وزارة التعليم العالي والبحث العلمي

UNIVERSITY OF ANBAR

جامعة الانبار
كلية علوم الحاسوب وتكنولوجيا المعلومات
قسم علوم الحاسبات

Department	علوم الحاسبات	القسم:
Subject Name:	**Logic Design**	أسم المادة :
Year of Study:	**2025-2024**	السنة الدراسية:
Course:	الكورس الاول	الكورس:
Title and No of lecture:	**Lecture 8:Karnaugh Map Methods I**	عنوان ورقم المحاضرة:
Instructor Name:	د. مصطفى معد حمدي	أسم التدريسي:

LECTURE EIGHT

KARNAUGH MAP METHODS I

Objectives:

1. Karnaugh Map definition.
2. Karnaugh map construction.
    - Two variables K-maps.
    - Three variables K-maps.
    - Four variables K-maps.
3. Summary

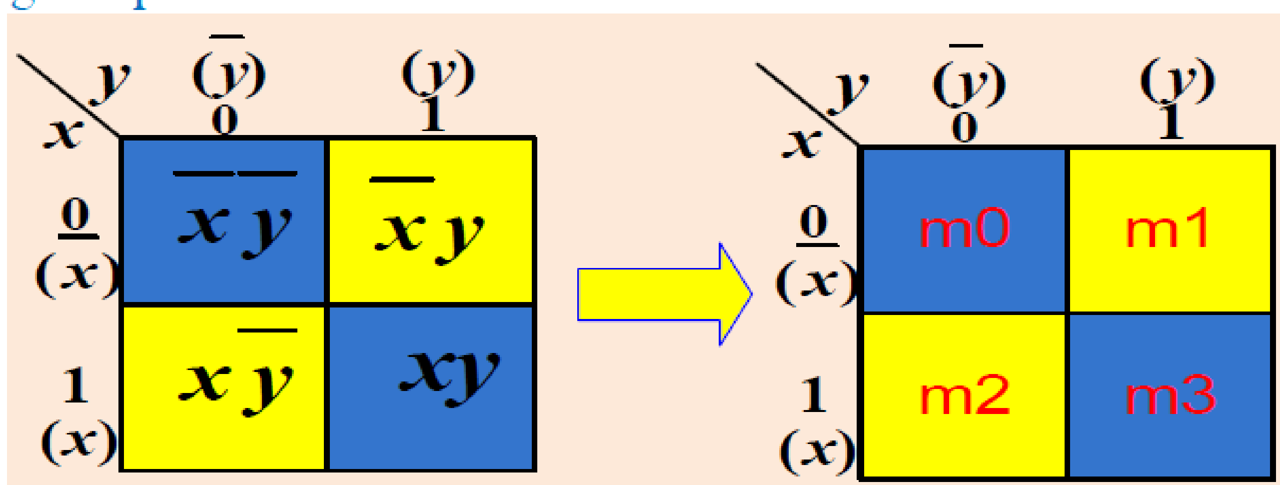
1) Karnaugh map definition

- The karnaugh map (k-map) is a ***graphical tool*** used to simplify (minimize) the logic functions, so that it can be implemented with minimum number of gates (minimum number of product terms and minimum number of literals).
- The K-map is used to ***convert*** a truth table to its corresponding logic circuit.

2) Karnaugh map construction

- **Two variables K-maps.**
  - A two- variable function has four possible minterms, we can rearrange these minterms into a karnaugh map:

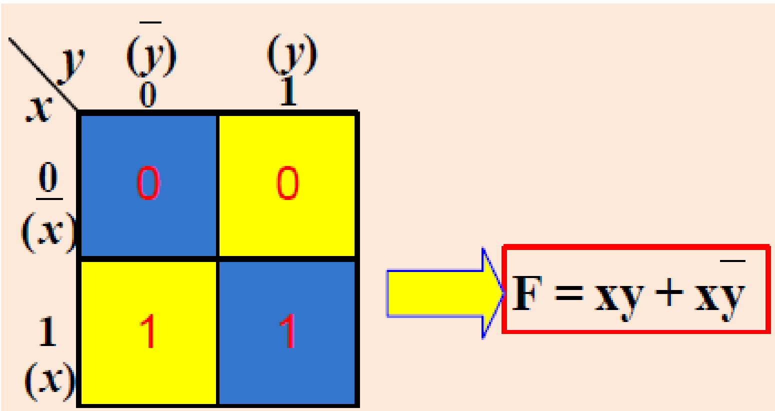
x	y	Minterms
0	0	$\bar{x}\bar{y}$
0	1	$\bar{x}y$
1	0	$x\bar{y}$
1	1	xy



