وزارة التعليم العالي والبحث العلمي

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كلية علوم الحاسوب وتكنولوجيا المعلومات
قسم علوم الحاسبات

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LECTURE SEVEN
BOOLEAN ALGEBRA II
SIMPLIFYING LOGIC CIRCUIT

Objectives:

1. Deriving of logical expression form truth tables.
2. Logical expression simplification methods:
  a. Algebraic manipulation.
  b. Karnaugh map (k-map).

1. Deriving of logical expression from truth tables

Definitions:

Literal: *Non-complemented* or *complemented* version of a variable (A or $\overline{A}$).

Product term: a ***series of literals*** related to one another through the ***AND*** operator (Example:$A \cdot \overline{B} \cdot C$).

Sum term: A ***series of literals*** related to one another through the ***OR*** operator (Example:($A + \overline{B} + \overline{C}$).

SOP form: ***(Sum –of-products form)***:

- The ***logic-circuit simplification*** require the logic expression to be in ***SOP*** form , for example:

$$AB + \overline{A}B\overline{C} + \overline{C}\overline{D}$$

POS form ***(product-of-sum form):***

- This form ***sometimes*** used in logic circuit, example:

$$(A + C) \cdot (C + \overline{D}) \cdot (\overline{B} + C)$$

- The methods of circuit simplification and design that will be used are based on ***SOP*** form.

Other Laws

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\overline{A} = 0$	$A + \overline{A} = 1$
Commutative law	$AB = BA$	$A + B = B + A$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption law	$A(A + B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\overline{B}$

Canonical and standard form

- ***Product terms*** that consist of the variables of function are called "***Canonical product terms***" or "***Minterms***".
- The term $AB\overline{C}$ is a ***minterm*** in a ***three variable logic function***, but will be a non-minterm in a four variable logic function.
- ***Sum terms*** which contain all the variables of a Boolean function are called "***Canonical sum terms***" or ***Maxterms***".

Example: $A + \overline{B} + C$ is a ***maxterm*** in a ***three variable logic function.***

- for two variables A and B, there are four combination: $\overline{A}\overline{B}, \overline{A}B, A\overline{B}, AB$ called minterms or standard products

A	B	**Minterm**
0	**0**	$\overline{A}\overline{B}$
0	**1**	$\overline{A}B$
1	**0**	$A\overline{B}$
1	**1**	AB

- for n variables there are 2^n minterms

***Example**: minterms and maxterms for 3 variables*:

inputs			Minterm	Designation	Maxterm	Designation
A	B	C				
0	0	0	$\bar{A}\bar{B}\bar{C}$	m0	$A + B + C$	M0
0	0	1	$\bar{A}\bar{B}C$	m1	$A + B + \bar{C}$	M1
0	1	0	$\bar{A}B\bar{C}$	m2	$A + \bar{B} + C$	M2
0	1	1	$\bar{A}BC$	m3	$A + \bar{B} + \bar{C}$	M3
1	0	0	$A\bar{B}\bar{C}$	m4	$\bar{A} + B + C$	M4
1	0	1	$A\bar{B}C$	m5	$\bar{A} + B + \bar{C}$	M5
1	1	0	$AB\bar{C}$	m6	$\bar{A} + \bar{B} + C$	M6
1	1	1	ABC	m7	$\bar{A} + \bar{B} + \bar{C}$	M7

Two important properties:

- *Any Boolean function can be expressed as a sum of minterms.*
- *Any Boolean function can be expressed as a product of minterms.*

Example: *Majority function: (for 3 variables)*

Output is one whenever majority of inputs is 1

inputs			Output	Minterms SOP Form
A	*B*	*C*	**F**	
0	0	0	0	------
0	0	1	0	------
0	1	0	0	------
0	1	1	1	$\bar{A}BC$
1	0	0	0	------
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

- Four product terms, because, there are 4 rows with a 1 output.
- ***Final expression***:

$$F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

inputs			Output	Maxterms POS Form
A	B	C	F	
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\overline{C}$
0	1	0	0	$A+\overline{B}+C$
0	1	1	1	------
1	0	0	0	$\overline{A}+B+C$
1	0	1	1	------
1	1	0	1	------
1	1	1	1	------

- Four sum terms, because, there are 4 rows with a 0 output.

