \text{V} - 0.7\ \text{V}}{2\ \text{k}\Omega} = \frac{3.3\ \text{V}}{2\ \text{k}\Omega} = 1.65\ \text{mA}$$

so that
$$r_e = \frac{26\ \text{mV}}{1.65\ \text{mA}} \cong \mathbf{15.76\ \Omega}$$

and
$$\beta r_e = 100(15.76\ \Omega) = 1576\ \Omega = \mathbf{1.576\ k\Omega}$$

Midband Gain
$$A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4\ \text{k}\Omega)\|(2.2\ \text{k}\Omega)}{15.76\ \Omega} \cong -90$$

C_s
$$R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$

$$f_{L_s} = \frac{1}{2\pi R_i C_s} = \frac{1}{(6.28)(1.32\ \text{k}\Omega)(10\ \mu\text{F})}$$

$$f_{L_s} \cong \mathbf{12.06\ Hz}$$

C_C
$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \quad \text{with} \quad R_o = R_C \| r_o \cong R_C$$

$$= \frac{1}{(6.28)(4\ \text{k}\Omega + 2.2\ \text{k}\Omega)(1\ \mu\text{F})}$$

$$\cong \mathbf{25.68\ Hz}$$

C_E

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e\right)$$
$$= 2\text{ k}\Omega \| \left(\frac{40\text{ k}\Omega \| 10\text{ k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\text{ k}\Omega \| \left(\frac{8\text{ k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\text{ k}\Omega \| (80\ \Omega + 15.76\ \Omega)$$
$$= 2\text{ k}\Omega \| 95.76\ \Omega$$
$$= 91.38\ \Omega$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(91.38\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{11{,}477.73} \cong \mathbf{87.13\ Hz}$$

Since $f_{L_E} \gg f_{L_C}$ or f_{L_S} the bypass capacitor C_E is determining the lower cutoff frequency of the amplifier.

9.8 IMPACT OF R_S ON THE BJT LOW-FREQUENCY RESPONSE

In this section we will investigate the impact of the source resistance on the various cutoff frequencies. In Fig. 9.32 a signal source and associated resistance have been added to the configuration of Fig. 9.25. The gain will now be between the output voltage V_o and the signal source V_s.

C_s The equivalent circuit at the input is now as shown in Fig. 9.33, with R_i continuing to be equal to $R_1 \| R_2 \| \beta r_e$.

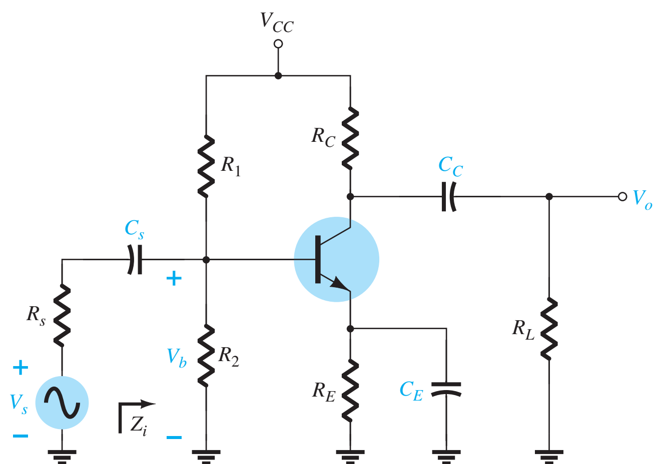

FIG. 9.32
Determining the effect of R_s on the low-frequency response of a BJT amplifier.

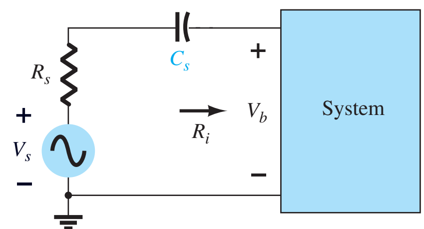

FIG. 9.33
Determining the effect of C_s on the low-frequency response.

Using the results of the last section it would appear we could simply find the total sum of the series resistors and plug it into Eq. (9.22). Doing so would result in the following equation for the cutoff frequency:

$$f_{L_s} = \frac{1}{2\pi(R_i + R_s)C_s} \tag{9.36}$$

However, it would be best to validate our assumption by first applying the voltage-divider rule in the following manner:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_s}{R_s + R_i - jX_{C_s}} \tag{9.37}$$

The cutoff frequency defined by C_s can be determined by manipulating the above equation into a standard form, as demonstrated below.

Rewriting Eq. (9.37):

$$\frac{\mathbf{V}_b}{\mathbf{V}_s} = \frac{R_i}{R_s + R_i - jX_{C_s}} = \frac{1}{1 + \frac{R_s}{R_i} - j\frac{X_{C_s}}{R_i}}$$

$$= \frac{1}{\left(1 + \frac{R_s}{R_i}\right)\left[1 - j\frac{X_{C_s}}{R_i}\left(\frac{1}{1 + \frac{R_s}{R_i}}\right)\right]} = \frac{1}{\left(1 + \frac{R_s}{R_i}\right)\left(1 - j\frac{X_{C_s}}{R_i + R_s}\right)}$$

The factor

$$\frac{X_{C_s}}{R_i + R_s} = \left(\frac{1}{2\pi f C_s}\right)\left(\frac{1}{R_i + R_s}\right) = \frac{1}{2\pi f(R_i + R_s)C_s}$$

Defining

$$f_{L_s} = \frac{1}{2\pi(R_i + R_s)C_s}$$

we have

$$\frac{\mathbf{V}_b}{\mathbf{V}_s} = \frac{1}{\left(\frac{1}{1 + \frac{R_s}{R_i}}\right)\left(1 - \frac{1}{1 - jf_{L_s/f}}\right)}$$

and finally

$$\mathbf{A}_v = \frac{\mathbf{V}_b}{\mathbf{V}_s} = \left[\frac{R_i}{R_i + R_s}\right]\left[\frac{1}{1 - j(f_{L_s}/f)}\right]$$

For the midband frequencies, the input network will appear as shown in Fig. 9.34.

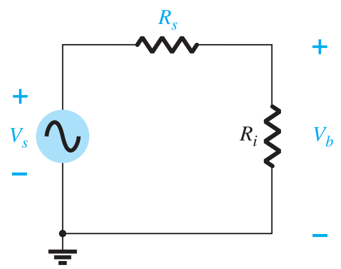

FIG. 9.34
Determining the effect of R_s on the gain A_{v_s}.

so that

$$A_{v_{\text{mid}}} = \frac{\mathbf{V}_b}{\mathbf{V}_s} = \frac{R_i}{R_i + R_s} \tag{9.38}$$

and

$$\frac{A_v}{A_{v_{\text{mid}}}} = \frac{1}{1 - j(f_{L_s}/f)}$$

Noting the similarities with Eq. (9.23) the cutoff frequency is defined by f_{L_s} above and

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s} \tag{9.39}$$

as assumed in the derivation of Eq. (9.36).

At f_{L_s}, the voltage V_o will be 70.7% of the midband value determined by Eq. (9.38), assuming the C_s is the only capacitive element controlling the low-frequency response.

C_C Reviewing the analysis of Section 9.7 for the coupling capacitor C_C, we find that the derivation of the equation for the cutoff frequency remains the same. That is,

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \tag{9.40}$$

C_E Again, following the analysis of Section 9.7 for the same capacitor, we find that R_s will affect the resistance level substituted into the cutoff equation so that

$$f_{L_E} = \frac{1}{2\pi R_e C_E} \tag{9.41}$$

with $$R_e = R_E \| \left(\frac{R'_s}{\beta} + r_e\right) \text{ and } R'_s = R_s \| R_1 \| R_2$$

In total, therefore, the introduction of the resistance R_s reduced the cutoff frequency defined by C_s and raised the cutoff frequency defined by C_E. The cutoff frequency defined by C_C remained the same. It is also important to note that the gain can be severely affected by the loss in signal voltage across the source resistance. This last factor will be demonstrated in the next example.

EXAMPLE 9.12

a. Repeat the analysis of Example 9.11 but with a source resistance R_s of 1 kΩ. The gain of interest will now be V_o/V_s rather than V_o/V_i. Compare results.
b. Sketch the frequency response using a Bode plot.
c. Verify the results using PSpice.

Solution: a. The dc conditions remain the same:

$$r_e = 15.76\ \Omega \text{ and } \beta r_e = 1.576\ \text{k}\Omega$$

Midband Gain $$A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} \cong -90 \text{ as before}$$

The input impedance is given by

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$
$$= 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega$$
$$\cong 1.32\ \text{k}\Omega$$

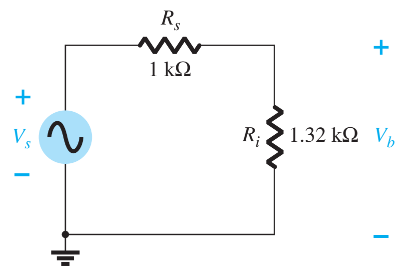

FIG. 9.35
Determining the effect of R_s on the gain A_{v_s}.

and from Fig. 9.35,

$$V_b = \frac{R_i V_s}{R_i + R_s}$$

or $$\frac{V_b}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\ \text{k}\Omega}{1.32\ \text{k}\Omega + 1\ \text{k}\Omega} = 0.569$$

so that $$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_b}{V_s} = (-90)(0.569)$$
$$= \mathbf{-51.21}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$
$$f_{L_S} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1\ \text{k}\Omega + 1.32\ \text{k}\Omega)(10\ \mu\text{F})}$$
$$f_{L_S} \cong 6.86\ \text{Hz vs. } 12.06\ \text{Hz without } R_s$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$
$$= \frac{1}{(6.28)(4\ \text{k}\Omega + 2.2\ \text{k}\Omega)(1\ \mu\text{F})}$$
$$\cong \mathbf{25.68\ Hz} \text{ as before}$$

C_E

$$R'_s = R_s \| R_1 \| R_2 = 1\ \text{k}\Omega \| 40\ \text{k}\Omega \| 10\ \text{k}\Omega \cong 0.889\ \text{k}\Omega$$
$$R_e = R_E \| \left(\frac{R'_s}{\beta} + r_e\right) = 2\ \text{k}\Omega \| \left(\frac{0.889\ \text{k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\ \text{k}\Omega \| (8.89\ \Omega + 15.76\ \Omega) = 2\ \text{k}\Omega \| 24.65\ \Omega \cong 24.35\ \Omega$$
$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{3058.36}$$
$$\cong \mathbf{327\ Hz} \text{ vs. } 87.13\ \text{Hz without } R_s.$$

The net result is a severe reduction in overall gain (almost 43%) but a corresponding reduction in the lower cutoff frequency. Recall that the highest of the low cutoff frequencies will determine the overall low cutoff frequency for the amplifier. The results point out that the internal series resistance can have a very strong impact on the midband gain, but on the other side of the coin it can improve the overall bandwidth. In this case it is clear that the loss in gain far outweighs any gain in bandwidth.

b. It was mentioned earlier that dB plots are usually normalized by dividing the voltage gain A_v by the magnitude of the midband gain. For Fig. 9.32, the magnitude of the midband gain is 51.21, and naturally the ratio $|A_v/A_{v_{mid}}|$ will be 1 in the midband region. The result is a 0-dB asymptote in the midband region as shown in Fig. 9.36. Defining f_{L_E} as our lower cutoff frequency f_L, we can draw an asymptote at -6 dB/octave as shown in Fig. 9.36 to form the Bode plot and our envelope for the actual response. At f_1, the actual curve is -3 dB down from the midband level as defined by the $0.707A_{v_{mid}}$ level, permitting a sketch of the actual frequency response curve as shown in Fig. 9.36. A -6-dB/octave asymptote was drawn at each frequency defined in the analysis above to demonstrate clearly that it is f_{L_E} for this network that will determine the -3-dB point. It is not until about -24 dB that f_{L_C} begins to affect the shape of the envelope. The magnitude plot shows that the slope of the resultant asymptote is the sum of the asymptotes having the same sloping direction in the same frequency interval. Note in Fig. 9.36 that the slope has dropped to -12 dB/octave for frequencies less than f_{L_C} and could drop to -18 dB/octave if the three defined cutoff frequencies of Fig. 9.36 were closer together. Using Eq. (9.9), the cutoff frequency for the low-frequency region is about 325 Hz.

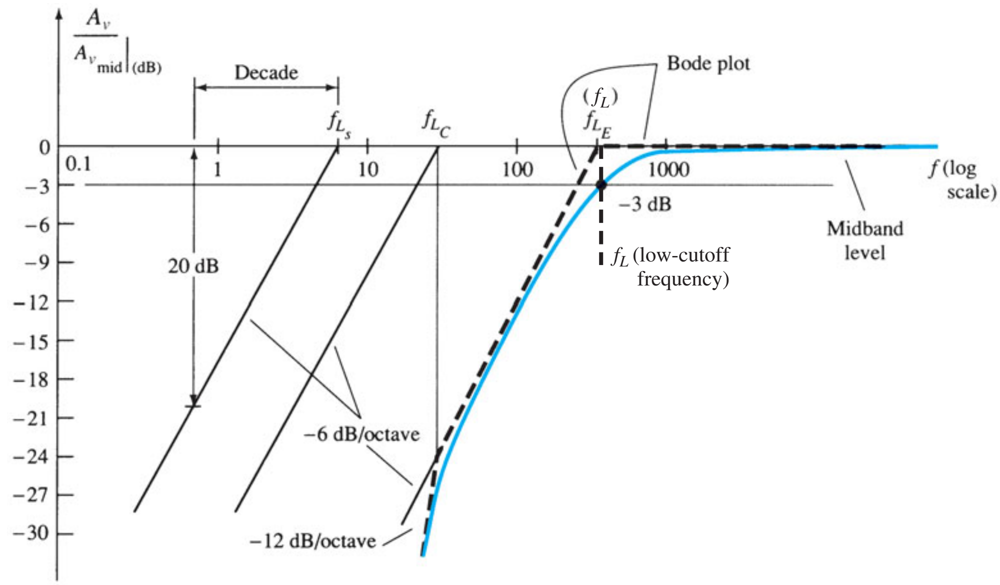

FIG. 9.36
Low-frequency plot for the network of Example 9.12.

c. The PSpice solution can be found in Section 9.15.

Keep in mind as we proceed to the next section that the analysis of this section is not limited to the networks of Figs. 9.25 and 9.32. For any transistor configuration it is simply necessary to isolate each *RC* combination formed by a capacitive element and determine the break frequencies. The resulting frequencies will then determine whether there is a strong interaction between capacitive elements in determining the overall response and which element will have the greatest effect on establishing the lower cutoff frequency. In fact, the analysis of the next section will parallel this section as we determine the low-cutoff frequencies for the FET amplifier.

9.9 LOW-FREQUENCY RESPONSE—FET AMPLIFIER

The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier of Section 9.7. There are again three capacitors of primary concern as appearing in the network of Fig. 9.37: C_G, C_C, and C_S. Although Fig. 9.37

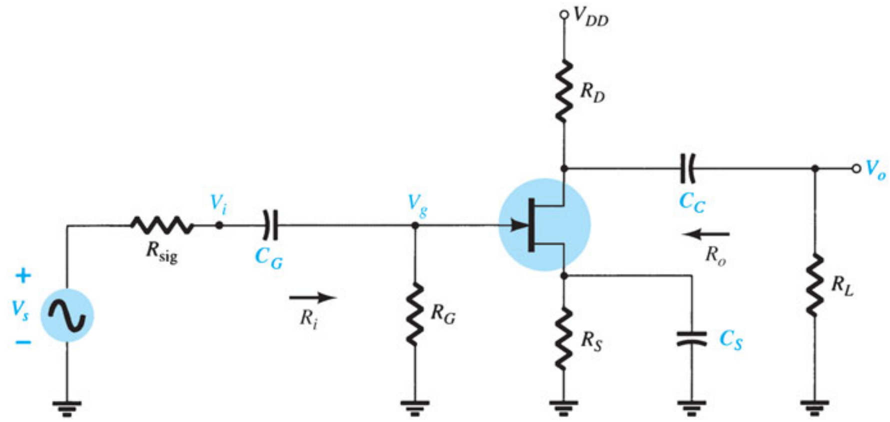

FIG. 9.37
Capacitive elements that affect the low-frequency response of a JFET amplifier.

will be used to establish the fundamental equations, the procedure and conclusions can be applied to any FET configuration. Most of the equations for impedance levels can be found in Table 8.2.

C_G For the coupling capacitor between the source and the active device, the ac equivalent network is as shown in Fig. 9.38. The cutoff frequency determined by C_G is

$$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G} \tag{9.42}$$

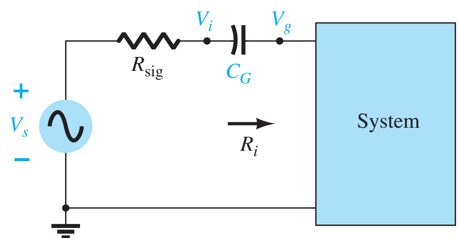

FIG. 9.38
Determining the effect of C_G on the low-frequency response.

which is an exact match of Eq. (9.39). For the network of Fig. 9.37,

$$R_i = R_G \tag{9.43}$$

Typically, $R_G \gg R_{sig}$, and the lower cutoff frequency is determined primarily by R_G and C_G. The fact that R_G is so large permits a relatively low level of C_G while maintaining a low cutoff frequency level for f_{L_G}.

C_C For the coupling capacitor between the active device and the load the network of Fig. 9.39 results, which is also an exact match of Fig. 9.27. The resulting cutoff frequency is

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \tag{9.44}$$

For the network of Fig. 9.37,

$$R_o = R_D \| r_d \tag{9.45}$$

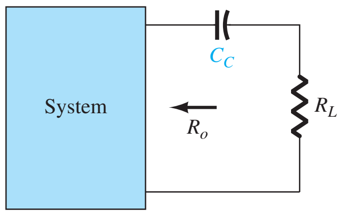

FIG. 9.39
Determining the effect of C_C on the low-frequency response.

C_S For the source capacitor C_S, the resistance level of importance is defined by Fig. 9.40. The cutoff frequency is defined by

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S} \tag{9.46}$$

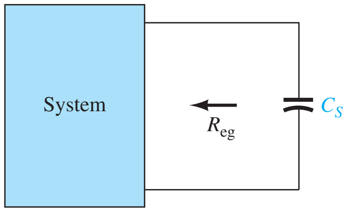

FIG. 9.40
Determining the effect of C_S on the low-frequency response.