Example:

$F = xy + x\overline{y}$

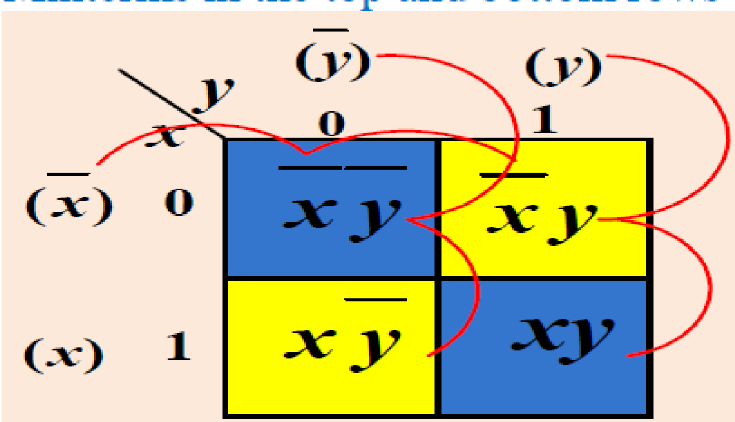
➢ The Karnaugh map for this function will be:



Note: we can easily from the k-map see which ***minterms contain common literal***:

- Minterms on the left and right sides contain $\overline{y}$ and y respectively.

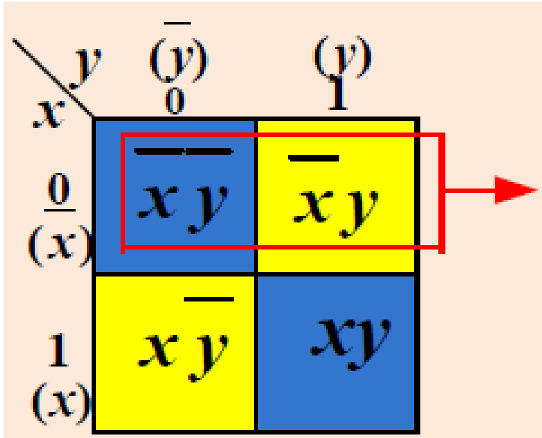
- Minterms in the top and bottom rows contain $\overline{x}$ and x respectively.



Note: each case in the truth table corresponds to a square in the K-map

- **Karnaugh map simplification:**
  ➢ The K-map squares labeled so that ***horizontally adjacent*** square differ only in ***<u>one variable.</u>***

Example 1:- ***Imagine a two-variable sum of minterms, both of these minterms appear in the top row of a karnaugh map, which means that they both contain the literal $\overline{x}$.***

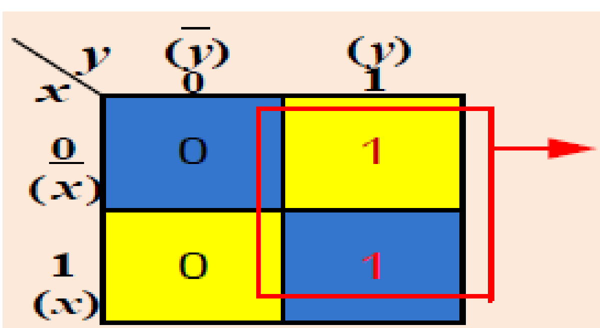


$$F=\overline{x}\,\overline{y}+\overline{x}\,y=\overline{x}\left(\overline{y}+y\right)=x$$

Example 2: $f=\overline{x}\,y+xy$ ***–minimize it using K-map.***

Solution:

- Both minterms appear in the right- side where y is uncomplemented.
- Thus, we can reduce $\overline{x}\,y+xy$ = just to y.