- ***Final expression***:

$$F=(A+B+C)\cdot(A+B+\overline{C})\cdot(A+\overline{B}+C)\cdot(\overline{A}+B+C)$$

Derivation of logical expression form truth tables

Summary

- ✓ *Sum-of-product (SOP) form.*
- ✓ *Product-of- sums (POS) form.*
  - *SOP form :*

- Write an ***AND*** term for each input combination that produces a 1 output.
- Write the variable if its value is 1 , complement otherwise.
- ***OR*** the ***AND*** terms to get the final expression.

2. logical expression simplification methods:

Two basic methods:

a) Algebraic manipulation:

- Use ***Boolean laws*** to simplify the expression: (difficult to use and don't know if you have the simplified form.

b) Karnaugh map (k-map) method:

- ***Graphical*** method.
- ***Ease*** to use.
- Can be used to simplify logical expressions ***with a few variables***.

a) Algebraic manipulation

Example 1:

Design a logic circuit that has three inputs, A, B and C and whose output will be high only when a majority of the input are high .(complete design procedure).

Solution:

Step 1: ***set up the truth table*** (see previous section)

Step 2: ***write the AND term for each case where the output is a 1*** :(see previous section)

Step 3: ***write the sum-of-products expression for the output***

$$F = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Step 4: ***simplify the output expression***

- Find the ***common*** term(s): ABC -common term.
- Use the common term (ABC) to ***factor*** with other terms:

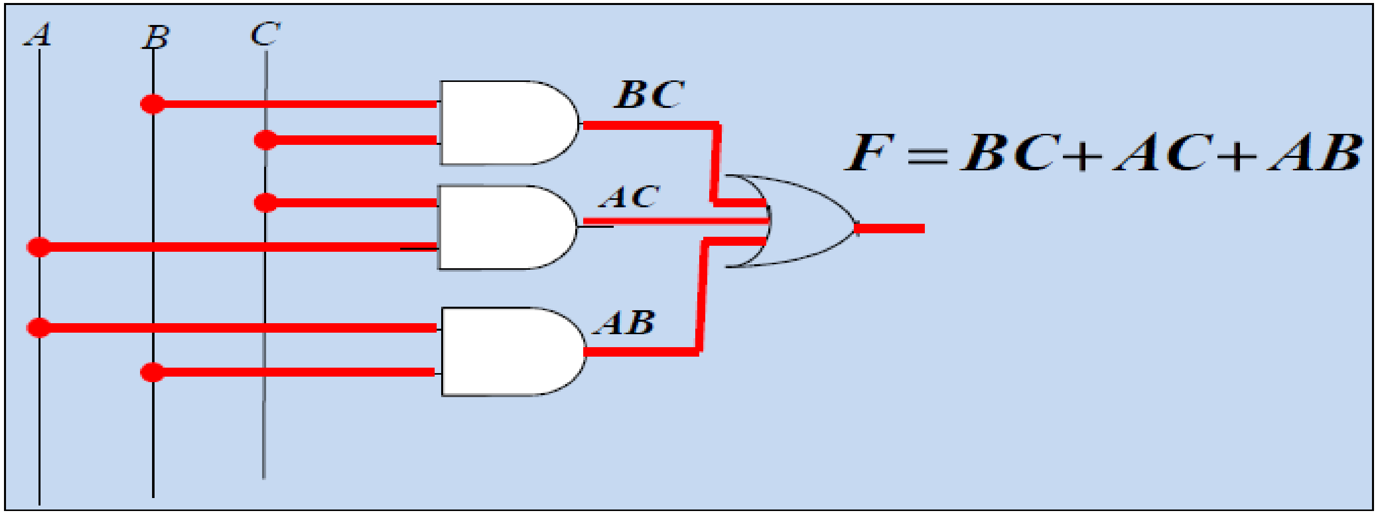
$$F = \overline{A}BC + ABC + A\overline{B}C + ABC + AB\overline{C} + ABC$$