For Fig. 9.37, the resulting value of R_{eq} is

$$R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)} \tag{9.47}$$

which for $r_d \cong \infty\,\Omega$ becomes

$$R_{eq} = R_S \| \frac{1}{g_m} \Bigg|_{r_d \cong \infty\Omega} \tag{9.48}$$

EXAMPLE 9.13

a. Determine the lower cutoff frequency for the network of Fig. 9.37 using the following parameters:

$$C_G = 0.01\,\mu\text{F}, \quad C_C = 0.5\,\mu\text{F}, \quad C_S = 2\,\mu\text{F}$$
$$R_{sig} = 10\,\text{k}\Omega, \quad R_G = 1\,\text{M}\Omega, \quad R_D = 4.7\,\text{k}\Omega, \quad R_S = 1\,\text{k}\Omega, \quad R_L = 2.2\,\text{k}\Omega$$
$$I_{DSS} = 8\,\text{mA}, \quad V_P = -4\,\text{V}, \quad r_d = \infty\,\Omega, \quad V_{DD} = 20\,\text{V}$$

b. Sketch the frequency response using a Bode plot.
c. Verify the results of part (b) using PSpice.
d. Perform a complete analysis of the network of Fig. 9.37 using Multisim.

Solution:

a. DC analysis: Plotting the transfer curve of $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ and superimposing the curve defined by $V_{GS} = -I_D R_S$ results in an intersection at $V_{GS_Q} = -2$ V and $I_{D_Q} = 2$ mA. In addition,

$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8\,\text{mA})}{4\,\text{V}} = 4\,\text{mS}$$

$$g_m = g_{m0}\left(1 - \frac{V_{GS_Q}}{V_P}\right) = 4\,\text{mS}\left(1 - \frac{-2\,\text{V}}{-4\,\text{V}}\right) = 2\,\text{mS}$$

C_G Eq. (9.36): $f_{L_G} = \dfrac{1}{2\pi(R_{sig} + R_i)C_G} = \dfrac{1}{2\pi(10\,\text{k}\Omega + 1\,\text{M}\Omega)(0.01\,\mu\text{F})} \cong$ **15.8 Hz**

C_C Eq. (9.38): $f_{L_C} = \dfrac{1}{2\pi(R_o + R_L)C_C} = \dfrac{1}{2\pi(4.7\,\text{k}\Omega + 2.2\,\text{k}\Omega)(0.5\,\mu\text{F})} \cong$ **46.13 Hz**

C_S $R_{eq} = R_S \| \dfrac{1}{g_m} = 1\,\text{k}\Omega \| \dfrac{1}{2\,\text{mS}} = 1\,\text{k}\Omega \| 0.5\,\text{k}\Omega = 333.33\,\Omega$

Eq. (9.40): $f_{L_S} = \dfrac{1}{2\pi R_{eq} C_S} = \dfrac{1}{2\pi(333.33\,\Omega)(2\,\mu\text{F})} =$ **238.73 Hz**

Because f_{L_s} is the largest of the three cutoff frequencies, it defines the low-cutoff frequency for the network of Fig. 9.37.

b. The midband gain of the system is determined by

$$A_{v_{mid}} = \frac{V_o}{V_i} = -g_m(R_D \| R_L) = -(2\,\text{mS})(4.7\,\text{k}\Omega \| 2.2\,\text{k}\Omega)$$
$$= -(2\,\text{mS})(1.499\,\text{k}\Omega)$$
$$\cong \mathbf{-3}$$

Using the midband gain to normalize the response for the network of Fig. 9.37 results in the frequency plot of Fig. 9.41.

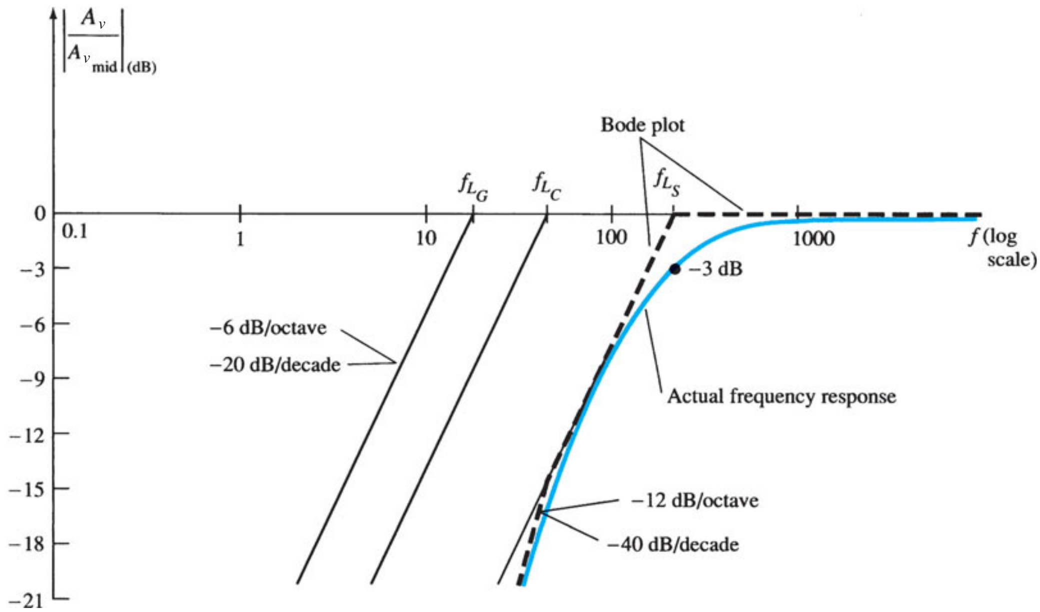

FIG. 9.41
Low-frequency response for the JFET configuration of Example 9.13.

c. and d. The computer solutions can be found in Section 9.15.

9.10 MILLER EFFECT CAPACITANCE

In the high-frequency region, the capacitive elements of importance are the interelectrode (between-terminals) capacitances internal to the active device and the wiring capacitance between leads of the network. The large capacitors of the network that controlled the low-frequency response are all replaced by their short-circuit equivalent due to their very low reactance levels.

For *inverting* amplifiers (phase shift of 180° between input and output, resulting in a negative value for A_v), the input and output capacitance is increased by a capacitance level sensitive to the interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier. In Fig. 9.42, this "feedback" capacitance is defined by C_f.

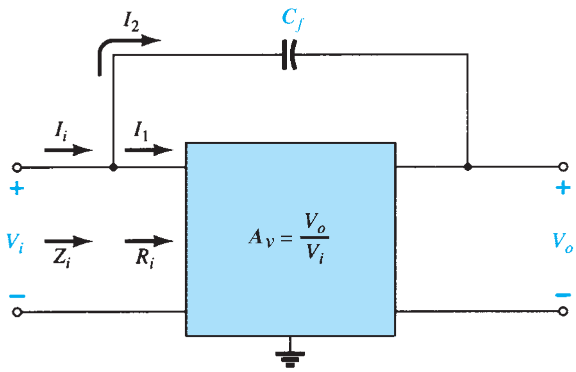

FIG. 9.42
Network employed in the derivation of an equation for the Miller input capacitance.

Applying Kirchhoff's current law gives

$$I_i = I_1 + I_2$$

Using Ohm's law yields

$$I_i = \frac{V_i}{Z_i}, \quad I_1 = \frac{V_i}{R_i}$$

and
$$I_2 = \frac{V_i - V_o}{X_{C_f}} = \frac{V_i - A_v V_i}{X_{C_f}} = \frac{(1 - A_v)V_i}{X_{C_f}}$$

Substituting, we obtain

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{(1 - A_v)V_i}{X_{C_f}}$$

and
$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_f}/(1 - A_v)}$$

but
$$\frac{X_{C_f}}{1 - A_v} = \frac{1}{\underbrace{\omega(1 - A_v)C_f}_{C_M}} = X_{C_M}$$

and
$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_M}}$$

establishing the equivalent network of Fig. 9.43. The result is an equivalent input impedance to the amplifier of Fig. 9.44 that includes the same R_i that we dealt with in previous chapters, with the addition of a feedback capacitor magnified by the gain of the amplifier. Any interelectrode capacitance at the input terminals to the amplifier will simply be added in parallel with the elements of Fig. 9.43.

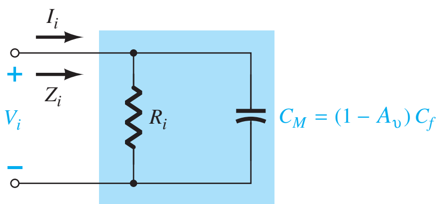

FIG. 9.43
Demonstrating the effect of the Miller effect capacitance.

In general, therefore, the Miller effect input capacitance is defined by

$$\boxed{C_{M_i} = (1 - A_v)C_f} \qquad \textbf{(9.49)}$$

This shows us that:

For any inverting amplifier, the input capacitance will be increased by a Miller effect capacitance sensitive to the gain of the amplifier and the interelectrode (parasitic) capacitance between the input and output terminals of the active device.

The dilemma of an equation such as Eq. (9.49) is that at high frequencies the gain A_v will be a function of the level of C_{M_i}. However, because the maximum gain is the midband value, using the midband value will result in the highest level of C_{M_i} and the worst-case scenario. In general, therefore, the midband value is typically employed for A_v in Eq. (9.49).

The reason for the constraint that the amplifier be of the inverting variety is now more apparent when one examines Eq. (9.49). A positive value for A_v would result in a negative capacitance (for $A_v > 1$).

The Miller effect will also increase the level of output capacitance, which must also be considered when the high-frequency cutoff is determined. In Fig. 9.44, the parameters of

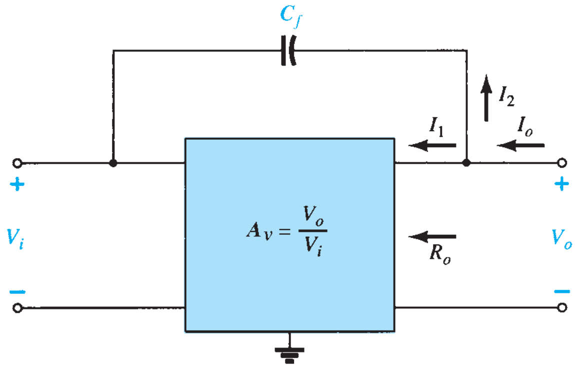

FIG. 9.44
Network employed in the derivation of an equation for the Miller output capacitance.

importance to determine the output Miller effect are in place. Applying Kirchhoff's current law results in

$$I_o = I_1 + I_2$$

with $$I_1 = \frac{V_o}{R_o} \quad \text{and} \quad I_2 = \frac{V_o - V_i}{X_{C_f}}$$

The resistance R_o is usually sufficiently large to permit ignoring the first term of the equation compared to the second term and assuming that

$$I_o \cong \frac{V_o - V_i}{X_{C_f}}$$

Substituting $V_i = V_o/A_v$ from $A_v = V_o/V_i$ results in

$$I_o = \frac{V_o - V_o/A_v}{X_{C_f}} = \frac{V_o(1 - 1/A_v)}{X_{C_f}}$$

and $$\frac{I_o}{V_o} = \frac{1 - 1/A_v}{X_{C_f}}$$

or $$\frac{V_o}{I_o} = \frac{X_{C_f}}{1 - 1/A_v} = \frac{1}{\omega C_f(1 - 1/A_v)} = \frac{1}{\omega C_{M_o}}$$

resulting in the following equation for the Miller output capacitance:

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_f \qquad \textbf{(9.50)}$$

For the usual situation where $A_v \gg 1$, Eq. (9.50) reduces to

$$C_{M_o} \cong C_f \quad _{|A_v| \gg 1} \qquad \textbf{(9.51)}$$

Examples of the use of Eq. (9.50) appear in the next two sections as we investigate the high-frequency responses of BJT and FET amplifiers.

For noninverting amplifiers such as the common-base and emitter-follower configurations, the Miller effect capacitance is not a contributing concern for high-frequency applications.

9.11 HIGH-FREQUENCY RESPONSE—BJT AMPLIFIER

At the high-frequency end, there are two factors that define the −3-dB cutoff point: the network capacitance (parasitic and introduced) and the frequency dependence of $h_{fe}(\beta)$.

Network Parameters

In the high-frequency region, the *RC* network of concern has the configuration appearing in Fig. 9.45. At increasing frequencies, the reactance X_C will decrease in magnitude, resulting in a shorting effect across the output and a decrease in gain. The derivation leading to the corner frequency for this R_C configuration follows along similar lines to that encountered for the low-frequency region. The most significant difference is in the following general form of A_v:

$$A_v = \frac{1}{1 + j(f/f_H)} \tag{9.52}$$

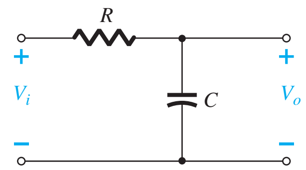

FIG. 9.45
RC combination that will define a high-cutoff frequency.

This results in a magnitude plot such as shown in Fig. 9.46 that drops off at 6 dB/octave with increasing frequency. Note that f_H is in the denominator of the frequency ratio rather than the numerator as occurred for f_L in Eq. (9.23).

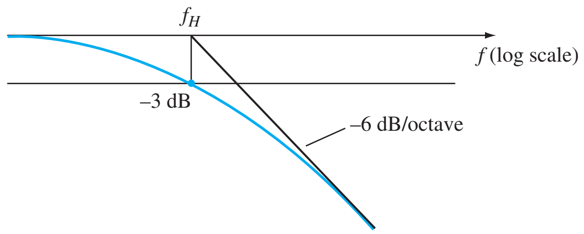

FIG. 9.46
Asymptotic plot as defined by Eq. (9.52).

In Fig. 9.47, the various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor are included with the wiring capacitances (C_{W_i}, C_{W_o}) introduced during construction. The high-frequency equivalent model for the network of Fig. 9.47 appears in Fig. 9.48. Note the absence of the capacitors C_s, C_C, and C_E, which are all assumed to be in the short-circuit state at these frequencies. The capacitance C_i includes the input wiring capacitance C_{W_i}, the transition capacitance C_{be}, and the Miller capacitance C_{M_i}. The capacitance C_o includes the output wiring capacitance C_{W_o}, the parasitic capacitance C_{ce}, and the output Miller capacitance C_{M_o}. In general, the capacitance C_{be} is the largest of the parasitic capacitances, with C_{ce} the smallest. In fact, most specification sheets simply provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of a particular type of transistor in a specific area of application.

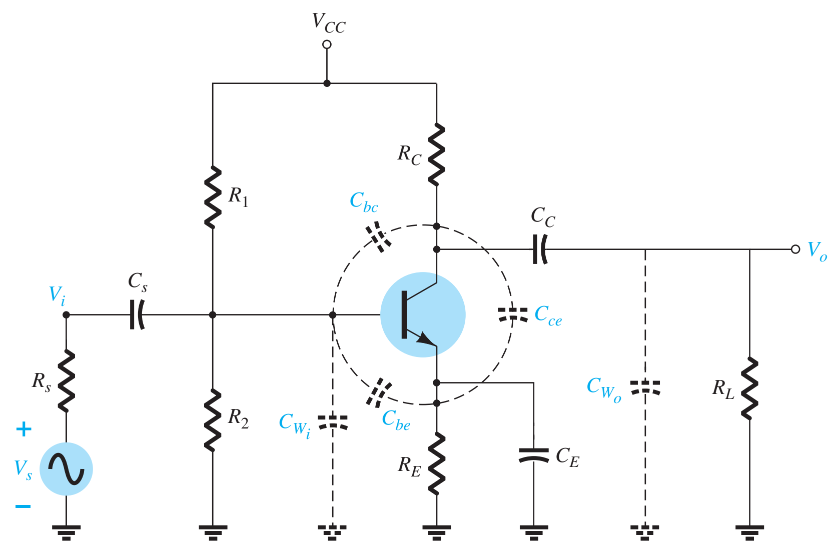

FIG. 9.47
Network of Fig. 9.25 with the capacitors that affect the high-frequency response.

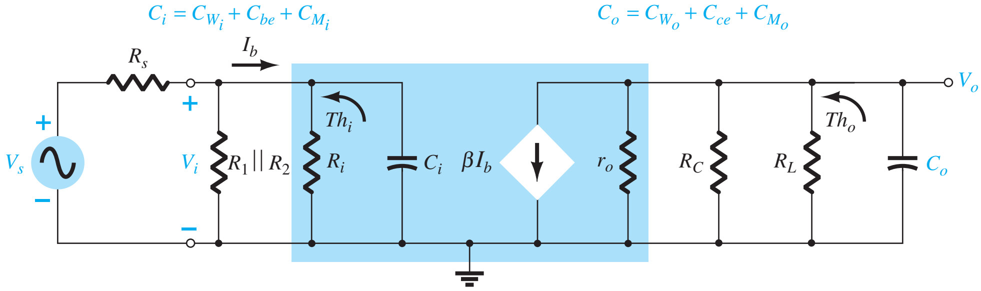

FIG. 9.48
High-frequency ac equivalent model for the network of Fig. 9.47.

Determining the Thévenin equivalent circuit for the input and output networks of Fig. 9.48 results in the configurations of Fig. 9.49. For the input network, the −3-dB frequency is defined by

$$f_{H_i} = \frac{1}{2\pi R_{\text{Th}_i} C_i} \tag{9.53}$$

with

$$R_{\text{Th}_i} = R_s \| R_1 \| R_2 \| \beta r_e \tag{9.54}$$

and

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc} \tag{9.55}$$

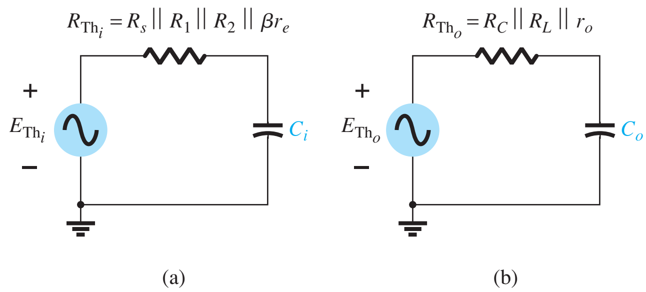

FIG. 9.49
Thévenin circuits for the input and output networks of the network of Fig. 9.48.

At very high frequencies, the effect of C_i is to reduce the total impedance of the parallel combination of R_1, R_2, βr_e, and C_i in Fig. 9.48. The result is a reduced level of voltage across C_i, a reduction in I_b, and a gain for the system.