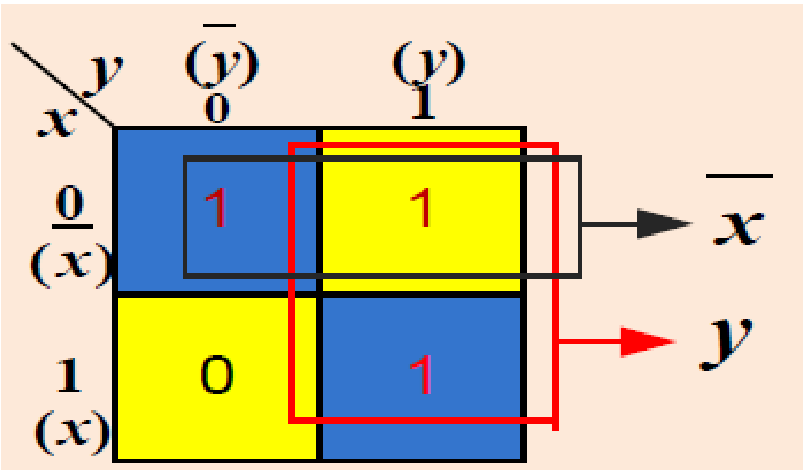


$$F=y$$

$$f=\overline{x}\,y+xy=y(x+\overline{x})=y$$

Example 3: $z = \bar{x}\,\bar{y} + \bar{x}\,y + xy$ ***–minimize it using K-map.***

Solution:



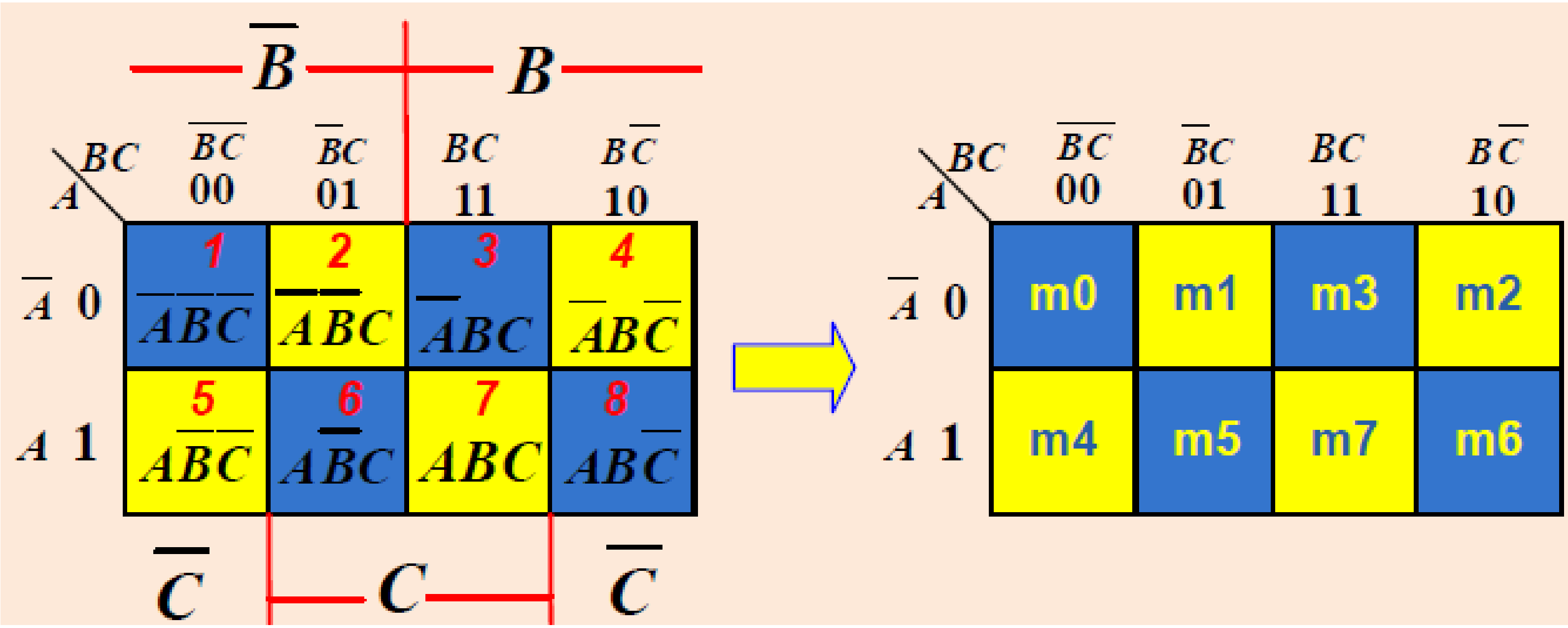
- We have $\bar{x}\,\bar{y} + \bar{x}y$ in the top row, corresponding to $\bar{x}$
- There's also $\bar{x}\,y + xy$ in the right side corresponding to y.
- The result $z = \bar{x} + y$.