- Factoring the appropriate pairs of terms:

$$F = BC(\overline{A} + A) + AC(B + \overline{B}) + AB(\overline{C} + C)$$

$$F = BC + AC + AB$$

Step 5: ***implement the circuit for the final expression.***



Example 2: *simplify the Boolean function.*

$$F = AB + \bar{A}C + BC$$

Solution:

$F = AB + \bar{A}C + BC.1$ **(Common term)**

$= AB + \bar{A}C + BC(A + \bar{A})$

$= AB + ABC + \bar{A}C + \bar{A}BC$

$= AB(1 + C) + \bar{A}C(1 + B)$

$= AB + \bar{A}C$ *(The result)*

Example 3: *simplify the following Boolean function to a minimum number of literals.*

$$X = BC + A\overline{C} + AB + BCD$$

Solution:

$$X = BC(1 + D) + A\overline{C} + AB$$

$$= BC + A\overline{C} + AB = BC + A(\overline{C} + B)$$

Example 4: *simplify the expression*.

$$Z = A\overline{B}\overline{C} + A\overline{B}C + ABC$$

Solution: *(the expression is in SOP form)*

$$Z = A\overline{B}(\overline{C} + C) + ABC$$

$$= A\overline{B}(1) + ABC = A\overline{B} + ABC$$

$$Z = A(\overline{B} + BC) = A(\overline{B} + B)(\overline{B} + C) = A(\overline{B} + C)$$

Example 5: *simplify the expression.*

$$Z = \bar{A}C\left(\overline{\bar{A}BD}\right) + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

Solution:

- ✓ *First, use Demorgan's theorem on the first term:*

$$Z = \bar{A}C\left(\bar{\bar{A}} + \bar{B} + \bar{D}\right) + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$Z = \bar{A}C(A + \bar{B} + \bar{D}) + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

- ✓ *Multiply, we get:*

$$Z = \bar{A}CA + \bar{A}C\bar{B} + \bar{A}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

($\bar{A}A = 0$) then

$$Z = \bar{A}\bar{B}C + \bar{A}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

- ✓ *Check for the largest common factor between any two or more product terms:*

$$Z = \bar{B}C(\bar{A} + A) + \bar{A}\bar{D}\,(C + B\bar{C})$$

$$(\bar{A} + A) \rightarrow 1 \qquad (C + B\bar{C}) \rightarrow C + B$$

- ✓ *We get:*

$$Z = \overline{B}C + \overline{A}\overline{D}\,(B + C)$$

- ✓ *The result can be obtained with other choices*

Example 6: ***Find the complement of the following Boolean function:***

$$Z = (B\overline{C} + \overline{A}D)(\,A\overline{B} + C\overline{D})$$

Solution:

- ✓ *Use Demorgan's theorem :*

$$\overline{Z} = \overline{(B\overline{C} + \overline{A}D)(\,A\overline{B} + C\overline{D})}$$

$$= \overline{(B\overline{C} + \overline{A}D)} + \overline{(\,A\overline{B} + C\overline{D})}$$

$$= \overline{B\overline{C}} \cdot \overline{\overline{A}D} + \overline{A\overline{B}} \cdot \overline{C\overline{D}}$$

$$= \left(\overline{B} + \overline{\overline{C}}\right) \cdot \left(\overline{\overline{A}} + \overline{D}\right) + \left(\overline{A} + \overline{\overline{B}}\right)\left(\overline{C} + \overline{\overline{D}}\right)$$

$$= (\overline{B} + C) \cdot (A + \overline{D}) + (\overline{A} + B) \cdot (\overline{C} + D)$$

- ✓ *It's not in standard form*

Example 7:

Express the following function in a sum of minterms and product of maxterms.

$$Z = (AB + C)(B + AC)$$

a) **Sum of minterms**:

✓ *Multiply, we get:*

$$Z = ABB + AABC + BC + ACC$$

$$= AB + ABC + BC + AC$$

$$= ABC + AB(C + \overline{C}) + BC(A + \overline{A}) + AC(B + \overline{B})$$

$$= ABC + ABC + AB\overline{C} + ABC + \overline{A}BC + ABC + A\overline{B}C$$

$$Z = AB\overline{C} + ABC + \overline{A}BC + A\overline{B}C$$

$$m6 \quad m7 \quad m3 \quad m5$$

$C + \overline{C} = 1$

$A + \overline{A} = 1$

$B + \overline{B} = 1$

$A + A = A$

✓ *So, we can write:*

$$Z(A, B, C) = \sum m(3, 5, 6, 7)$$

inputs			Minterm	z
A	*B*	*C*		
0	0	0	m0	0
0	0	1	m1	0
0	1	0	m2	0
0	1	1	m3	1
1	0	0	m4	0
1	0	1	m5	1
1	1	0	m6	1
1	1	1	m7	1

Sum of minterms

b) Sum of maxterms

- ✓ *Using distributive law:*

$$(A + BC) = (A + B)(A + C)$$

- ✓ *In the*

$$Z = (AB + C)(B + AC)$$

- ✓ *We get:*

$$= (A + C)(B + C)(B + A)(B + C)$$

$$= (A + B)(B + C)(C + A)$$

$$Z = (A + B)(B + C)(C + A)$$

- *In* $A + B$ *we need* C
- *In* $B + C$ *we need* A
- *In* $C + A$ *we need* B

✓ *We can write*

$$(A + B) = (A + B + 0) = (A + B + C\overline{C})$$

$$(A + B) = (A + B + C)(A + B + \overline{C})$$

$$(B + C) = (B + C + 0) = (B + C + A)(B + C + \overline{A})$$

$$(C + A) = (C + A + B)(C + A + \overline{B})$$

✓ *Substitute all of these term in* Z *,we get:*

$$Z = (A + B)(B + C)(C + A)$$

$$= (A + B + C)(A + B + \bar{C})\,(B + C + A)(B + C + \bar{A})\,(C + A + B)(C + A + \bar{B})$$

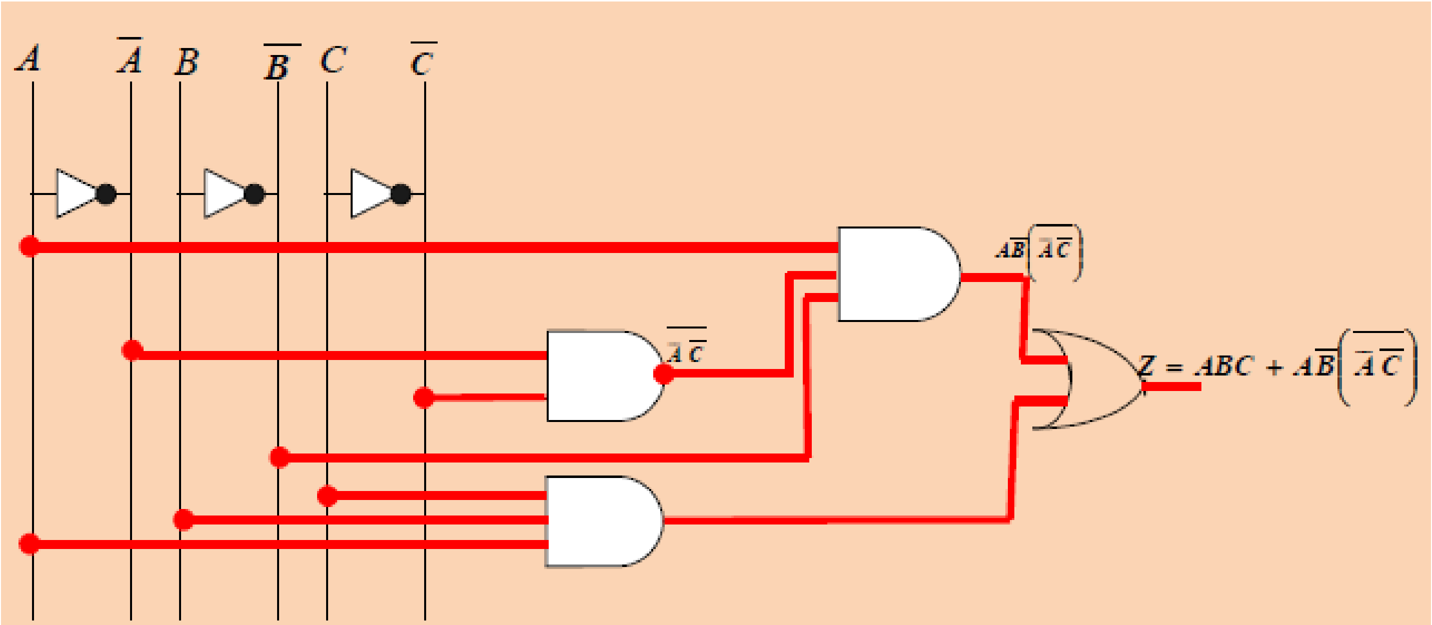
$$Z = (A + B + C)(A + B + \bar{C})\,(B + C + \bar{A})(C + A + \bar{B})$$