For the output network,

$$f_{H_o} = \frac{1}{2\pi R_{\text{Th}_o} C_o} \tag{9.56}$$

with

$$R_{\text{Th}_o} = R_C \| R_L \| r_o \tag{9.57}$$

and

$$C_o = C_{W_o} + C_{ce} + C_{M_o} \tag{9.58}$$

or

$$C_o = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc}$$

For A_v large (typical):

$$1 \gg 1/A_v$$

and

$$C_o \cong C_{W_o} + C_{ce} + C_{bc} \tag{9.59}$$

At very high frequencies, the capacitive reactance of C_o will decrease and consequently reduce the total impedance of the output parallel branches of Fig. 9.48. The net result is that V_o will also decline toward zero as the reactance X_C becomes smaller. The frequencies

f_{H_i} and f_{H_o} will each define a −6-dB/octave asymptote such as depicted in Fig. 9.46. If the parasitic capacitors were the only elements to determine the high-cutoff frequency, the lowest frequency would be the determining factor. However, the decrease in h_{fe} (or β) with frequency must also be considered as to whether its break frequency is lower than f_{H_i} or f_{H_o}.

h_{fe} (or β) Variation

The variation of h_{fe} (or β) with frequency will approach, with some degree of accuracy, the following relationship:

$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)} \tag{9.60}$$

The use of h_{fe} rather than β in some of this descriptive material is due primarily to the fact that manufacturers typically use the hybrid parameters when covering this issue in their specification sheets and so on.

The only undefined quantity, f_β, is determined by a set of parameters employed in the *hybrid* π or *Giacoletto* model of Fig. 9.50 introduced in Section 5.22. The resistance r_b includes the base contact, base bulk, and base spreading resistance. The first is due to the actual connection to the base. The second includes the resistance from the external terminal to the active region of the transistors, and the last is the actual resistance within the active base region. The resistances r_π, r_o, and r_u are the resistances between the indicated terminals when the device is in the active region. The same is true for the capacitances C_{bc} and C_{be}, although the former is a transition capacitance, whereas the latter is a diffusion capacitance. A more detailed explanation of the frequency dependence of each can be found in a number of readily available texts.

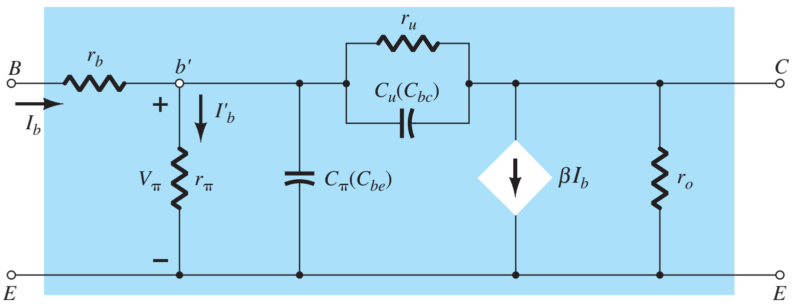

FIG. 9.50
Giacoletto (or hybrid π) high-frequency transistor small-signal ac equivalent circuit.

If we remove the base resistance r_b, the base-to-collector resistance r_u, and all the parasitic capacitances, the result is an ac equivalent circuit that matches the small-signal equivalent for the common-emitter configuration used in Chapter 5. The base-to-emitter resistance r_π is βr_e and the output resistance r_o is simply a value provided through the hybrid parameter h_{oe}. The controlled source is also βI_b as used in Chapter 5. However, if we include the resistance r_u (usually quite large) between base and collector, there is a feedback loop between output and input circuits to match the contribution of h_{r_e} for the hybrid equivalent circuit. Recall from Chapter 5 that the feedback term is normally inconsequential for most applications, but if a particular application puts it at the forefront, then the model of Fig. 9.50 will bring it into play. The resistance r_u is a result of the fact that the base current is somewhat sensitive to the collector-to-base voltage. Because the base-to-emitter voltage is linearly related to the base current through Ohm's law and the output voltage is equal to the difference between the base-to-emitter voltage and collector-to-base voltage, we can conclude that the base current is sensitive to the changes in output voltage as revealed by the hybrid parameter h_{r_e}.

In terms of these parameters,

$$f_\beta(\text{often appearing as } f_{h_{fe}}) = \frac{1}{2\pi r_\pi(C_\pi + C_u)} \tag{9.61}$$

or, because $r_\pi = \beta r_e = h_{fe_{\text{mid}}} r_e$,

$$f_\beta = \frac{1}{h_{fe_{\text{mid}}}} \frac{1}{2\pi r_e(C_\pi + C_u)} \tag{9.62}$$

or

Equation (9.62) clearly reveals that because r_e is a function of the network design:

f_β is a function of the bias configuration.

The basic format of Eq. (9.60) is exactly the same as Eq. (9.52) if we extract the multiplying factor $h_{fe_{\text{mid}}}$, revealing that h_{fe} will drop off from its midband value with a 6-dB/octave slope as shown in Fig. 9.51. The same figure has a plot of h_{fb} (or α) versus frequency. Note the small change in h_{fb} for the chosen frequency range, revealing that the common-base configuration displays improved high-frequency characteristics over the common-emitter configuration. Recall also the absence of the Miller effect capacitance due to the noninverting characteristics of the common-base configuration. For this very reason, common-base high-frequency parameters rather than common-emitter parameters are often specified for a transistor—especially those designed specifically to operate in the high-frequency regions.

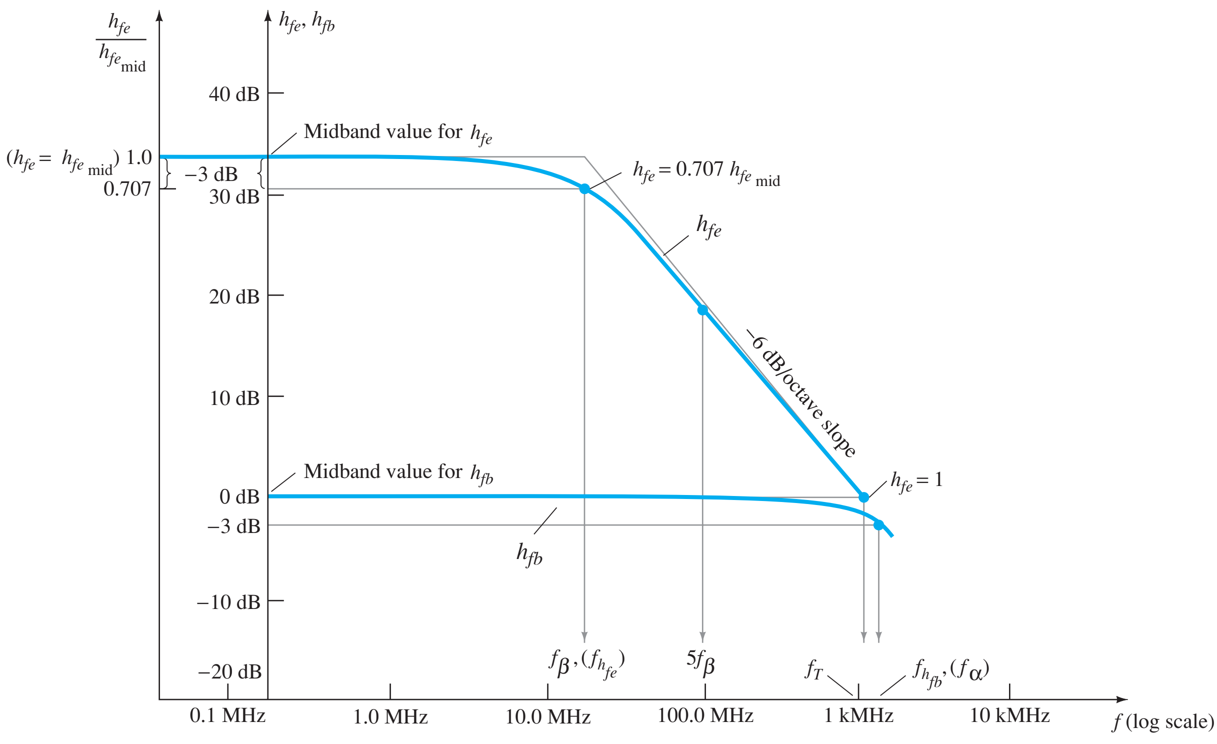

FIG. 9.51
h_{fe} and h_{fb} versus frequency in the high-frequency region.

The following equation permits a direct conversion for determining f_β if f_α and α are specified:

$$f_\beta = f_\alpha(1 - \alpha) \tag{9.63}$$

Gain-Bandwidth Product

There is a **Figure of Merit** applied to amplifiers called the **Gain-Bandwidth Product (GBP)** that is commonly used to initiate the design process of an amplifier. It provides important information about the relationship between the gain of the amplifier and the expected operating frequency range.

In Fig. 9.52 the frequency response of an amplifier with a gain of 100, a low cutoff frequency of 250 Hz, and an upper cutoff frequency of 1 MHz has been plotted on a linear scale rather than the typical log scale. Note that because a linear scale was chosen for the horizontal axis it is impossible to show the low cutoff frequency, and the curve appears as essentially a straight vertical line at $f = 0$ Hz. Because $f = 0$ Hz represents a dc situation,

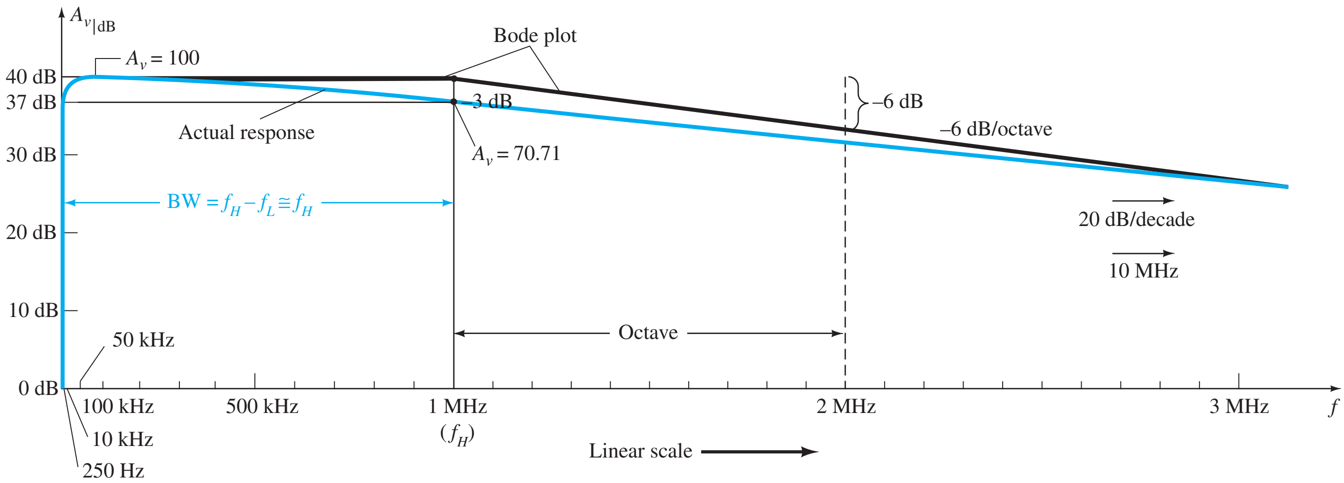

FIG. 9.52
Plotting the dB gain of an amplifier in a linear-frequency plot.

the gain at the low end of an amplifier is often called the DC gain.

Note also that the use of a linear horizontal axis results in a very slow decline in gain with frequency past the breakpoint. It would take many pages to show the full frequency plot at the high end.

It is also clear from Fig. 9.52 that the bandwidth is essentially defined by the upper cutoff frequency because the low cutoff frequency is so small in comparison.

If Fig. 9.52 were plotted using a log scale for the horizontal axis, the plot of Fig. 9.53 would result.

The low end is expanded and the frequency response at the upper end is complete with a boundary defined by the 20-dB drop per decade slope. The upper breakpoint frequency is labeled f_H with the lower breakpoint frequency labeled f_L.

At $A_v = A_{v_{mid}} = 100$ the bandwidth as shown in Fig. 9.53 is approximately 1 MHz.

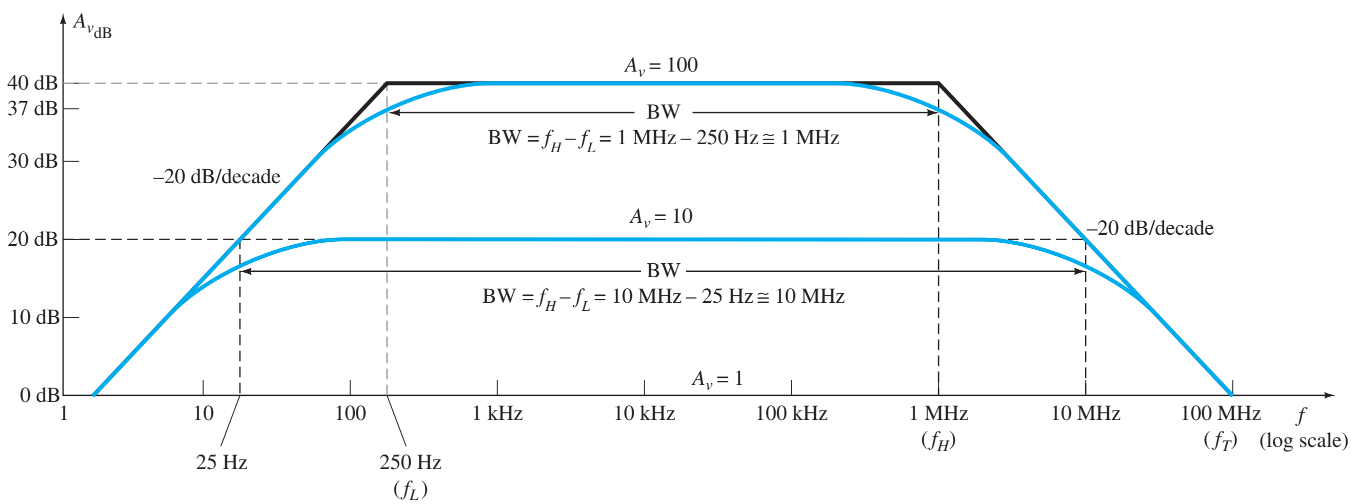

FIG. 9.53
Finding the bandwidth at two different gain levels.

The gain-bandwidth product is

$$\boxed{\text{GBP} = A_{v_{\text{mid}}}\text{BW}} \tag{9.64}$$

which for this example is

$$\text{GBP} = (100)(1\text{ MHz}) = 100\text{ MHz}$$

At $A_v = 10$, $20\log_{10}10 = 20$ and the bandwidth as shown in Fig. 9.53 is approximately 10 MHz.

The resulting gain-bandwidth product is now

$$\text{GBP} = (10)(10\text{ MHz}) = 100\text{ MHz}$$

In fact, at any level of gain the product of the two remains a constant.

At $A_v = 1$ or $A_v|_{dB} = 0$ bandwidth is defined as f_T in Fig. 9.53.

In general,

the frequency f_T is called the unity-gain frequency and is always equal to the product of the midband gain of an amplifier and the bandwidth at any level of gain.

That is,

$$\boxed{f_T = A_{v_{\text{mid}}} f_H} \quad \text{(Hz)} \tag{9.65a}$$

The result is that the expected bandwidth of an amplifier for any level of gain can be found quite directly. Consider an amplifier with a given f_T of 120 MHz. At a gain of 80 the expected f_H or bandwidth is $f_T/A_{v_{\text{mid}}} = 120\text{ MHz}/80 = 1.5\text{ MHz}$. At a gain of 60 the bandwidth is 120 MHz/60 = 2 MHz and so on—a very useful tool.

For transistors themselves, where a voltage gain has not been defined by a configuration, specification sheets will provide a value of f_T that relates to the transistor only. That is,

$$\boxed{f_T = h_{fe_{\text{mid}}} f_\beta} \quad \text{(Hz)} \tag{9.65b}$$

The dB plot would appear as shown in Fig. 9.49.

The general equation for the h_{f_e} variation with frequency is defined by Eq. 9.60. For the amplifier it is defined by

$$\boxed{A_v = \frac{A_{v_{\text{mid}}}}{1 + j(f/f_H)}} \tag{9.66}$$

Note that in each case the frequency f_H defines the corner frequency.

Substituting Eq. (9.62) for f_β in Eq. (9.65) gives

$$f_T = h_{fe_{\text{mid}}}\frac{1}{2\pi h_{fe_{\text{mid}}} r_e(C_\pi + C_u)}$$

and

$$\boxed{f_T \cong \frac{1}{2\pi r_e(C_\pi + C_u)}} \tag{9.67}$$

EXAMPLE 9.14 Use the network of Fig. 9.47 with the same parameters as in Example 9.12, that is,

$$R_s = 1\text{ k}\Omega, R_1 = 40\text{ k}\Omega, R_2 = 10\text{ k}\Omega, R_E = 2\text{ k}\Omega, R_C = 4\text{ k}\Omega, R_L = 2.2\text{ k}\Omega$$
$$C_s = 10\ \mu\text{F}, C_C = 1\ \mu\text{F}, C_E = 20\ \mu\text{F}$$
$$h_{fe} = 100, r_o = \infty\ \Omega, V_{CC} = 20\text{ V}$$

with the addition of

$$C_\pi(C_{be}) = 36\text{ pF}, C_u(C_{bc}) = 4\text{ pF}, C_{ce} = 1\text{ pF}, C_{W_i} = 6\text{ pF}, C_{W_o} = 8\text{ pF}$$

a. Determine f_{H_i} and f_{H_o}.
b. Find f_β and f_T.

c. Sketch the frequency response for the low- and high-frequency regions using the results of Example 9.12 and the results of parts (a) and (b).
d. Obtain the PSpice response for the full frequency spectrum and compare with the results of part (c).