Using algebraic simplification:

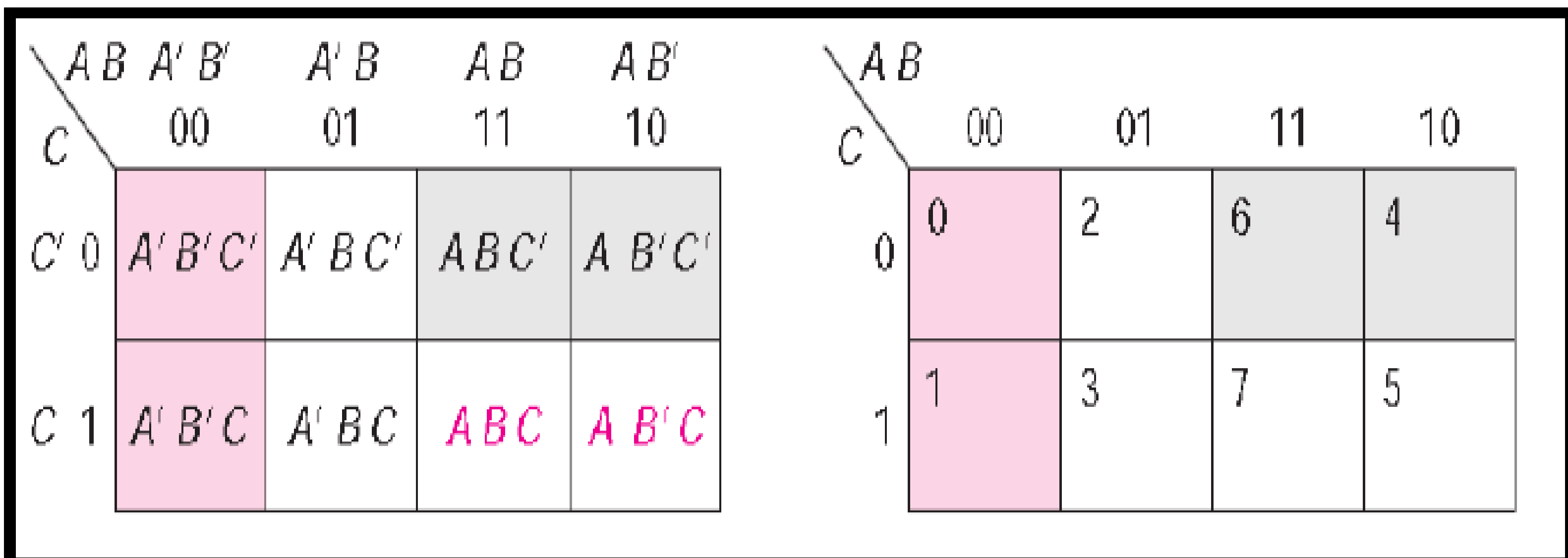
$$z = \bar{x}\,\bar{y} + \bar{x}\,y + xy = \bar{x}\,(\bar{y} + y) + xy$$
$$= \bar{x} + xy = (x + \bar{x})\,(\bar{x} + y) = \bar{x} + y$$

- **Three- variables K-map**
  - For $F(A, B, C)$ there are $2^3 = 8$ minterms.
  - *Representation* truth table using K-map

- *Different versions:*



Or



Grouping (ordering, looping)

- The groups can be ***2, 4, or 8 adjacent squares***:
  - ✓ **2 squares** ⟶ 1 variable can be canceled.
  - ✓ **4 squares** ⟶ 2 variables can be canceled.
  - ✓ **8 squares** ⟶ 3 variables can be canceled.