$$Z = (A + B + C)(A + B + \bar{C})\,(\bar{A} + B + C)(A + \bar{B} + C)$$

$$Z = M0 \cdot M1 \cdot M4 \cdot M2$$

inputs			Z	Maxterm
A	B	C		
0	0	0	0	M0
0	0	1	0	M1
0	1	0	0	M2
0	1	1	1	M3
1	0	0	0	M4
1	0	1	1	M5
1	1	0	1	M6
1	1	1	1	M7

***Example 8**: simplify the logic circuit shown in the following figure.*



Solution:

- ✓ *The output of the circuit is*

$$Z = abc + a\bar{b}.(\overline{\bar{a}\bar{c}})$$

- ✓ *Using Demorgan's law and multiply out all terms:*

$$Z = abc + a\bar{b}(\bar{\bar{a}} + \bar{\bar{c}})$$

$$Z = abc + a\overline{b}(a + c)$$

$$Z = abc + a\overline{b}a + a\overline{b}c$$

$$Z = abc + a\overline{b}a + a\overline{b}c$$

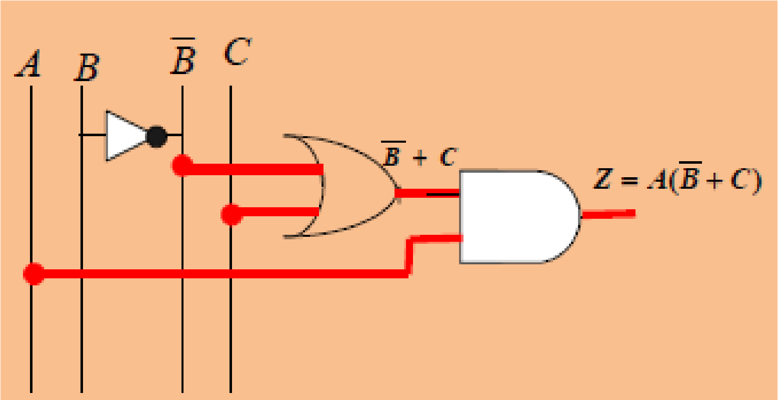
$$Z = abc + a\overline{b} + a\overline{b}c$$

SOP Form

$$Z = abc + a\overline{b} + a\overline{b}c$$

$$Z = ac(b + \overline{b}) + a\overline{b}$$

$$Z = ac.1 + a\overline{b} = ac + a\overline{b} = a(c + \overline{b})$$



MORE EXAMPLES

Ex1: Determine the value of A, B, C, and D that make the product term $\boldsymbol{A\overline{B}C\overline{D}}$ equal to 1.