Solution:

a. From Example 9.12:

$$\beta r_e = 1.576\text{ k}\Omega, \qquad A_{v_{\text{mid}}}(\text{amplifier—not including effects of } R_s) = -90$$

and
$$R_{\text{Th}_i} = R_s \| R_1 \| R_2 \| \beta r_e = 1\text{ k}\Omega \| 40\text{ k}\Omega \| 10\text{ k}\Omega \| 1.576\text{ k}\Omega \cong 0.57\text{ k}\Omega$$

with
$$C_i = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6\text{ pF} + 36\text{ pF} + [1 - (-90)]4\text{ pF} = 406\text{ pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{\text{Th}_i} C_i} = \frac{1}{2\pi(0.57\text{ k}\Omega)(406\text{ pF})} = \mathbf{687.73\text{ kHz}}$$

$$R_{\text{Th}_o} = R_C \| R_L = 4\text{ k}\Omega \| 2.2\text{ k}\Omega = 1.419\text{ k}\Omega$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = 8\text{ pF} + 1\text{ pF} + \left(1 - \frac{1}{-90}\right)4\text{ pF} = 13.04\text{ pF}$$

$$f_{H_o} = \frac{1}{2\pi R_{\text{Th}_o} C_o} = \frac{1}{2\pi(1.419\text{ k}\Omega)(13.04\text{ pF})} = \mathbf{8.6\text{ MHz}}$$

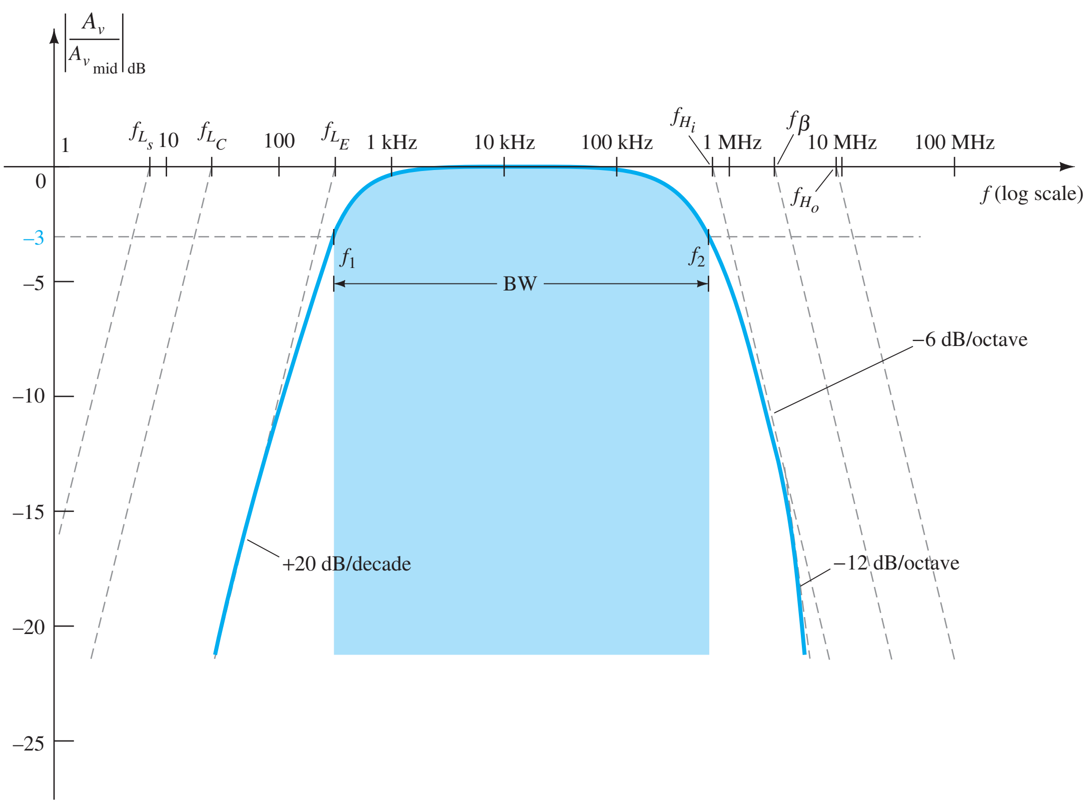

FIG. 9.54
Full frequency response for the network of Fig. 9.47.

b. Applying Eq. (9.63) gives

$$f_\beta = \frac{1}{2\pi h_{fe_{\text{mid}}} r_e (C_{be} + C_{bc})}$$

$$= \frac{1}{2\pi(100)(15.76\ \Omega)(36\text{ pF} + 4\text{ pF})} = \frac{1}{2\pi(100)(15.76\ \Omega)(40\text{ pF})}$$

$$= \mathbf{2.52\ MHz}$$

$$f_T = h_{fe_{\text{mid}}} f_\beta = (100)(2.52\text{ MHz})$$

$$= \mathbf{252\ MHz}$$

c. See Fig. 9.54. The corner frequency f_{H_i} will determine the high cutoff frequency and the bandwidth of the amplifier. The upper cutoff frequency is very close to 600 kHz.

d. The PSpice analysis will appear in Section 9.15.

9.12 HIGH-FREQUENCY RESPONSE—FET AMPLIFIER

The analysis of the high-frequency response of the FET amplifier will proceed in a very similar manner to that encountered for the BJT amplifier. As shown in Fig. 9.55, there are interelectrode and wiring capacitances that will determine the high-frequency characteristics of the amplifier. The capacitors C_{gs} and C_{gd} typically vary from 1 pF to 10 pF, whereas the capacitance C_{ds} is usually quite a bit smaller, ranging from 0.1 pF to 1 pF.

Because the network of Fig. 9.55 is an inverting amplifier, a Miller effect capacitance will appear in the high-frequency ac equivalent network appearing in Fig. 9.56. At high frequencies, C_i will approach a short-circuit equivalent and V_{gs} will drop in value and reduce the overall gain. At frequencies where C_o approaches its short-circuit equivalent, the parallel output voltage V_o will drop in magnitude.

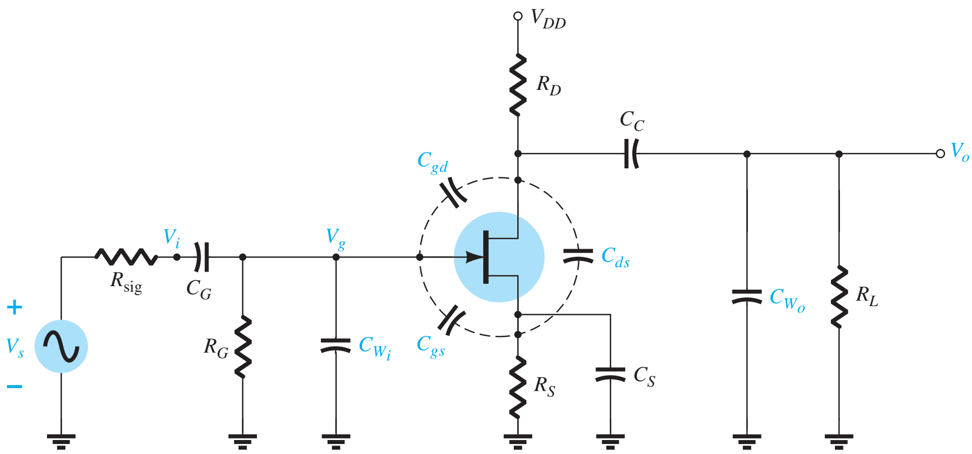

FIG. 9.55
Capacitive elements that affect the high-frequency response of a JFET amplifier.

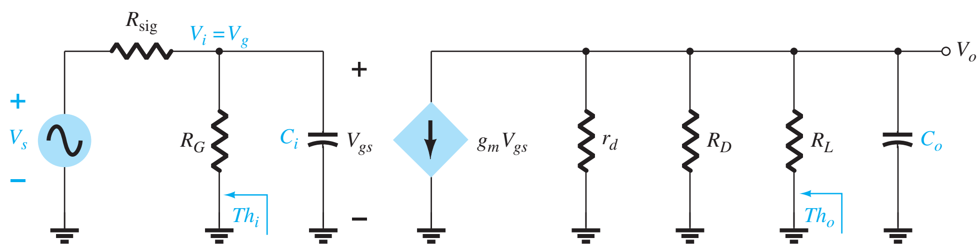

FIG. 9.56
High-frequency ac equivalent circuit for Fig. 9.55.

The cutoff frequencies defined by the input and output circuits can be obtained by first finding the Thévenin equivalent circuits for each section as shown in Fig. 9.57. For the input circuit,

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} \tag{9.68}$$

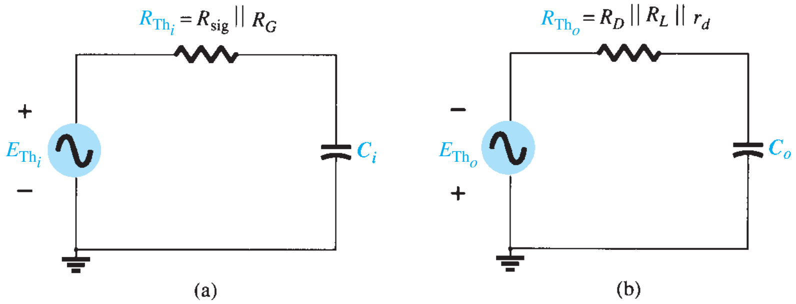

FIG. 9.57
The Thévenin equivalent circuits for: (a) the input circuit and (b) the output circuit.

and

$$R_{Th_i} = R_{sig} \| R_G \tag{9.69}$$

with

$$C_i = C_{W_i} + C_{gs} + C_{M_i} \tag{9.70}$$

and

$$C_{M_i} = (1 - A_v)C_{gd} \tag{9.71}$$

for the output circuit,

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} \tag{9.72}$$

with

$$R_{Th_o} = R_D \| R_L \| r_d \tag{9.73}$$

and

$$C_o = C_{W_o} + C_{ds} + C_{M_o} \tag{9.74}$$

and

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_{gd} \tag{9.75}$$

EXAMPLE 9.15

a. Determine the high-cutoff frequencies for the network of Fig. 9.55 using the same parameters as Example 9.13:

$$C_G = 0.01\ \mu\text{F}, \quad C_C = 0.5\ \mu\text{F}, \quad C_S = 2\ \mu\text{F}$$
$$R_{sig} = 10\ \text{k}\Omega, \quad R_G = 1\ \text{M}\Omega, \quad R_D = 4.7\ \text{k}\Omega, \quad R_S = 1\ \text{k}\Omega, \quad R_L = 2.2\ \text{k}\Omega$$
$$I_{DSS} = 8\ \text{mA}, \quad V_P = -4\ \text{V}, \quad r_d = \infty\ \Omega, \quad V_{DD} = 20\ \text{V}$$

with the addition of

$$C_{gd} = 2\ \text{pF}, \quad C_{gs} = 4\ \text{pF}, \quad C_{ds} = 0.5\ \text{pF}, \quad C_{W_i} = 5\ \text{pF}, \quad C_{W_o} = 6\ \text{pF}$$

b. Obtain a PSpice response for the full frequency range and note whether it supports the conclusions of Example 9.13 and the calculations above.

Solution:

a. $R_{\text{Th}_i} = R_{\text{sig}} \| R_G = 10\,\text{k}\Omega \| 1\,\text{M}\Omega = 9.9\,\text{k}\Omega$

From Example 9.13, $A_v = -3$. We have

$$\begin{aligned} C_i &= C_{W_i} + C_{gs} + (1 - A_v)C_{gd} \\ &= 5\text{ pF} + 4\text{ pF} + (1 + 3)2\text{ pF} \\ &= 9\text{ pF} + 8\text{ pF} \\ &= 17\text{ pF} \end{aligned}$$

$$\begin{aligned} f_{H_1} &= \frac{1}{2\pi R_{\text{Th}_i} C_i} \\ &= \frac{1}{2\pi(9.9\,\text{k}\Omega)(17\text{ pF})} = \mathbf{945.67\ kHz} \end{aligned}$$

$$\begin{aligned} R_{\text{Th}_o} &= R_D \| R_L \\ &= 4.7\,\text{k}\Omega \| 2.2\,\text{k}\Omega \\ &\cong 1.5\,\text{k}\Omega \end{aligned}$$

$$C_o = C_{W_o} + C_{ds} + C_{M_o} = 6\text{ pF} + 0.5\text{ pF} + \left(1 - \frac{1}{-3}\right)2\text{ pF} = 9.17\text{ pF}$$

$$f_{H_o} = \frac{1}{2\pi(1.5\,\text{k}\Omega)(9.17\text{ pF})} = \mathbf{11.57\ MHz}$$

The results above clearly indicate that the input capacitance with its Miller effect capacitance will determine the upper cutoff frequency. This is typically the case due to the smaller value of C_{ds} and the resistance levels encountered in the output circuit.

b. The PSpice analysis will appear in Section 9.15.

Even though the analysis of the last few sections has been limited to two configurations, the general procedure for determining the cutoff frequencies should support the analysis of any other transistor configuration. Keep in mind that the Miller capacitance is limited to inverting amplifiers and that f_α is significantly greater than f_β if the common-base configuration is encountered. There is a great deal more literature on the analysis of single-stage amplifiers that goes beyond the coverage of this chapter. However, the content of this chapter should provide a firm foundation for any analysis of frequency effects.

9.13 MULTISTAGE FREQUENCY EFFECTS

For a second transistor stage connected directly to the output of a first stage, there will be a significant change in the overall frequency response. In the high-frequency region, the output capacitance C_o must now include the wiring capacitance (C_{W_1}), parasitic capacitance (C_{be}), and Miller capacitance (C_{M_i}) of the following stage. Furthermore, there will be additional low-frequency cutoff levels due to the second stage, which will further reduce the overall gain of the system in this region. For each additional stage, the upper cutoff frequency will be determined primarily by the stage having the lowest cutoff frequency. The low-frequency cutoff is primarily determined by that stage having the highest low-frequency cutoff frequency. Obviously, therefore, one poorly designed stage can offset an otherwise well-designed cascaded system.

The effect of increasing the number of *identical* stages can be clearly demonstrated by considering the situations indicated in Fig. 9.58. In each case, the upper and lower cutoff frequencies of each of the cascaded stages are identical. For a single stage, the cutoff frequencies are f_L and f_H as indicated. For two identical stages in cascade, the drop-off rate in the high- and low-frequency regions has increased to −12 dB/octave or −40 dB/decade. At f_L and f_H, therefore, the decibel drop is now −6 dB rather than the defined band frequency gain level of −3 dB. The −3-dB point has shifted to f'_L and f'_H as indicated, with a resulting drop in the bandwidth. A −18-dB/octave or −60-dB/decade slope will result for a three-stage system of identical stages with the indicated reduction in bandwidth (f''_L and f''_H).

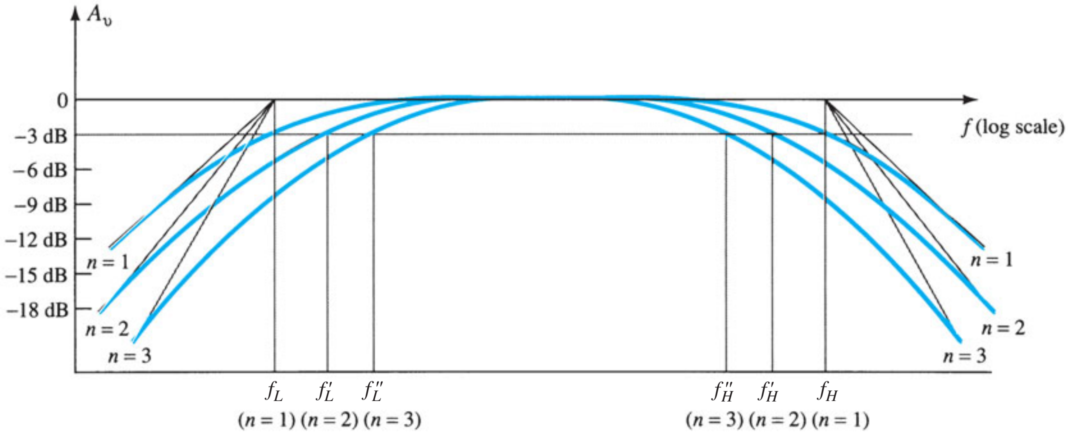

FIG. 9.58

Effect of an increased number of stages on the cutoff frequencies and the bandwidth.

Assuming identical stages, we can determine an equation for each band frequency as a function of the number of stages (n) in the following manner: For the low-frequency region,

$$A_{v_{\text{low, (overall)}}} = A_{v_{1_{\text{low}}}} A_{v_{2_{\text{low}}}} A_{v_{3_{\text{low}}}} \cdots A_{v_{n_{\text{low}}}}$$

but because all stages are identical, $A_{v_{1_{\text{low}}}} = A_{v_{2_{\text{low}}}} =$ etc., and

$$A_{v_{\text{low, (overall)}}} = (A_{v_{1_{\text{low}}}})^n$$

or

$$\frac{A_{v_{\text{low}}}}{A_{v_{\text{mid}}}}\text{(overall)} = \left(\frac{A_{v_{\text{low}}}}{A_{v_{\text{mid}}}}\right)^n = \frac{1}{(1 - jf_L/f)^n}$$

Setting the magnitude of this result equal to $1/\sqrt{2}$(−3 dB level) results in

$$\frac{1}{\sqrt{[1 + (f_L/f'_L)^2]^n}} = \frac{1}{\sqrt{2}}$$

or

$$\left\{\left[1 + \left(\frac{f_L}{f'_L}\right)^{2-}\right]^{1/2}\right\}^n = \left\{\left[1 + \left(\frac{f_L}{f'_L}\right)^2\right]^n\right\}^{1/2} = (2)^{1/2}$$

so that

$$\left[1 + \left(\frac{f_L}{f'_L}\right)^2\right]^n = 2$$

and

$$1 + \left(\frac{f_L}{f'_L}\right)^2 = 2^{1/n}$$

with the result that

$$f'_L = \frac{f_L}{\sqrt{2^{1/n} - 1}} \qquad \textbf{(9.76)}$$

In a similar manner, it can be shown that for the high-frequency region,

$$f'_H = (\sqrt{2^{1/n} - 1})f_H \qquad \textbf{(9.77)}$$

Note the presence of the same factor $\sqrt{2^{1/n} - 1}$ in each equation. The magnitude of this factor for various values of n is listed below.

n	$\sqrt{2^{1/n} - 1}$
2	0.64
3	0.51
4	0.43
5	0.39

For $n = 2$, consider that the upper cutoff frequency $f'_H = 0.64f_H$, or 64% of the value obtained for a single stage, whereas $f'_L = (1/0.64)f_L = 1.56f_L$. For $n = 3, f'_H = 0.51f_H$, or approximately one-half the value of a single stage, and $f'_L = (1/0.51)f_L = 1.96f_L$, or approximately *twice* the single-stage value.

For the *RC*-coupled transistor amplifier, if $f_H = f_\beta$, or if they are close enough in magnitude for both to affect the upper 3-dB frequency, the number of stages must be increased by a factor of 2 when determining f'_H due to the increased number of factors $1/(1 + jf/f_x)$.

A decrease in bandwidth is not always associated with an increase in the number of stages if the midband gain can remain fixed and independent of the number of stages. For instance, if a single-stage amplifier produces a gain of 100 with a bandwidth of 10,000 Hz, the resulting gain–bandwidth product is $10^2 \times 10^4 = 10^6$. For a two-stage system the same gain can be obtained by having two stages with a gain of 10 ($10 \times 10 = 100$). The bandwidth of each stage would then increase by a factor of 10 to 100,000 due to the lower gain requirement and fixed gain–bandwidth product of 10^6. Of course, the design must be such as to permit the increased bandwidth and establish the lower gain level.

9.14 SQUARE-WAVE TESTING

A sense for the frequency response of an amplifier can be determined experimentally by applying a square-wave signal to the amplifier and noting the output response. The shape of the output waveform will reveal whether the high or low frequencies are being properly amplified. Using *square-wave testing* is significantly less time consuming than applying a series of sinusoidal signals at different frequencies and magnitudes to test the frequency response of the amplifier.