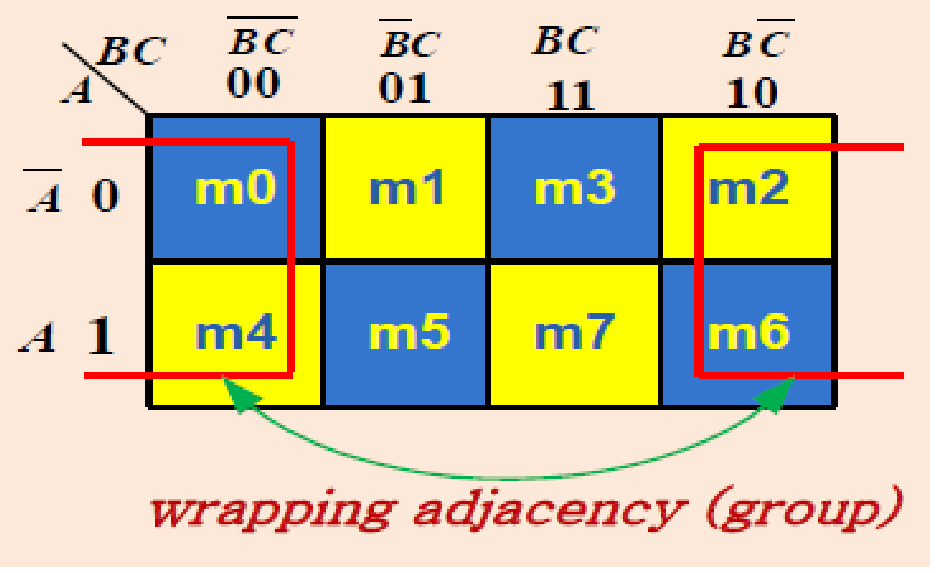
- ***Examples***:-

Squares	*Common Literal (s)*
(1) and (2)	$\bar{A}\,\bar{B}$
(2) and (3)	$\bar{A}C$
(1) and (4)	$\bar{A}\,\bar{C}$
(1), (2), (5) and (6)	$\bar{B}$
(3), (4), (7) and (8)	B
(1), (2), (3) and (4)	$\bar{A}$
(5), (6), (7) and (8)	A
(1), (5), (4) and (8) ***(Wrapping case is also adjacent)***	$\bar{C}$

- To *proof* the wrapping case (the last one in the table) algebraically, we can write:

$$F = \bar{A}\,\bar{B}\,\bar{C} + A\,\bar{B}\,\bar{C} + \bar{A}\,B\,\bar{C} + A\,B\,\bar{C}$$
$$= \bar{C}(\bar{A}\,\bar{B} + A\bar{B} + \bar{A}\,B + AB)$$
$$= \bar{C}\,(\bar{B}\,(\bar{A} + A) + B\,(\bar{A} + A))$$
$$= \bar{C}\,(\bar{B} + B) = \bar{C}$$

- "*Adjacency*" includes wrapping around the left and right side.



Example 1: ***Simplify the following logical function using K-map.***

$$F(x, y, z) = x\,y + \bar{y}\,z + x\,z$$

Solution:

Step 1: the *expression must be in a sum of minterms form*, so we should convert it:(two ways to do that):

1. *Using logical rules (algebraically).*

$$F = x\,y + \bar{y}\,z + x\,z$$
$$= x\,z\,(z + \bar{z}) + \bar{y}\,z(x + \bar{x}) + x\,z\,(y + \bar{y})$$
$$= x\,y\,z + x\,y\,\bar{z} + x\,\bar{y}\,z + \bar{x}\,\bar{y}\,z + x\,y\,z + x\,\bar{y}\,z$$
$$= x\,y\,z + x\,y\,\bar{z} + x\,\bar{y}\,z + \bar{x}\,\bar{y}\,z$$
$$= m_1 + m_5 + m_6 + m_7$$