Ex2: Apply DeMorgan's theorems to each of the following expressions:

$\boldsymbol{a})\ \overline{(\boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C})\boldsymbol{D}}$

b) $\overline{\boldsymbol{ABC}+\boldsymbol{DEF}}$

c) $\overline{\boldsymbol{A\overline{B}}+\boldsymbol{\overline{C}D}+\boldsymbol{EF}}$

Ex3: Simplify the Boolean expression: $[\boldsymbol{A\overline{B}}(\boldsymbol{C}+\boldsymbol{BD})$
$+\ \boldsymbol{\overline{A}\overline{B}}]\boldsymbol{C}$

Ex4: Simplify the Boolean expression : $\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$

Ex5: Simplify the Boolean expression: $\overline{AB + AC} + \overline{A}\,\overline{B}C$

Ex6: Convert the Boolean expression into standard SOP form: $A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$

Ex7: Convert the Boolean expression into standard POS form: $(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$

Ex8: Convert the following SOP expression to an equivalent POS expression: $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$

Converting a sum term to standard POS :

Step 1 : Add to each nonstandard product term a term made up of the product of the missing variable and its complement .

Step 2 : Apply rule 12 : A+BC = (A+B)(A+C).

Step 3 : Repeat step -1- until all resulting sum terms contains all variables in the domain in either complemented or un-complemented form .

Converting Standard SOP to Standard POS

Step 1 :Evaluate each product sum in the SOP expression. That is determine the binary numbers that represent the product form.

Step 2 : Determine all of the binary numbers not included in the evaluation in step 1 .

Step 3 : Write the equivalent sum term for each binary numbers from step and express in pos form.

Ex1: Determine the value of A, B, C, and D that make the product term $\boldsymbol{A\overline{B}C\overline{D}}$ equal to 1.

Answer:
The correct option is A = 1, B = 0, C = 1, D = 0

Ex2: Apply DeMorgan's theorems to each of the following expressions:

$\boldsymbol{a})\ \overline{(\boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C})\boldsymbol{D}}$

b) $\overline{\boldsymbol{ABC}+\boldsymbol{DEF}}$

c) $\overline{\boldsymbol{A}\overline{\boldsymbol{B}}+\overline{\boldsymbol{C}}\boldsymbol{D}+\boldsymbol{EF}}$

Answer:

(a)

Let, $A+B+C=X$ and $D=Y$.

The expression $\overline{(A+B+C)D}$ is of the form $\overline{XY} = \bar{X}+\bar{Y}$ and can be rewritten as,

$$\overline{(A+B+C)D} = \overline{A+B+C} + \bar{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A+B+C}$.

$$\overline{A+B+C} + \bar{D} = \bar{A}\bar{B}\bar{C} + \bar{D}$$

(b)

Let, $ABC = X$ and $DEF = Y$.

The expression $\overline{ABC + DEF}$ is of the form $\overline{X+Y} = \bar{X}\bar{Y}$ and can be rewritten as,

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms, $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\bar{A} + \bar{B} + \bar{C})(\bar{D} + \bar{E} + \bar{F})$$

(c)

Let, $A\bar{B} = X, \bar{C}D = Y$, and $EF = Z$.

The expression $\overline{A\bar{B} + \bar{C}D + EF}$ is of the form $\overline{X + Y + Z} = \bar{X}\bar{Y}\bar{Z}$ and can be rewritten as,

$$\overline{A\bar{B} + \bar{C}D + EF} = (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms, $\overline{A\bar{B}}, \overline{\bar{C}D}$, and $\overline{EF}$.

$$(\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF}) = (\bar{A} + B)(C + \bar{D})(\bar{E} + \bar{F})$$

Ex3: Simplify the Boolean expression: $[A\overline{B}(C + BD) + \overline{A}\overline{B}]C$

Answer:

Step 1. Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\overline{B})C$$

Step 2. Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\overline{B})C$$

Step 3. Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\overline{B})C$$

Step 4. Apply rule 1(drop the 0) within the parentheses. $(A\overline{B}C + \overline{A}\overline{B})C$

Step 5. Apply the distributive law. $A\overline{B}CC + \overline{A}\overline{B}C$

Step 6. Apply rule 7 ($CC = C$) to the first term. $A\overline{B}C + \overline{A}\overline{B}C$

Step 7. Factor out $\overline{B}C$. $\overline{B}C(A + \overline{A})$

Step 8. Apply rule 6 ($A + \overline{A} = 1$). $\overline{B}C \cdot 1$

Step 9. Apply rule 4 (drop the 1). $\overline{B}C$

Ex4: Simplify the Boolean expression : $\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$

Answer:

Step 1. Factor BC out of the first and last terms. $BC(\bar{A} + A) + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

Step 2. Apply rule 6 ($\bar{A} + A = 1$) to the term in parentheses, $BC \cdot 1 + A\bar{B}(\bar{C} + C) + \bar{A}\bar{B}\bar{C}$

Step 3. Apply rule 4 (drop the 1) to the first term and rule 6 ($\bar{C} + C = 1$) to the tern in parentheses.

$$BC + A\bar{B} \cdot 1 + \bar{A}\bar{B}\bar{C}$$

Step 4. Apply rule 4 (drop the 1) to the second term. $BC + A\bar{B} + \bar{A}\bar{B}\bar{C}$

Step 5. Factor $\bar{B}$ from the second and third terms. $BC + \bar{B}(A + \bar{A}\bar{C})$

Step 6. Apply rule 11 ($A + \bar{A}\bar{C} = A + \bar{C}$) to the term in parentheses. $BC + \bar{B}(A + \bar{C})$

Step 7. Use the distributive and commutative laws to get the following expression: $BC + A\bar{B} + \bar{B}\bar{C}$

Ex5: Simplify the Boolean expression: $\overline{AB+AC}+\bar{A}\bar{B}C$

Answer:

Step 1. Apply DeMorgan's theorem to the first term. $(\overline{AB})(\overline{AC}) + \bar{A}\bar{B}C$

Step 2. Apply DeMorgan's theorem to each term in parentheses.

$$(\bar{A} + \bar{B})(\bar{A} + \bar{C}) + \bar{A}\bar{B}C$$

Step 3. Apply the distributive law to the two terms in parentheses.

$$\bar{A}\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$$

Step 4. Apply rule 7 ($\bar{A}\bar{A} = \bar{A}$) to the first term, and apply rule 10 [$\bar{A}\bar{B} + \bar{A}\bar{B}C = \bar{A}\bar{B}(1 + C) = \bar{A}\bar{B}$] to the third and last terms.

$$\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C}$$

Step 5. Apply rule 10 [$\bar{A} + \bar{A}\bar{C} = \bar{A}(1 + \bar{C}) = \bar{A}$] to the first and second terms.

$$\bar{A} + \bar{A}\bar{B} + \bar{B}\bar{C}$$

Step 6. Apply rule 10 [$\bar{A} + \bar{A}\bar{B} = \bar{A}(1 + \bar{B}) = \bar{A}$] to the first and second terms.

$$\bar{A} + \bar{B}\bar{C}$$

Ex6: Convert the Boolean expression into standard SOP form: $\boldsymbol{A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D}$

Answer:

The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D + \overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D + \overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result.

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C + \overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D + \overline{D}$ as follows:

$$\begin{aligned}\overline{A}\,\overline{B} &= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D}) \\ &= \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\overline{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Ex7: Convert the Boolean expression into standard POS form: $(\boldsymbol{A} + \overline{\boldsymbol{B}} + \boldsymbol{C})(\overline{\boldsymbol{B}} + \boldsymbol{C} + \overline{\boldsymbol{D}})(\boldsymbol{A} + \overline{\boldsymbol{B}} + \overline{\boldsymbol{C}} + \boldsymbol{D})$

Answer:

The domain of this POS expression is A, B, C, D. Take one term at a time. The first term, $A + \overline{B} + C$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12 as follows:

$$A + \overline{B} + C = A + \overline{B} + C + D\overline{D} = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})$$

The second term, $\overline{B} + C + \overline{D}$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply rule 12 as follows:

$$\overline{B} + C + \overline{D} = \overline{B} + C + \overline{D} + A\overline{A} = (A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

The third term, $A + \overline{B} + \overline{C} + D$, is already in standard form. The standard POS form of the original expression is as follows:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Ex8: Convert the following SOP expression to an equivalent POS expression: $\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$

Answer:

The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Since there are three variables in the domain of this expression, there are a total of eight (2^3) possible combinations. The SOP expression contains five of these combinations, so the POS must contain the other three which are 001, 100, and 110.

Remember, these are the binary values that make the sum term 0. The equivalent POS expression is

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$