The reason for choosing a square-wave signal for the testing process is best described by examining the *Fourier series* expansion of a square wave composed of a series of sinusoidal components of different magnitudes and frequencies. The summation of all the terms of the series will result in the original waveform. In other words, even though a waveform may not be sinusoidal, it can be reproduced by a series of sinusoidal terms of different frequencies and magnitudes.

The Fourier series expansion for the square wave of Fig. 9.59 is

$$v = \frac{4}{\pi}V_m\Big(\underbrace{\sin 2\pi f_s t}_{\text{fundamental}} + \underbrace{\frac{1}{3}\sin 2\pi(3f_s)t}_{\text{third harmonic}} + \underbrace{\frac{1}{5}\sin 2\pi(5f_s)t}_{\text{fifth harmonic}} + \underbrace{\frac{1}{7}\sin 2\pi(7f_s)t}_{\text{seventh harmonic}} + \underbrace{\frac{1}{9}\sin 2\pi(9f_s)t}_{\text{ninth harmonic}} + \cdots + \underbrace{\frac{1}{n}\sin 2\pi(nf_s)t}_{n\text{th harmonic}}\Big) \tag{9.78}$$

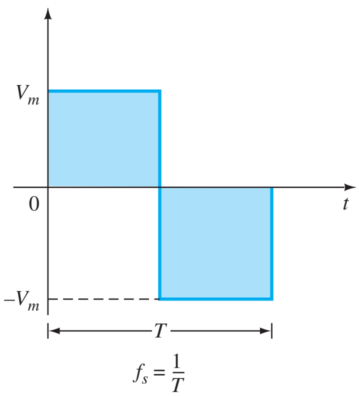

FIG. 9.59
Square wave.

The first term of the series is called the *fundamental* term and in this case has the same frequency, f_s, as the square wave. The next term has a frequency equal to three times the fundamental and is referred to as the *third harmonic.* Its magnitude is one-third the magnitude of the fundamental term. The frequencies of the succeeding terms are odd multiples of the fundamental term, and the magnitude decreases with each higher harmonic. Figure 9.58 demonstrates how the summation of terms of a Fourier series can result in a nonsinusoidal waveform. The generation of the square wave of Fig. 9.59 would require an infinite number of terms. However, the summation of just the fundamental term and the third harmonic in Fig. 9.60a clearly results in a waveform that is beginning to take on the appearance of a square wave. Including the fifth and seventh harmonics as in Fig. 9.60b takes us a step closer to the waveform of Fig. 9.59.

Because the ninth harmonic has a magnitude greater than 10% of the fundamental term [$\frac{1}{9}(100\%) = 11.1\%$], the terms from the fundamental term through the ninth harmonic are the major contributors to the Fourier series expansion of the square-wave function. It is therefore reasonable to assume that if the application of a square wave of a particular frequency results in a nice clean square wave at the output, then the terms from the fundamental through the ninth harmonic are being amplified without visual distortion by the amplifier. For instance, if an audio amplifier with a bandwidth of 20 kHz (audio range is from 20 Hz to 20 kHz) is to be tested, the frequency of the applied signal should be at least 20 kHz/9 = 2.22 kHz.

If the response of an amplifier to an applied square wave is an undistorted replica of the input, the frequency response (or BW) of the amplifier is obviously sufficient for the applied frequency. If the response is as shown in Fig. 9.61a and b, the low frequencies are not being amplified properly and the low cutoff frequency has to be investigated. If the waveform has the appearance of Fig. 9.61c and d, the high-frequency components are not receiving sufficient amplification and the high-cutoff frequency (or BW) has to be reviewed.

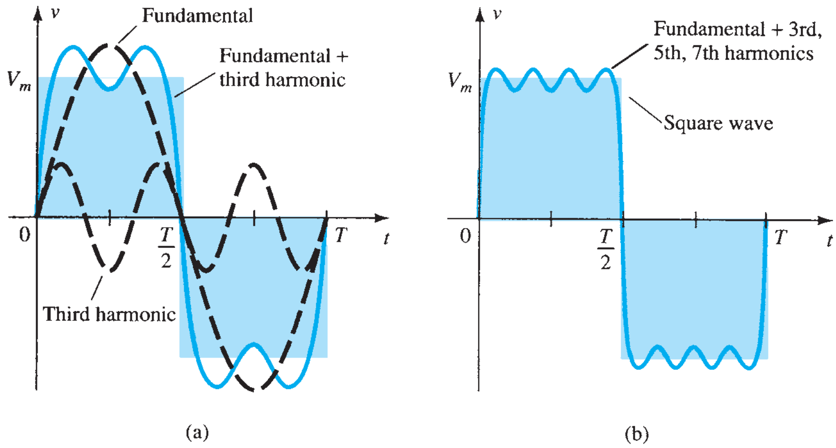

FIG. 9.60
Harmonic content of a square wave.

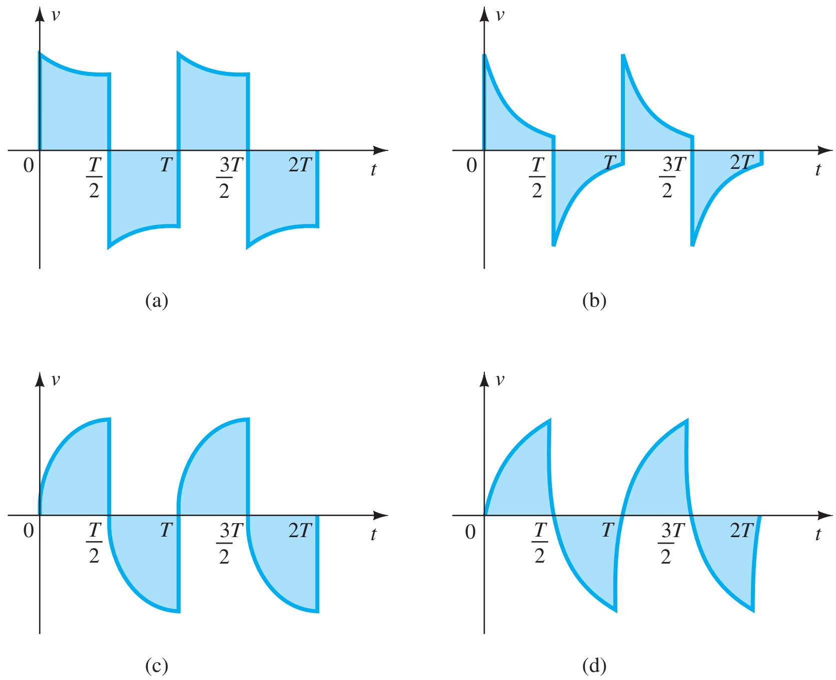

FIG. 9.61
(a) Poor low-frequency response; (b) very poor low-frequency response; (c) poor high-frequency response; (d) very poor high-frequency response.

The actual high-cutoff frequency (or BW) can be determined from the output waveform by carefully measuring the rise time defined between 10% and 90% of the peak value, as shown in Fig. 9.62. Substituting into the following equation will provide the upper cutoff frequency, and because $\text{BW} = f_{H_i} - f_{L_o} \cong f_{H_i}$, the equation also provides an indication of the BW of the amplifier:

$$\text{BW} \cong f_{H_i} = \frac{0.35}{t_r} \tag{9.79}$$

The low-cutoff frequency can be determined from the output response by carefully measuring the tilt of Fig. 9.62 and substituting into one of the following equations:

$$\% \text{ tilt} = P\% = \frac{V - V'}{V} \times 100\% \tag{9.80}$$

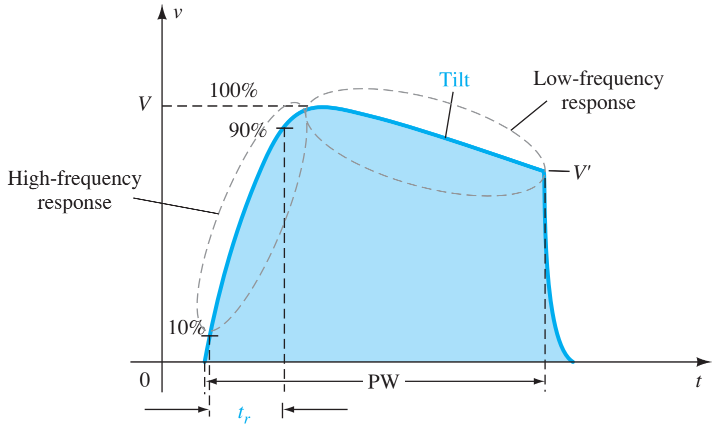

FIG. 9.62
Defining the rise time and tilt of a square wave response.

$$\text{tilt} = P = \frac{V - V'}{V} \quad \text{(decimal form)} \tag{9.81}$$

The low-cutoff frequency is then determined from

$$f_{L_o} = \frac{P}{\pi} f_s \tag{9.82}$$

EXAMPLE 9.16 The application of a 1-mV, 5-kHz square wave to an amplifier resulted in the output waveform of Fig. 9.63.

a. Write the Fourier series expansion for the square wave through the ninth harmonic.
b. Determine the bandwidth of the amplifier.
c. Calculate the low-cutoff frequency.

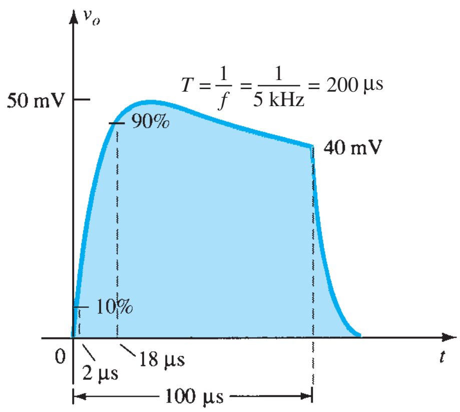

FIG. 9.63
Example 9.16.

Solution:

a. $$v_i = \frac{4\text{ mV}}{\pi}\left(\sin 2\pi\,(5 \times 10^3)t + \frac{1}{3}\sin 2\pi(15 \times 10^3)t + \frac{1}{5}\sin 2\pi(25 \times 10^3)t + \frac{1}{7}\sin 2\pi(35 \times 10^3)t + \frac{1}{9}\sin 2\pi(45 \times 10^3)t\right)$$

b. $t_r = 18\ \mu s - 2\ \mu s = 16\ \mu s$

$$\text{BW} \cong \frac{0.35}{t_r} = \frac{0.35}{16\ \mu\text{s}} = \mathbf{21{,}875\ Hz} \cong 4.4 f_s$$

c. $P = \dfrac{V - V'}{V} = \dfrac{50\text{ mV} - 40\text{ mV}}{50\text{ mV}} = 0.2$

$$f_{L_o} = \frac{P}{\pi} f_s = \left(\frac{0.2}{\pi}\right)(5\text{ kHz}) = \mathbf{318.31\ Hz}$$

9.15 SUMMARY

Important Conclusions and Concepts

1. The logarithm of a number gives the **power to which the base must be brought to obtain the same number.** If the base is 10, it is referred to as the **common logarithm;** if the base is $e = 2.71828\ldots$, it is called the **natural logarithm.**
2. Because the decibel rating of any piece of equipment is a **comparison between levels,** a reference level must be selected for each area of application. For audio systems the reference level is generally accepted as **1 mW.** When using voltage levels to determine the gain in dB between two points, any difference in resistance level is generally ignored.
3. The dB gain of cascaded systems is simply the **sum** of the dB gains of each stage.
4. It is the **capacitive elements** of a network that determine the **bandwidth** of a system. The **larger** capacitive elements of the basic design determine the **low**-cutoff frequency, whereas the **smaller** parasitic capacitors determine the **high**-cutoff frequencies.
5. The frequencies at which the gain drops to 70.7% of the midband value are called the **cutoff, corner, band, break,** or **half-power** frequencies.
6. The **narrower** the bandwidth, the **smaller** is the range of frequencies that will permit a transfer of power to the load that is at least 50% of the midband level.
7. A change in frequency by a factor of **two,** equivalent to **one octave,** results in a **6-dB change in gain.** For a **10:1** change in frequency, equivalent to **one decade,** there is a **20-dB change in gain.**
8. For any **inverting** amplifier, the input capacitance will be increased by a **Miller effect** capacitance determined by the **gain** of the amplifier and the **interelectrode** (parasitic) capacitance between the input and output terminals of the active device.
9. A **3-dB drop in beta** (h_{fe}) will occur at a frequency defined by f_β that is sensitive to the **dc operating conditions** of the transistor. This variation in beta can define the upper cutoff frequency of the design.
10. The **high- and low-cutoff frequencies** of an amplifier can be determined by the response of the system to a **square-wave input.** The general appearance will immediately reveal whether the low- or high-frequency response of the system is too limited for the applied frequency, whereas a more detailed examination of the response will reveal the actual bandwidth of the amplifier.

Equations

Logarithms:

$$a = b^x, \qquad x = \log_b a, \qquad \log_{10}\frac{a}{b} = \log_{10} a - \log_{10} b$$

$$\log_{10} ab = \log_{10} a + \log_{10} b, \qquad G_{\text{dB}} = 10\log_{10}\frac{P_2}{P_1} = 20\log_{10}\frac{V_2}{V_1}$$

$$G_{\text{dB}_T} = G_{\text{dB}_1} + G_{\text{dB}_2} + G_{\text{dB}_3} + \cdots + G_{\text{dB}_n}$$

Low-frequency response:

$$A_v = \frac{1}{1 - j(f_L/f)}, \qquad f_L = \frac{1}{2\pi RC}$$

BJT low-frequency response:

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s}, \qquad R_i = R_1 \| R_2 \| \beta r_e$$

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}, \qquad R_o = R_C \| r_o$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E}, \qquad R_e = R_E \| \left(\frac{R_s'}{\beta} + r_e\right), \qquad R_s' = R_s \| R_1 \| R_2$$

FET low-frequency response:

$$f_{L_G} = \frac{1}{2\pi(R_{\text{sig}} + R_i)C_G}, \qquad R_i = R_G$$

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}, \qquad R_o = R_D \| r_d$$

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S}, \qquad R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)} \cong R_S \left\| \frac{1}{g_m} \right|_{r_d \cong \infty \Omega}$$

Miller effect capacitance:

$$C_{M_i} = (1 - A_v)C_f, \qquad C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_f$$

BJT high-frequency response:

$$A_v = \frac{1}{1 + j(f/f_H)}, \qquad f_{H_i} = \frac{1}{2\pi R_{\text{Th}_i} C_i}, \qquad R_{\text{Th}_i} = R_s \| R_1 \| R_2 \| R_i,$$

$$C_i = C_{W_i} + C_{be} + C_{M_i}$$

$$f_{H_o} = \frac{1}{2\pi R_{\text{Th}_o} C_o}, \qquad R_{\text{Th}_o} = R_C \| R_L \| r_o, \qquad C_o = C_{W_o} + C_{ce} + C_{M_o},$$

$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}$$

$$f_\beta \cong \frac{1}{2\pi \beta_{\text{mid}} r_e (C_{be} + C_{bc})}$$

$$f_T \cong h_{fe_{\text{mid}}} f_\beta$$

FET high-frequency response:

$$f_{H_i} = \frac{1}{2\pi R_{\text{Th}_i} C_i}, \qquad R_{\text{Th}_i} = R_{\text{sig}} \| R_G, \qquad C_i = C_{W_i} + C_{gs} + C_{M_i},$$

$$C_{M_i} = (1 - A_v)C_{gd}$$

$$f_{H_o} = \frac{1}{2\pi R_{\text{Th}_o} C_o}, \qquad R_{\text{Th}_o} = R_D \| R_L \| r_d, \qquad C_o = C_{W_o} + C_{ds} + C_{M_o},$$

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_{gd}$$

Multistage effects:

$$f_L' = \frac{f_L}{\sqrt{2^{1/n} - 1}}, \qquad f_H' = (\sqrt{2^{1/n} - 1})f_H$$

Square-wave testing:

$$\text{BW} \cong f_{H_i} = \frac{0.35}{t_r}, \qquad f_{L_o} = \frac{P}{\pi} f_s, \qquad P = \frac{V - V'}{V}$$

9.16 COMPUTER ANALYSIS

The computer analysis of this section will verify the results of a number of examples appearing in this chapter.

Low-Frequency BJT Response

The network of Example 9.12 with its various capacitors appears in Fig. 9.64. The sequence **Edit-PSpice Model** was used to set I_s to 2E-15A and beta to 100. The remaining parameters of the **PSpice Model** for the transistor were removed to idealize the response to the greatest degree possible. In the **Simulation Settings** dialog box **AC Sweep/Noise** was selected under the **Analysis type** heading, and **Linear** was chosen under the **AC Sweep Type.** The **Start Frequency** was set at 10 kHz, the **End Frequency** at 10 kHz, and the

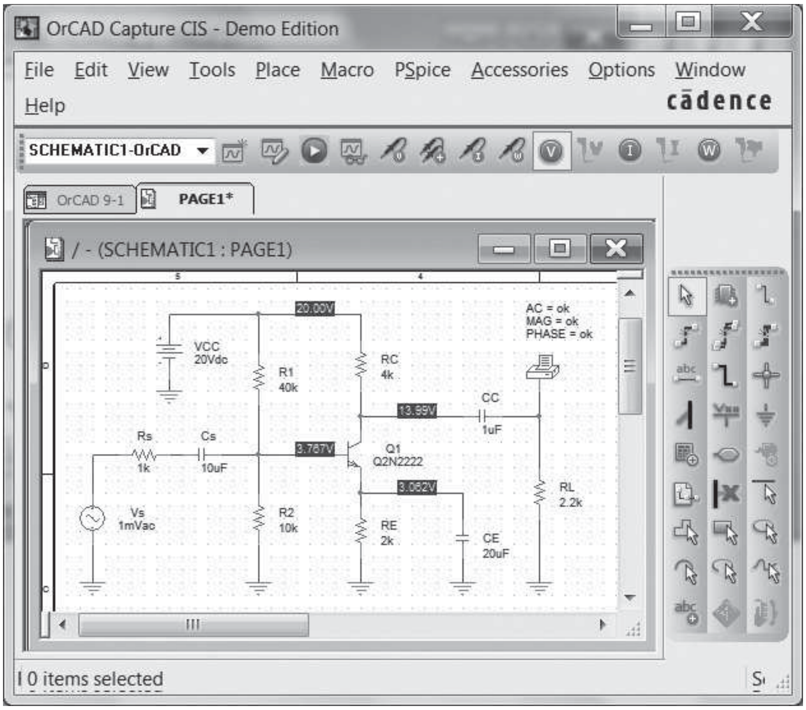

FIG. 9.64
Network of Fig. 9.32 with assigned values.

number of **Points** at 1. A **Simulation** resulted in the dc bias voltage levels of Fig. 9.64. Note that V_B is 3.767 V, compared to the calculated level of 4 V, and that V_E is 3.062 V, compared to the calculated level of 3.3 V. These values are very close when you consider that the approximate model was used to represent the transistor. The output file reveals that the ac voltage across the load at a frequency of 10 kHz is 49.69 mV, resulting in a gain of 49.69, which is very close to the calculated level of 51.21.