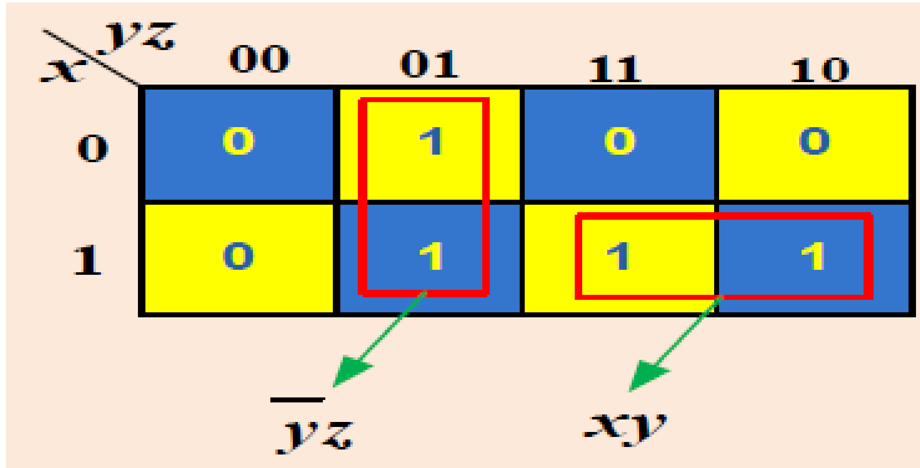
2. *Make the truth table and read the minterms.*

$$F = x\,y + \bar{y}\,z + x\,z$$

inputs			Output	Terms replacement
x	y	z	$F(x,y,z)$	
0	0	0	0	
0	0	1	1	$\bar{y}z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	xz and $\bar{y}z$
1	1	0	1	xy
1	1	1	1	xy and xz

$$F(x,y,z) = \bar{x}\bar{y}z + x\,\bar{y}\,z + x\,y\,\bar{z} + x\,y\,z$$
$$= m_1 + m_5 + m_6 + m_7$$

Step 2: *fill one's (for the minterms) in karnaugh map; zero's for other squares.*



Step 3: *grouping* (looping):

2 groups: **(m1) and (m5)**
(m6) and (m7)

Step 4: *simplify*:

$$F = x\,y + \bar{y}\,z$$

- To ***proof*** the result:

$$F = \bar{x}\bar{y}z + x\bar{y}z + xyz + xy\bar{z}$$
$$= \bar{y}z(x + \bar{x}) + xy(z + \bar{z})$$
$$= \bar{y}z + xy$$

Grouping the minterms:-

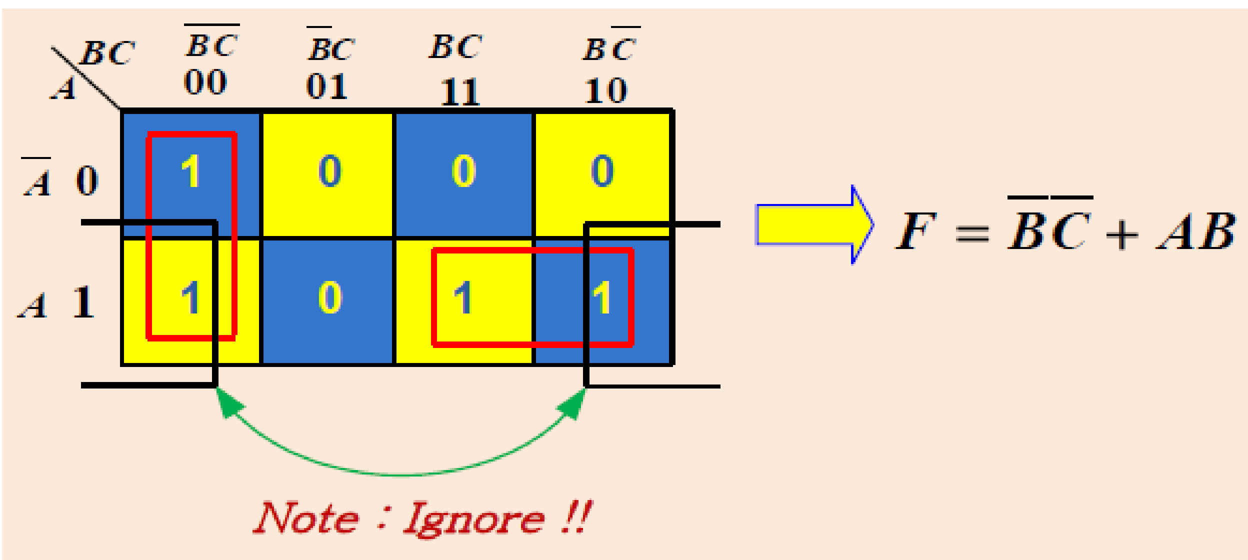
- Grouping together all the **1_s** in the K-map.
  - ***Make rectangles*** of 2^n (1, 2, 4, …).
  - All the **1_s** in the map should be included in at least one rectangle.
  - ***Do not*** include any of the **0_s**.
  - Each group corresponds to one product term.
  - Make each rectangle ***as large as possible***.
  - We can ***overlap the rectangles***, if that makes them lager.