A plot of the gain versus frequency will now be obtained with only C_s as a determining factor. The other capacitors, C_C and C_E, will be set to very high values, so they are essentially short circuits at any of the frequencies of interest. Setting C_C and C_E to 1 F will remove any effect they might have on the response in the low-frequency region. Here, however, one must be careful because the program does not recognize 1 F as one farad. It must be entered as 1E6uF. Because the plot desired is gain versus frequency, we must set the **Simulation** to run through a range of frequencies, not as in the first **Simulation** where the frequency was fixed at 10 kHz. This is accomplished by first selecting the **New Simulation key**, giving the run a new **Name,** and proceeding to the **Simulation Settings** dialog box. Under **Analysis type, AC Sweep/Noise** is selected, and under **AC Sweep Type, Linear** is chosen, followed by a **Start Frequency** of 1 Hz, an **End Frequency** of 100 Hz, and **Points** set at 1000. The **Start Frequency** is set at 1 Hz because 0 Hz is an invalid entry. If one is really concerned about what happens between 0 Hz and 1 Hz, one could choose the start frequency as 0.001 Hz and work from there. However, 1 Hz is only 1/100 of the full scale and will be fine for this analysis. The **End Frequency** was selected as 100 Hz because we limit our interest to the low-frequency range. With 1000 points there will be sufficient data points to provide a smooth plot throughout the frequency range. Once **Simulation** is enacted followed by **Trace-Add Trace-V(RL:1)**, a plot appears extending to 120 Hz. Note also that the computer selected a log scale even though we called for a **Linear** plot. If we choose **Plot-Axis Settings-X-Axis-Linear,** we get a linear plot to 120 Hz, but the curve of interest is in the low end—the log axis obviously provided a better plot for our region of interest. Returning to **Plot-Axis Settings-X-Axis-Log** returns the original plot. Our interest lies in the region of 1 Hz to 50 Hz, so the remaining frequencies to 1 kHz should be removed with **Plot-Axis Settings-User Defined-1 Hz to 100 Hz-OK.** The vertical axis also goes to 60 mV, and we want to limit the range to 50 mV for this frequency range. This is accomplished through **Plot-Axis Settings-Y-Axis User Defined-0V to 50 mV-OK,** after which the plot of Fig. 9.65 will be obtained.

Note how closely the curve approaches 50 mV in this range. The cutoff level is determined by 0.707(49.69 mV) = 35.13 mV, which can be found using the **Cursor** option.

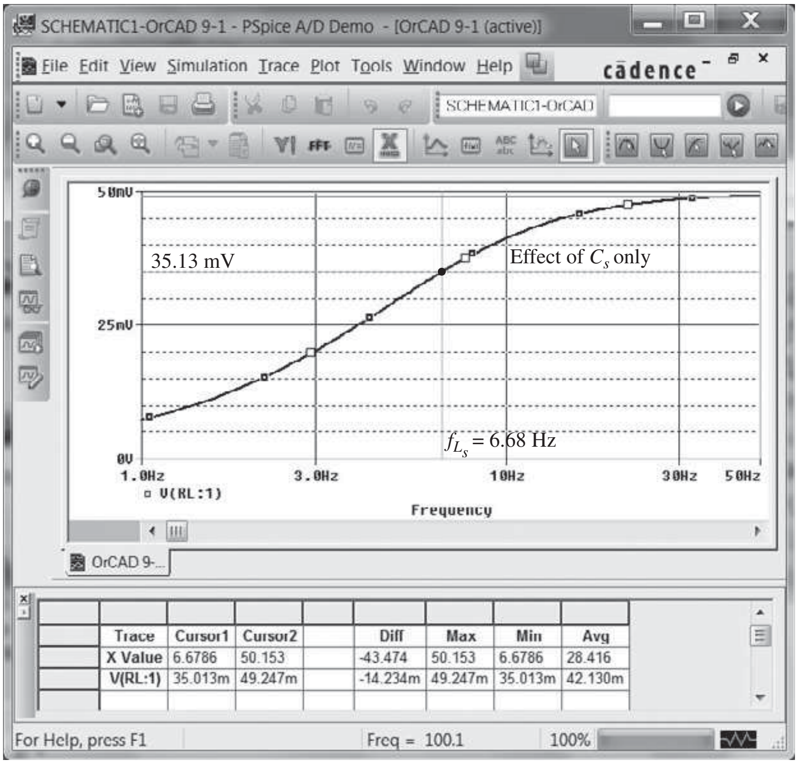

FIG. 9.65

Low-frequency response due to C_s.

Going to **Trace-Cursor** results in intersecting lines whose horizontal and vertical values at the intersection appear in the **Probe Cursor** box in the bottom right of the plot. Moving **Cursor 1** along the curve until we are as close to the 35.13-mV level as possible results in the intersection shown in Fig. 9.65 at 35.13 mV. Note that the corresponding frequency is 6.6786 Hz, which corresponds very closely to the predicted value of 6.69 Hz. **Cursor 2**

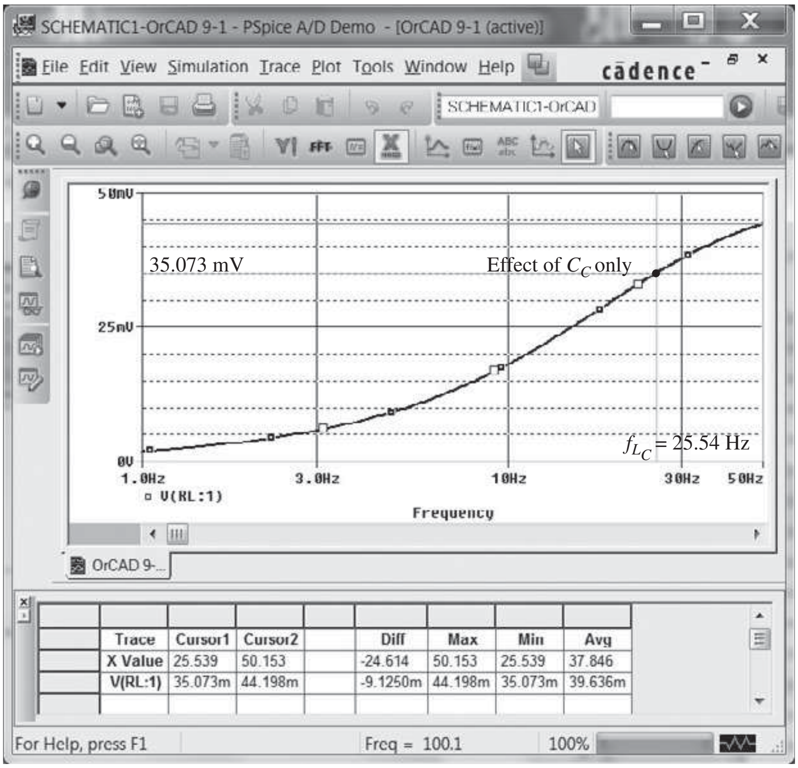

FIG. 9.66

Low-frequency response to C_C.

was placed close to 50 Hz to obtain a level of 49.247 mV. The labels were added using the **Tools-Label-Text** option.

To investigate the effects of C_C on the lower cutoff frequency, both C_S and C_E must be set to 1 F as described above. Following the procedure outlined above results in the plot of Fig. 9.66, with a cutoff frequency of 25.539 Hz, providing a close match with the calculated level of 25.68 Hz.

The effect of C_E can be examined using PSpice Windows by setting both C_s and C_C to 1 F. In addition, because the frequency range is greater, the start frequency has to be changed to 10 Hz and the final frequency to 1 kHz. The result is the plot of Fig. 9.67, with a cutoff frequency of 320 Hz, providing a close match with the calculated value of 327 Hz.

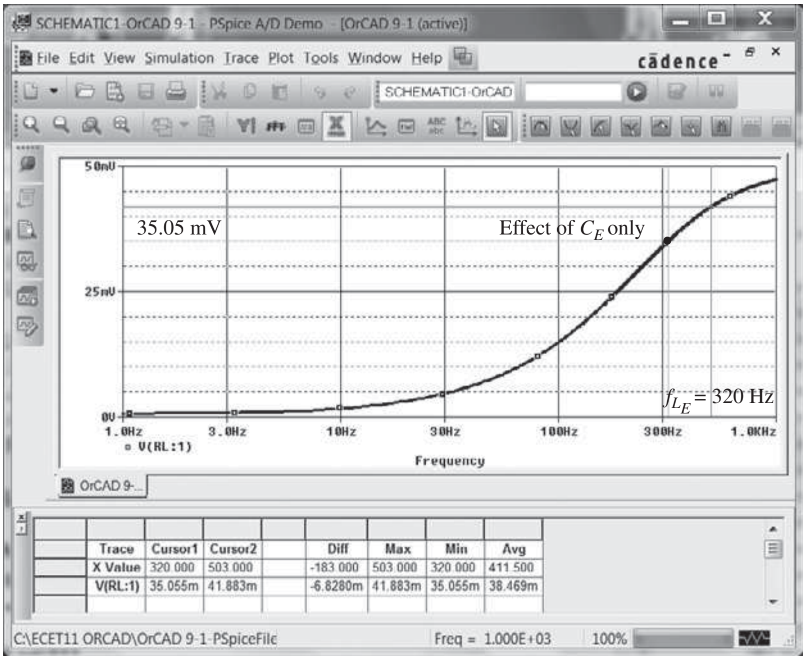

FIG. 9.67
Low-frequency response due to C_E.

The fact that f_{L_E} is significantly higher than f_{L_S} or f_{L_C} suggests that it will be the predominant factor in determining the low-frequency response for the complete system. To test the accuracy of our hypothesis, the network is simulated with all the initial values of capacitance level to obtain the results of Fig. 9.68. Note the strong similarity with the waveform of Fig. 9.67, with the only visible difference being the higher gain at lower frequencies on Fig. 9.67. Without question, the plot supports the fact that the highest of the low cutoff frequencies will have the most impact on the low cutoff frequency for the system. The result is that $f_L \cong 327$ Hz.

A dB plot of the low-frequency response can be obtained by creating a **Simulation** for the frequency range and then, when the **Add Traces** dialog box appears, creating the desired **Trace Expression** using the provided listings. For a plot of $20 \log_{10}|A_v/A_{v_{\text{mid}}}|$ the ratio $A_v/A_{v_{\text{mid}}}$ can also be written as $(V_o/V_i)/(V_{o_{\text{mid}}}/V_i) = V_o/V_{o_{\text{mid}}}$, resulting in the following expression for the dB gain:

$$20 \log_{10} |A_v/A_{v_{\text{mid}}}| = 20 \log_{10} |V_o/V_{o_{\text{mid}}}| = \text{dB}(V_o/V_{o_{\text{mid}}}) = \text{dB}(V_{R_L}/49.7 \text{ mV})$$

The **Trace Expression** can be created by first selecting **DB** from the **Function** list and then selecting **V(RL:1)** from the **Simulation Output Variable** list. Note that the second selection will appear within the parentheses of the first. Then be sure to enter the division sign and the number 0.0497 V = 49.7 mV within the parentheses. Of course, the entire expression can be written directly if you prefer not to use the listings. Once the expression is properly written, select **OK** and the plot of Fig. 9.69 will result. The plot clearly reveals the change in slope of the asymptote at f_{L_C} and how the actual curve follows the envelope created by the Bode plot. In addition, note the 3-dB drop at f_L.

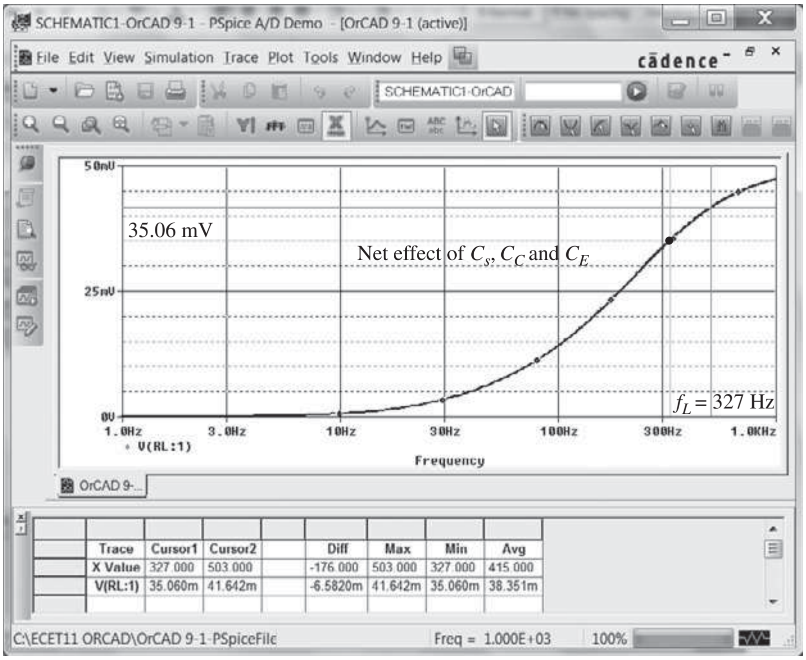

FIG. 9.68

Low-frequency response due to C_S, C_E, and C_C.

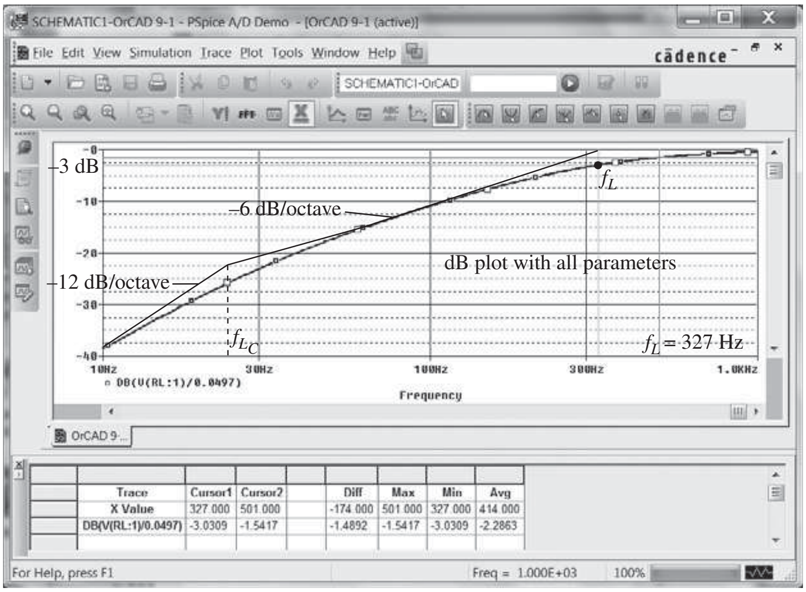

FIG. 9.69

dB plot of the low-frequency response of the BJT amplifier of Fig. 9.32.

Low-Frequency JFET Response

PSpice Applying PSpice to the network of Fig. 9.37 results in the display of Fig. 9.70. The JFET parameters were set at **Beta** = 0.5 mA/V^2 and **Vto** at −4 V with all other parameters in the model listing deleted. The frequency of interest is 10 kHz. The resulting dc levels confirm that V_{GS} is −2 V with V_D at 10.60 V, which should be in the middle of the linear active region because $V_{GS} = \frac{1}{2} V_D$ and $V_{DS} = \frac{1}{2} V_{DD}$. The ac response reveals that the output voltage is 2.993 mV for a gain of 2.993, which is essentially equal to the calculated gain of 3.

If we establish a **New Simulation** and set the **Analysis type** to **AC Sweep/Noise,** we can generate a plot for the low-frequency region. The **Start Frequency** is set at 10 Hz, the

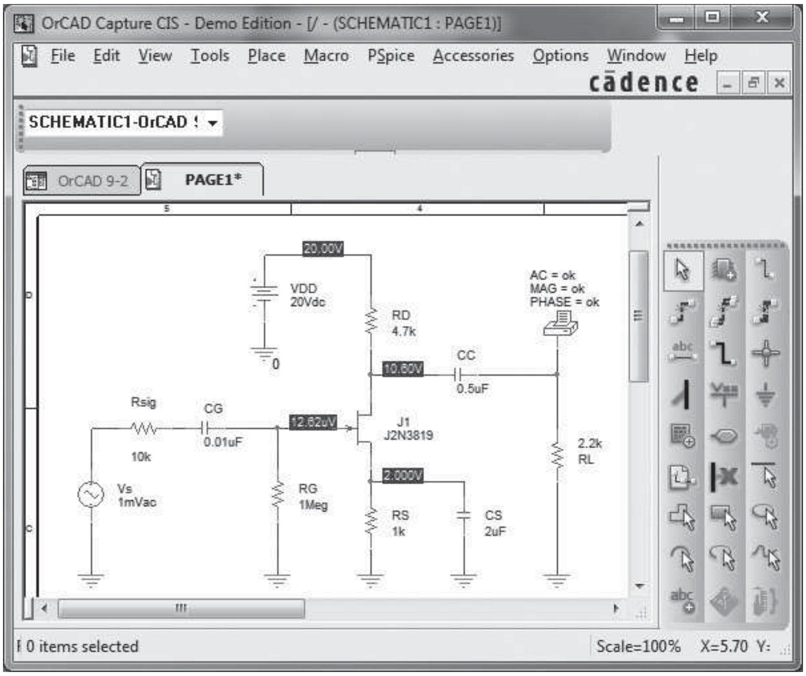

FIG. 9.70
Schematic network for Example 9.13.

End Frequency at 10 kHz, and the number of **Points** at 1000. The sequence **Simulation-Trace-Add Trace** then permits establishing the **Trace Expression DB(V(RL:1)/2.993 mV),** which, following an **OK,** results in the plot of Fig. 9.71. The low cutoff frequency of 221.29 Hz was primarily determined by the capacitance **CS.**

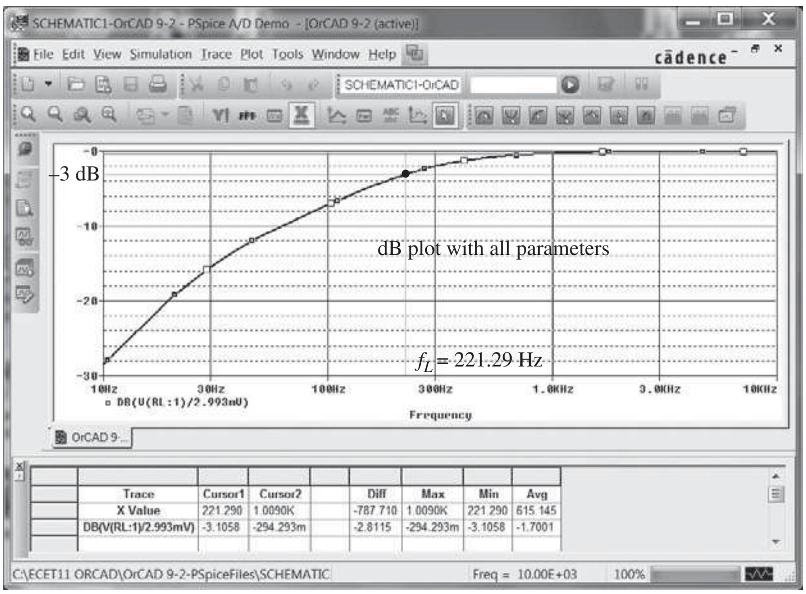

FIG. 9.71
dB response for the low-frequency region in the network of Example 9.13.

Multisim Multisim can also provide a frequency plot of the gain and phase response of a BJT or a JFET network by first constructing the network or calling it up from storage. Because the network of Fig. 9.70 is the same as that analyzed using Multisim in Chapter 8,

Fig. 8.63 is retrieved and displayed as Fig. 9.72 with its dc levels at the drain and source terminals. Next the sequence **Simulate-Analyses-AC Analysis** is applied to obtain the **AC Analysis** dialog box. Under **Frequency Parameters,** the **Start frequency** is selected as **10 Hz** and the **Stop frequency** as **10 kHz** to match the plot of Fig. 9.71. The **Sweep type** is left at the default selection of **decade,** and the **Number of points** per decade is also left at **100.** Finally, the vertical scale is set in the linear mode because it is the magnitude of the output voltage versus frequency rather than the dB gain as in Fig. 9.71.

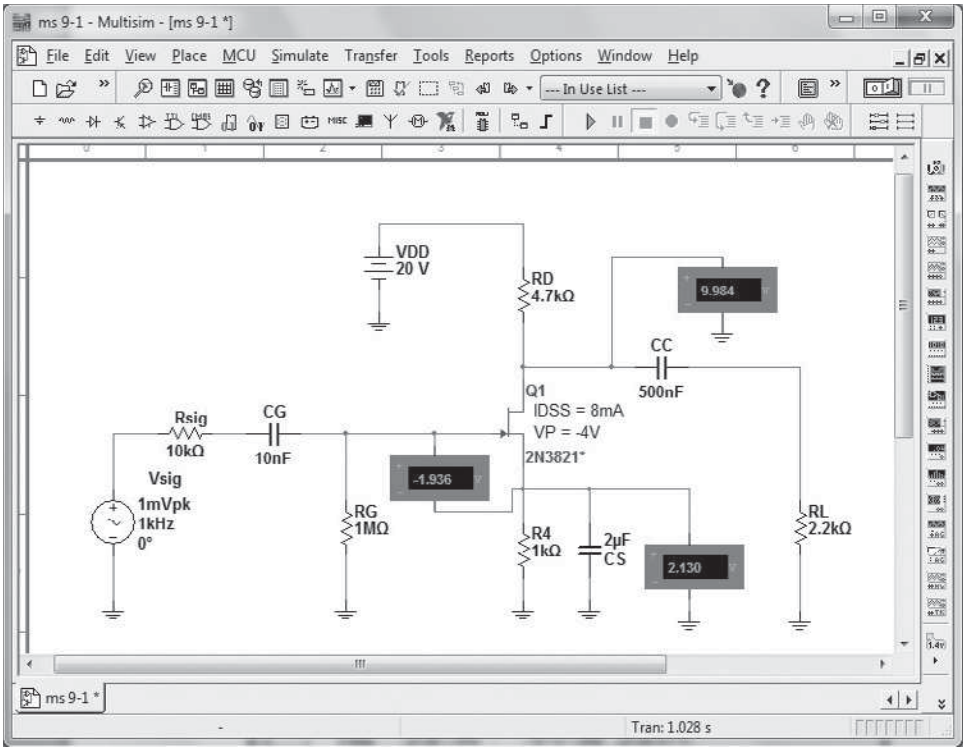

FIG. 9.72
Examining the network of Fig. 9.37 (Example 9.13) using Multisim.

Next, **Output variables** are selected in the dialog box. Under the heading **Variables in circuit,** select **Voltage** to reduce the number of options. Because we want a plot of the output voltage versus frequency, we select **$24** under **Variables in circuit,** followed by **Add** to place it in the **Selected variables for analysis.** We then choose **Simulate,** and the plot of Fig. 9.73 results.

At first, the plot may appear without a grid structure to help define the levels at each frequency. This is corrected by the sequence **View-Show/Hide Grid** as shown in Fig. 9.73. Always be aware that the red arrow along the left vertical column defines the plot under review. To add the grid to the phase plot, simply click on the lower graph at any point, and the red arrow will drop down. Then follow with the same sequence as above to establish the grid structure. If you want the graph to fill the entire screen, simply select the full-screen option at the top right corner of the **Analysis Graphs.**

Finally, cursors can be added to define the level of the plotted function at any frequency. Simply select **View-Show/Hide Cursors,** and the cursors will appear on the selected graph (which is the magnitude plot in Fig. 9.73). Then click on cursor 1, and the **AC Analysis** dialog box on the screen will reveal the level of the voltage and the frequency. By clicking on cursor 1 and moving it to the right, we can find an **x1** value of 227.65 to match the −3-dB point of Fig. 9.71. At this frequency the output voltage **(y1)** is 2.41 V, which is very close to the 0.707 level of the 2.93 gain (actually 2.07 V) obtained in Chapter 8. Cursor 2 was moved to an **x2** value of 10 kHz to obtain a voltage of 3.67 V. Before leaving Fig. 9.73, note that the higher the frequency, the closer is the phase shift to 180° as the relatively large, low-frequency capacitors lose their effect.

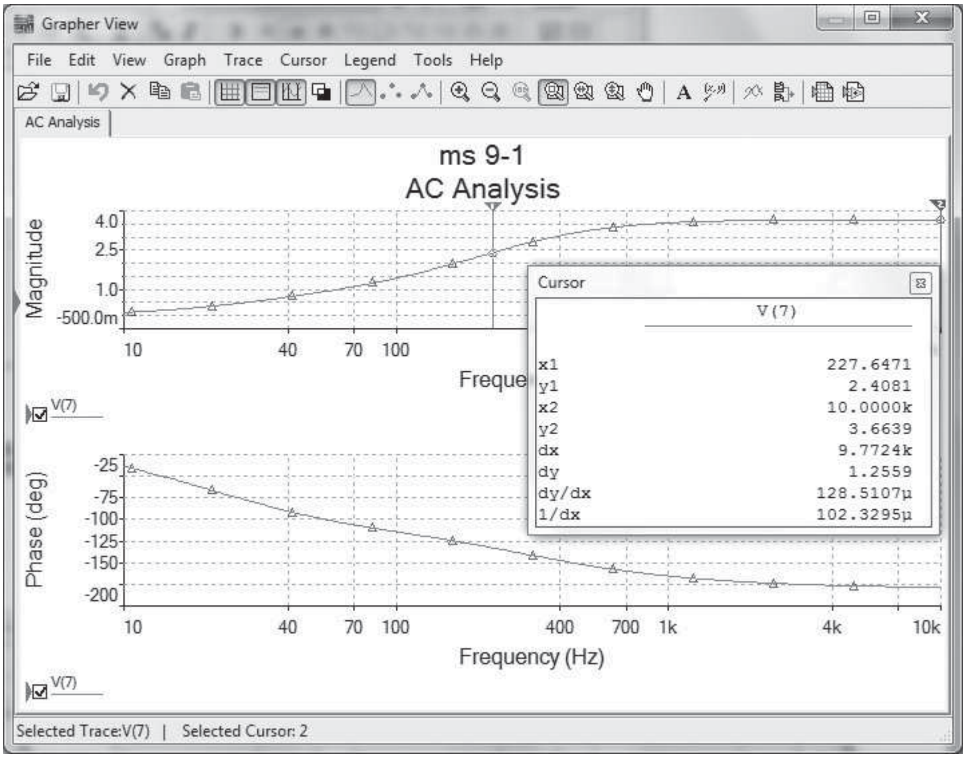

FIG. 9.73
Multisim plot for Example 9.13.

Full-Frequency BJT Response

PSpice To obtain a PSpice analysis for the full frequency range for the network of Fig. 9.32, the parasitic capacitances have been added to the network as shown in Fig. 9.74.

An **Analysis** will result in the plot of Fig. 9.75 using the **Trace Expression** appearing at the bottom of the plot. The vertical scale was changed from −60 to 0 dB to −30 to 0 dB to highlight the area of interest using the **Y-Axis Settings.** The low-cutoff frequency

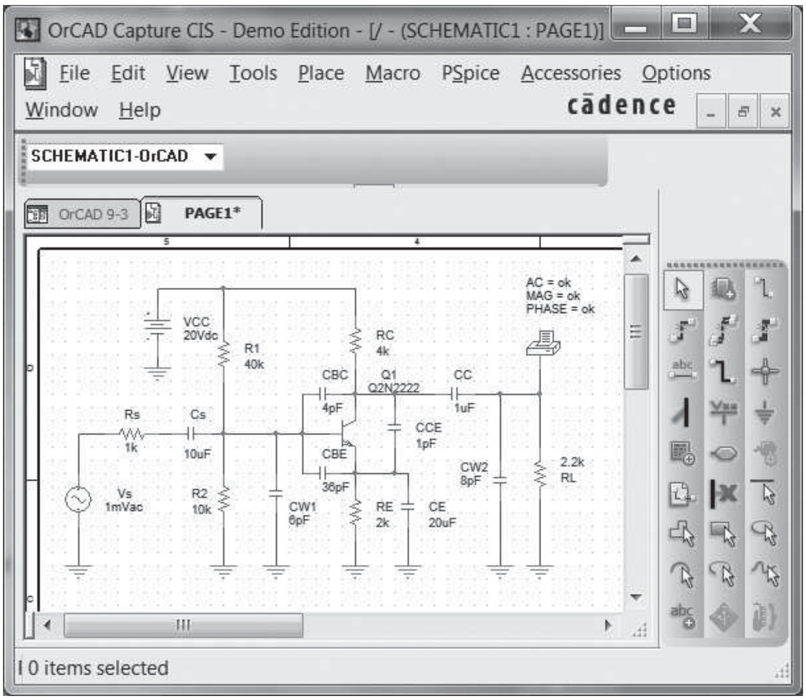

FIG. 9.74
Network of Fig. 9.32 with parasitic capacitances in place.

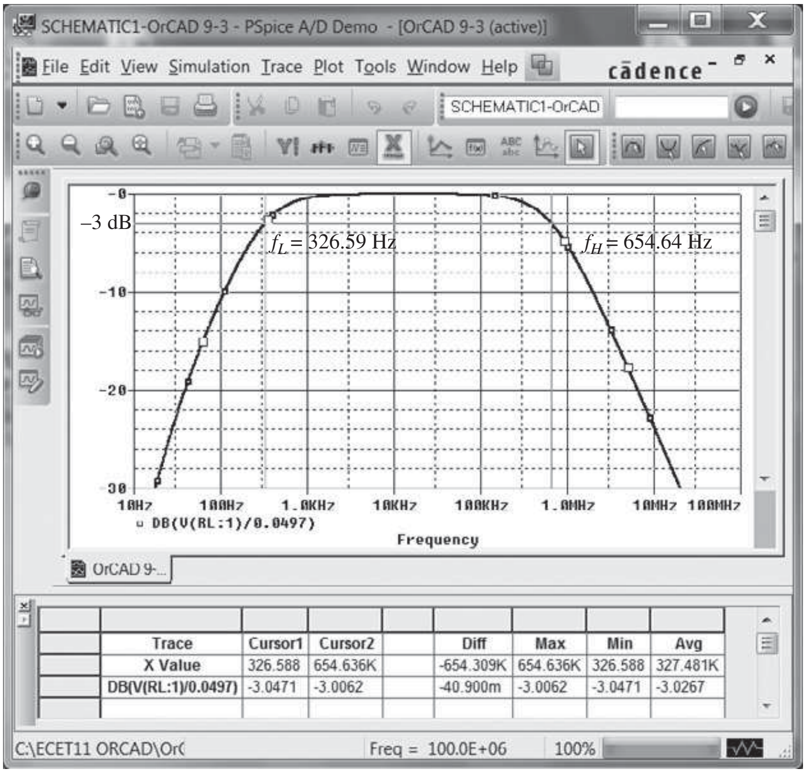

FIG. 9.75

Full frequency response for the network of Fig. 9.74.

of 326.59 Hz is as determined primarily by f_{L_E}, and the high-cutoff frequency is near 654.64 kHz. Even though f_{H_o} is more than a decade higher than f_{H_i}, it will have an effect on the high-cutoff frequency. In total, however, the PSpice analysis is a welcome verification of the handwritten approach.

Full-Frequency JFET Response

PSpice The schematic for the network of Fig. 9.55 appears as shown in Fig. 9.76 with the parasitic capacitances in place.

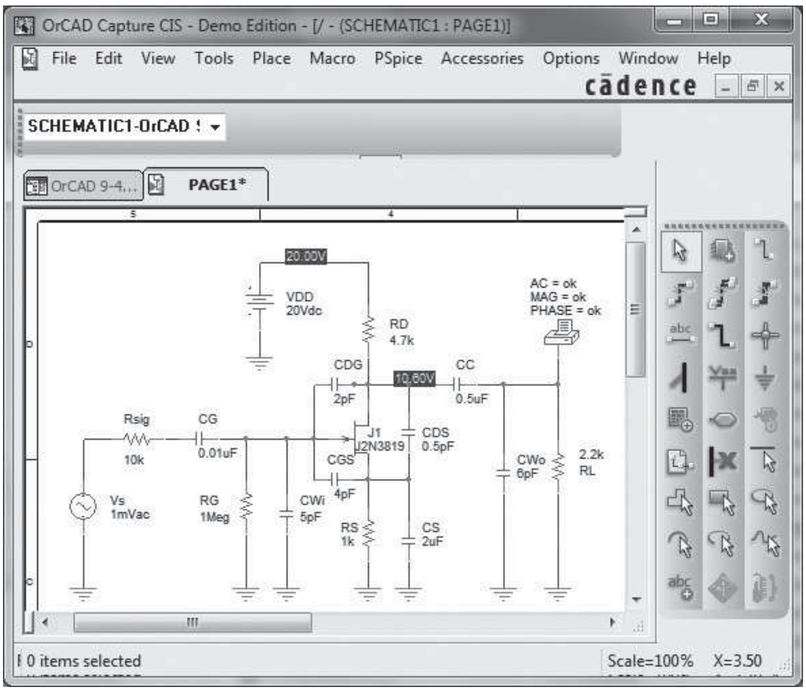

FIG. 9.76

Network of Fig. 9.55 with assigned values.

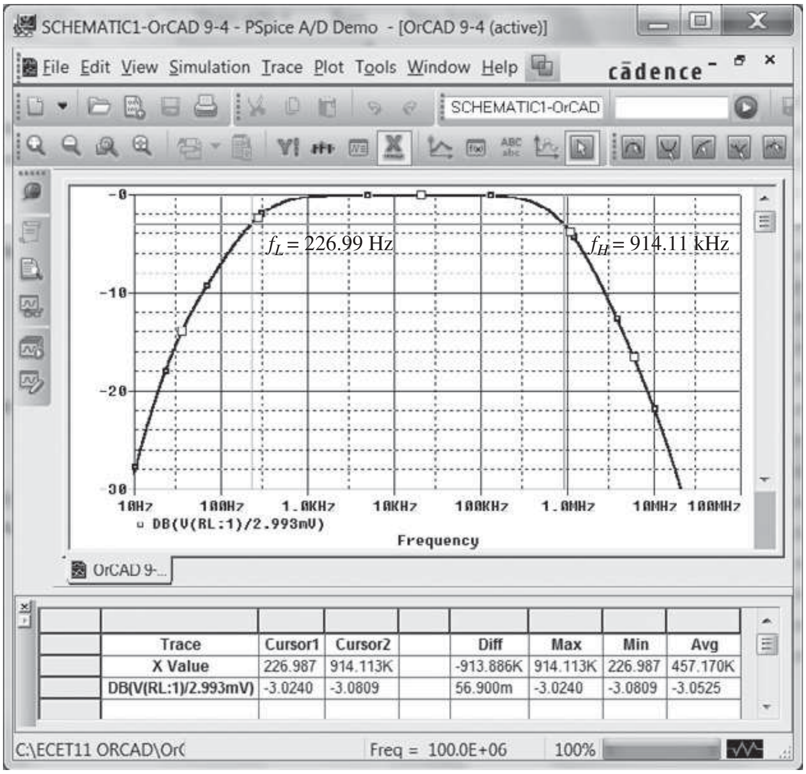

FIG. 9.77
Frequency response for the network of Example 9.15.

For the full frequency response the **Start Frequency** is set at 10 Hz and the **End Frequency** at 10 MHz, and 1000 **Points** is selected. The **Trace Expression** is set as **DB(V(RL:1)/2.993 mV)** to obtain the plot of Fig. 9.77. Consider how much time it would take to sketch the curve of Fig. 9.77 using a handheld calculator. We often forget how computer methods can save us an enormous amount of time.

Using the cursor, we find the lower and upper cutoff frequencies to be 226.99 Hz and 914.11 kHz, respectively, providing a nice match with the calculated values.

PROBLEMS

**Note*: Asterisks indicate more difficult problems.

9.2 Logarithms

1. **a.** Determine the common logarithm of the following numbers: 10^3, 50, and 0.707.
   b. Determine the natural logarithm of the numbers appearing in part (a).
   c. Compare the solutions of parts (a) and (b).
2. **a.** Determine the common logarithm of the number 0.24×10^6.
   b. Determine the natural logarithm of the number of part (a) using Eq. (9.4).
   c. Determine the natural logarithm of the number of part (a) using natural logarithms and compare with the solution of part (b).
3. Determine:
   a. $20 \log_{10}(\frac{84}{6})$ using Eq. (9.6) and compare with $20 \log_{10} 14$.
   b. $10 \log_{10}(\frac{1}{250})$ using Eq. (9.7) and compare with $10 \log_{10} 4 \times 10^{-3}$.
   c. $\log_{10}(40)(0.2)$ using Eq. (9.8) and compare with $\log_{10} 8$.
4. Calculate the power gain in decibels for each of the following cases.
   a. $P_o = 100$ W, $P_i = 5$ W.
   b. $P_o = 100$ mW, $P_i = 5$ mW.
   c. $P_o = 100$ mW, $P_i = 20\ \mu$W.
5. Determine G_{dBm} for an output power level of 25 W.
6. Two voltage measurements made across the same resistance are $V_1 = 110$ V and $V_2 = 220$ V. Calculate the power gain in decibels of the second reading over the first reading.