Example 2: Simplify the following logical function using K-map.

$$Z = AB + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$$

Solution:

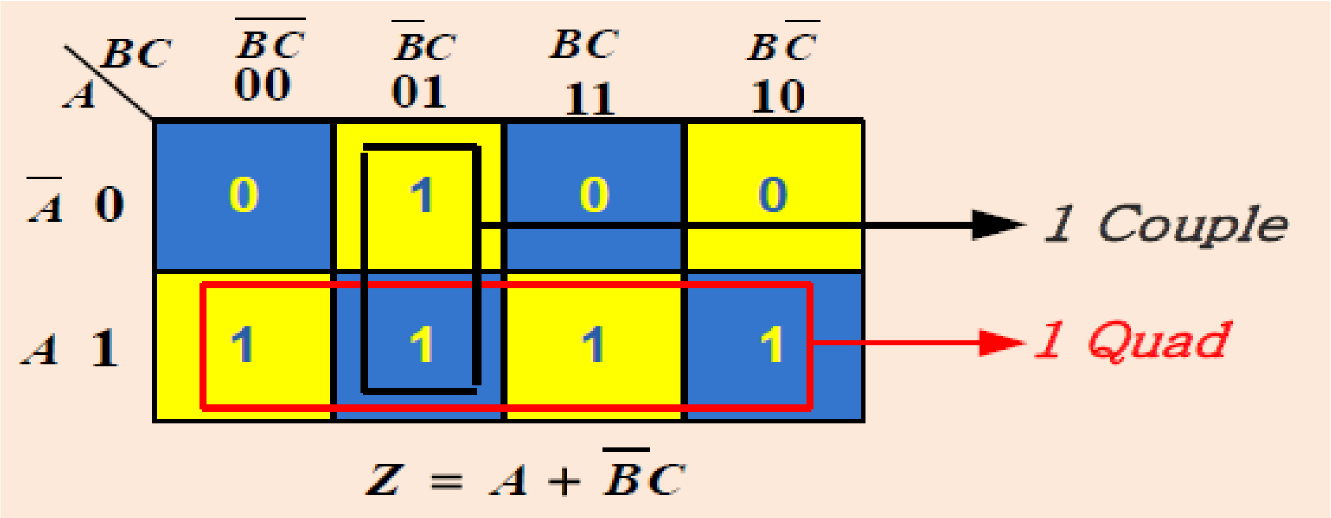
$$Z = AB + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C} = AB\overline{C} + ABC + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$$



Example 3: **Simplify the following logical function using K-map.**

$$Z = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

Solution:



Quad:

$$A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC = A(\bar{B}C + \bar{B}\bar{C} + B\bar{C} + BC)$$

$$= A(\bar{B}(C + \bar{C}) + B(\bar{C} + C)) = A(\bar{B} + B) = A$$

Couple:

$$\bar{A}\bar{B}C + A\bar{B}C = \bar{B}C(A + A) = \bar{B}C$$