7. Input and output voltage measurements of $V_i = 10$ mV and $V_o = 25$ V are made. What is the voltage gain in decibels?

***8.** **a.** The total decibel gain of a three-stage system is 120 dB. Determine the decibel gain of each stage if the second stage has twice the decibel gain of the first and the third has 2.7 times the decibel gain of the first.
b. Determine the voltage gain of each stage.

***9.** If the applied ac power to a system is 5 μW at 100 mV and the output power is 48 W, determine:
a. The power gain in decibels.
b. The voltage gain in decibels if the output impedance is 40 kΩ.
c. The input impedance.
d. The output voltage.

9.4 General Frequency Considerations

10. Given the characteristics of Fig. 9.78, sketch:
a. The normalized gain.
b. The normalized dB gain (and determine the bandwidth and cutoff frequencies).

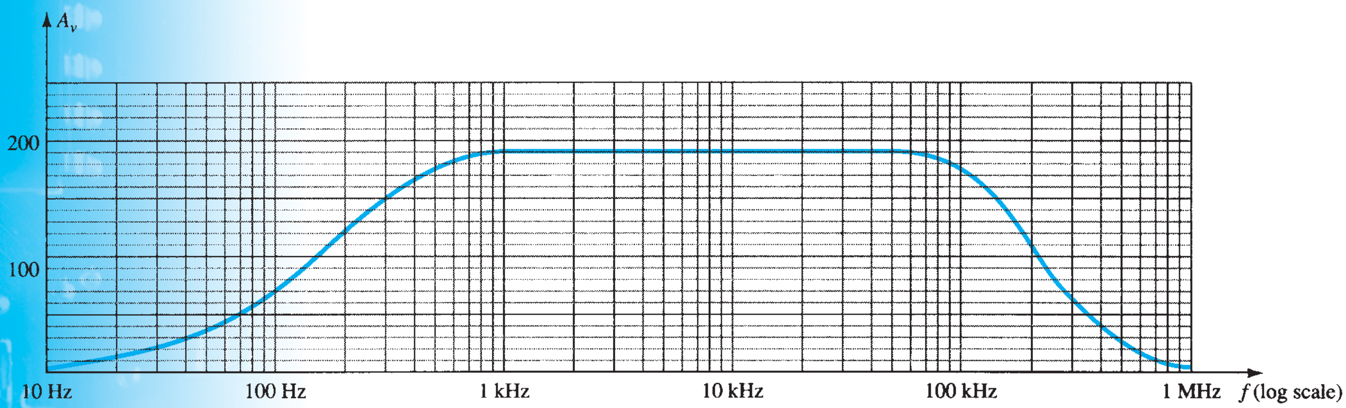

FIG. 9.78
Problem 10.

9.6 Low-Frequency Analysis—Bode Plot

11. For the network of Fig. 9.79:
a. Determine the mathematical expression for the magnitude of the ratio V_o/V_i.
b. Using the results of part (a), determine V_o/V_i at 100 Hz, 1 kHz, 2 kHz, 5 kHz, and 10 kHz, and plot the resulting curve for the frequency range of 100 Hz to 10 kHz. Use a log scale.
c. Determine the break frequency.
d. Sketch the asymptotes and locate the −3-dB point.
e. Sketch the frequency response for V_o/V_i and compare to the results of part (b).

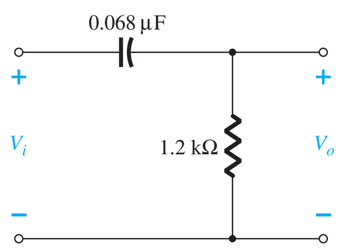

FIG. 9.79
Problems 11, 12, and 37.

12. For the network of Fig. 9.79:
a. Determine the mathematical expression for the angle by which V_o leads V_i.
b. Determine the phase angle at $f = 100$ Hz, 1 kHz, 2 kHz, 5 kHz, and 10 kHz, and plot the resulting curve for the frequency range of 100 Hz to 10 kHz.
c. Determine the break frequency.
d. Sketch the frequency response of θ for the frequency spectrum of part (b) and compare results.

13. **a.** What frequency is one octave above 5 kHz?
b. What frequency is one decade below 10 kHz?
c. What frequency is two octaves below 20 kHz?
d. What frequency is two decades above 1 kHz?

9.7 Low-Frequency Response—BJT Amplifier with R_L

14. Repeat the analysis of Example 9.11 with $r_o = 40\ \text{k}\Omega$. What is the effect on $A_{v_{\text{mid}}}$, f_{L_S}, f_{L_C}, f_{L_E}, and the resulting cutoff frequency?

15. For the network of Fig. 9.80:
a. Determine r_e.
b. Find $A_{v_{\text{mid}}} = V_o/V_i$.
c. Calculate Z_i.
d. Determine f_{L_S}, f_{L_C}, and f_{L_E}.
e. Determine the low cutoff frequency.
f. Sketch the asymptotes of the Bode plot defined by the cutoff frequencies of part (d).
g. Sketch the low-frequency response for the amplifier using the results of part (e).

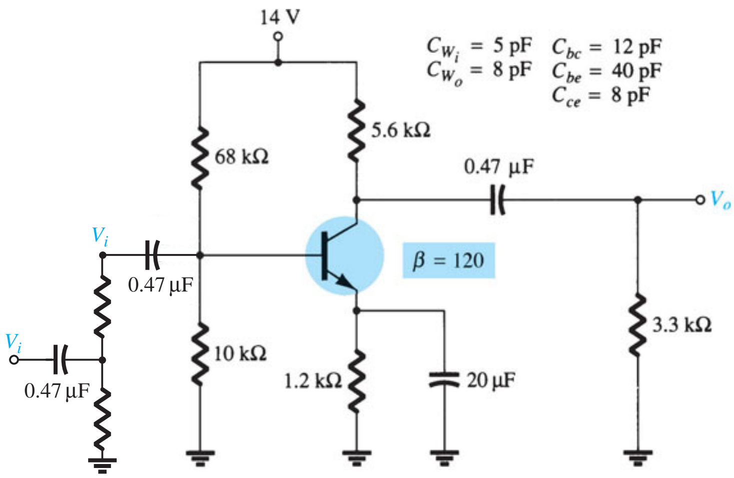

FIG. 9.80
Problems 15, 19, 27, and 38.

***16.** Repeat Problem 15 for the emitter-stabilized network of Fig. 9.81.

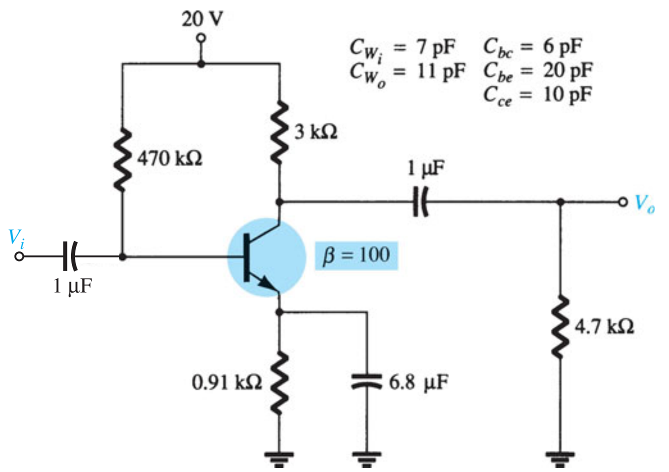

FIG. 9.81
Problems 16, 20, and 28.

***17.** Repeat Problem 15 for the emitter-follower network of Fig. 9.82.

***18.** Repeat Problem 15 for the common-base configuration of Fig. 9.83. Keep in mind that the common-base configuration is a noninverting network when you consider the Miller effect.

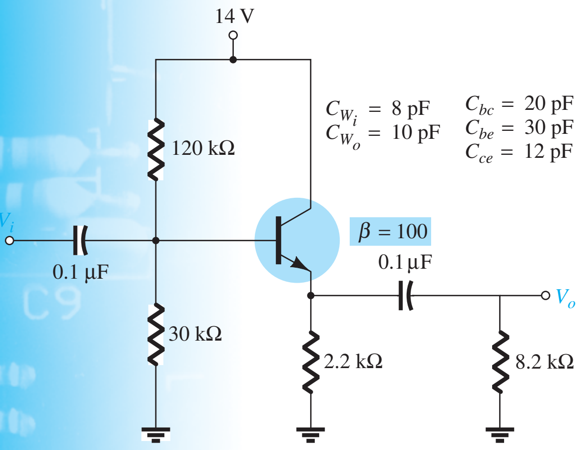

FIG. 9.82
Problems 17, 21, and 29.

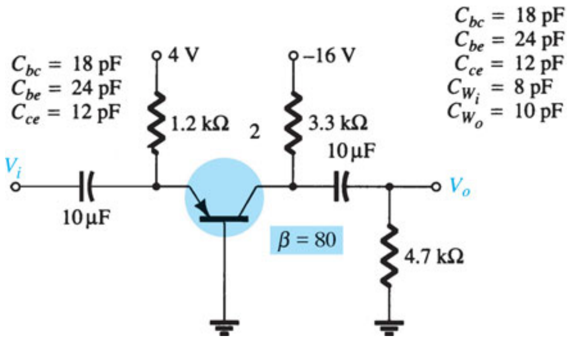

FIG. 9.83
Problems 18, 22, and 39.

9.8 Impact of R_s on the BJT Low-Frequency Response

19. Repeat the analysis of problem 15 for the network of Fig. 9.80 with the addition of a source resistance and signal source as shown in Fig. 9.84. Plot the gain $A_{v_s} = \frac{V_o}{V_s}$ and comment on the change in low-frequency cutoff as compared to problem 15.

20. Repeat the analysis of problem 15 for the network of Fig. 9.81 with the addition of a source resistance and signal source as shown in Fig. 9.85. Plot the gain $A_{v_s} = \frac{V_o}{V_s}$ and comment on the change in low-frequency cutoff as compared to problem 16.

21. Repeat the analysis of problem 15 for the network of Fig. 9.82 with the addition of a source resistance and signal source as shown in Fig. 9.86. Plot the gain $A_{v_s} = \frac{V_o}{V_s}$ and comment on the change in low-frequency cutoff as compared to problem 17.

22. Repeat the analysis of problem 15 for the network of Fig. 9.83 with the addition of a source resistance and signal source as shown in Fig. 9.87. Plot the gain $A_{v_s} = \frac{V_o}{V_s}$ and comment on the change in low-frequency cutoff as compared to problem 18.

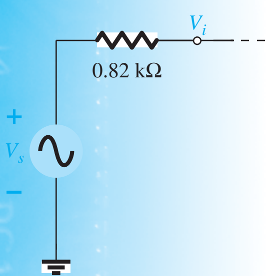

FIG. 9.84
Modification of Fig. 9.80. Problem 19.

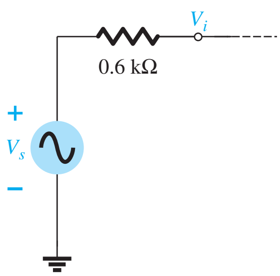

FIG. 9.85
Modification of Fig. 9.81. Problem 20.

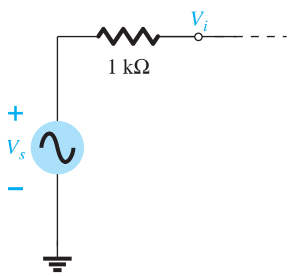

FIG. 9.86
Modification of Fig. 9.82. Problem 21.

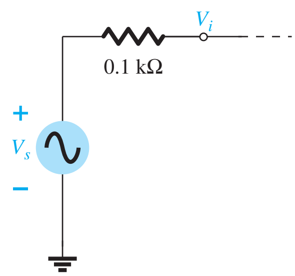

FIG. 9.87
Modification of Fig. 9.83. Problem 22.

9.9 Low-Frequency Response—FET Amplifier

23. For the network of Fig. 9.88:
- **a.** Determine V_{GS_Q} and I_{D_Q}.
- **b.** Find g_{m0} and g_m.
- **c.** Calculate the midband gain of $A_v = V_o/V_i$.
- **d.** Determine Z_i.
- **e.** Calculate $A_{v_s} = V_o/V_s$.
- **f.** Determine f_{L_G}, f_{L_C}, and f_{L_S}.
- **g.** Determine the low-cutoff frequency.
- **h.** Sketch the asymptotes of the Bode plot defined by part (f).
- **i.** Sketch the low-frequency response for the amplifier using the results of part (f).

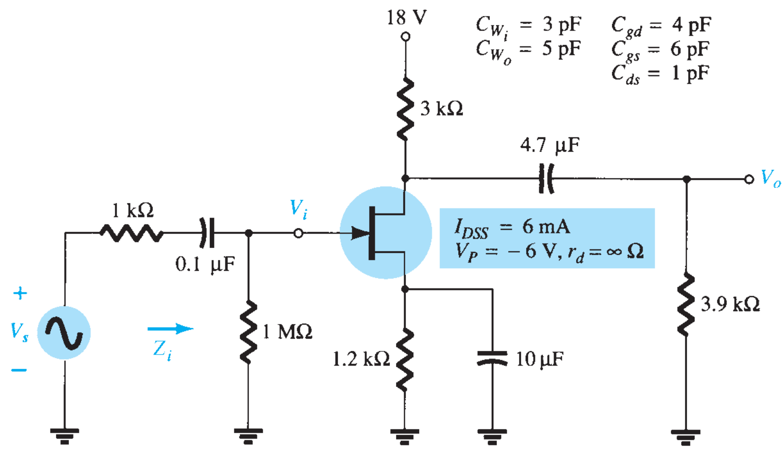

FIG. 9.88
Problems 23, 24, 31, and 40.

***24.** Repeat the analysis of Problem 23 with $r_d = 100\ \text{k}\Omega$. Does it have an effect of any consequence on the results? If so, which elements?

***25.** Repeat the analysis of Problem 23 for the network of Fig. 9.89. What effect does the voltage-divider configuration have on the input impedance and the gain A_{v_s} compared to the biasing arrangement of Fig. 9.88?

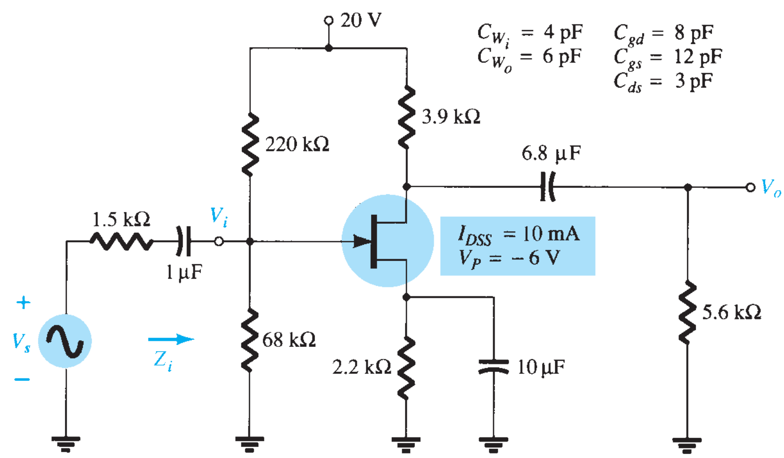

FIG. 9.89
Problems 25 and 32.

9.10 Miller Effect Capacitance

26. **a.** The feedback capacitance of an inverting amplifier is 10 pF. What is the Miller capacitance at the input if the gain of the amplifier is −120?
b. What is the Miller capacitance at the output of the amplifier?
c. Is it a good approximation to assume $C_{M_i} \cong |A_v|C_f$ and $C_{M_o} \cong C_f$?

9.11 High-Frequency Response—BJT Amplifier

***27.** For the network of Fig. 9.80 with R_s and V_s of Fig. 9.84:
a. Determine f_{H_i} and f_{H_o}.
b. Find f_β and f_T.
c. Sketch the frequency response for the high-frequency region using a Bode plot and determine the cutoff frequency.
d. What is the gain-bandwidth product of the amplifier?

***28.** Repeat the analysis of Problem 27 for the network of Fig. 9.81 with R_s and V_s of Fig. 9.85.

***29.** Repeat the analysis of Problem 27 for the network of Fig. 9.82 with R_s and V_s of Fig. 9.86.

***30.** Repeat the analysis of Problem 27 for the network of Fig. 9.83 with R_s and V_s of Fig. 9.87.

9.12 High-Frequency Response—FET Amplifier

31. For the network of Fig. 9.88:

a. Determine g_{m0} and g_m.

b. Find A_v and A_{v_s} in the mid-frequency range.

c. Determine f_{H_i} and f_{H_o}.

d. Sketch the frequency response for the high-frequency region using a Bode plot and determine the cutoff frequency.

e. What is the gain-bandwidth product of the amplifier?

***32.** Repeat the analysis of Problem 31 for the network of Fig. 9.89.

9.13 Multistage Frequency Effects

33. Calculate the overall voltage gain of four identical stages of an amplifier, each having a gain of 20.

34. Calculate the overall upper 3-dB frequency for a four-stage amplifier having an individual stage value of $f_2 = 2.5$ MHz.

35. A four-stage amplifier has a lower 3-dB frequency for an individual stage of $f_1 = 40$ Hz. What is the value of f_1 for this full amplifier?

9.14 Square-Wave Testing

***36.** The application of a 10-mV, 100-kHz square wave to an amplifier resulted in the output waveform of Fig. 9.90.

a. Write the Fourier series expansion for the square wave through the ninth harmonic.

b. Determine the bandwidth of the amplifier to the accuracy available by the waveform of Fig. 9.90.

c. Calculate the low-cutoff frequency.

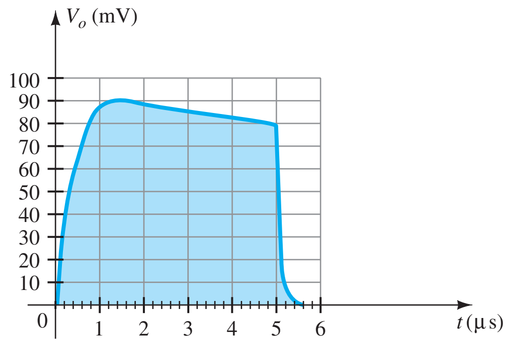

FIG. 9.90
Problem 36.

9.16 Computer Analysis

37. Using PSpice Windows, determine the frequency response of V_o/V_i for the high-pass filter of Fig. 9.45 of $R = 8.2\ \text{k}\Omega$ and $C = 4.7\ \mu\text{F}$.

38. Using PSpice Windows, determine the frequency response of V_o/V_s for the BJT amplifier of Fig. 9.87.

39. Repeat Problem 38 for the network of Fig. 9.83 using Multisim.

40. Repeat Problem 38 for the JFET configuration of Fig. 9.88 using Multisim.