

زكاة التبع العجايب والبعث العليمي



جامعة الانبار كلية علوم الحاسوب وتكنولوجيا المعلومات قسم علوم الحاسبات

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اسم المادة: التصميم المنطقي

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LECTURE FIVE

BINARY SYSTEM OPERATIONS & NEGATIVE NUMBERS REPRESENTATION

Objective:

- 1. Binary system operations.**
- 2. Representation of negative numbers.**
- 3. Two's complement addition and subtraction.**
- 4. One's complement addition and subtraction.**

1) Binary system operations

a) Binary Addition:

Example: -

$$\begin{array}{r}
 \leftarrow \text{carries} \\
 \\
 + \\
 \hline
 1
 \end{array}$$

Addition table

	<u>Sum</u>	<u>Carry</u>
0 + 0 =	0	0
0 + 1 =	1	0
1 + 0 =	1	0
1 + 1 =	0	1

b) Binary subtraction:

Example: -

First number → X → 229 → 11100101
 Second number → Y → 46 → 00101110
 Borrow → 001111100

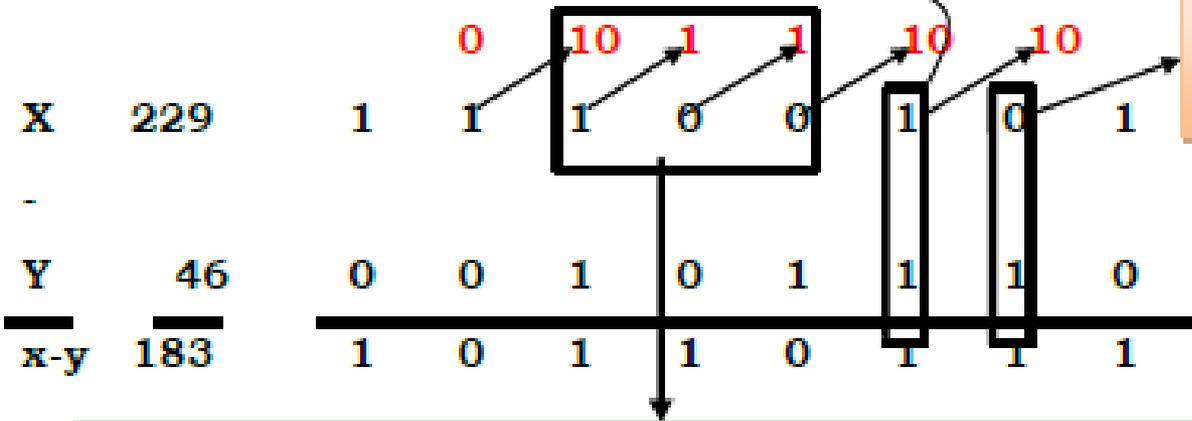
$$\begin{array}{r}
 11100101 \\
 - 00101110 \\
 \hline
 10110111
 \end{array}$$
 x - y = 183

Subtraction table

0 - 0 =	0
1 - 0 =	1
1 - 1 =	0
0 - 1 =	1 (and the borrow = 1)

After the first borrow, the new subtraction for this column is 0-1, so we must borrow again.

Must borrow 1, yielding the new subtraction 10-1 = 1



The borrow goes through three columns to reach a borrowable 1.

100 = 011 (the modified bits), + 1 (the borrow)

c) Binary multiplication:

Example :-

$$\begin{array}{r} 10111 \\ * 10110 \\ \hline 00000 \\ 00111 \\ 00000 \\ 10111 \\ \hline 11100110 \text{ (result)} \end{array}$$

Multiplication table

$$0 * 0 = 0$$

$$0 * 1 = 0$$

$$1 * 0 = 0$$

$$1 * 1 = 1$$

2) Representation of negative numbers

- There are many ways to represent negative numbers:
 - ✓ Signed-magnitude system.
 - ✓ Complement number systems.
 - **Signed-magnitude representation:**
 - In signed-magnitude system, the number consists of a *magnitude* and a *symbol* indicating whether the magnitude is positive or negative.
 - In binary system: *extra bit position to represent the sign* (sign bit): (*MSB*) is used.

Sign bit: 0 = plus, 1 = minus.

Example:-

$$0\ 1010101_2 = +85_{10}, \quad 11010101_2 = -85_{10}$$

Sign bit magnitude

$$01111111_2 = +127_{10}$$

$$11111111_2 = -127_{10}$$

$$00000000_2 = +0_{10}$$

$$10000000_2 = -0_{10}$$

- **Complement number systems:**

- Complement number system negates a number by taking its *complement* as defined by the system.
- There are two complement number systems that can be used:
 - *Two's complement system*
 - *One's complement system.*

Two's complement system:

- The two's complement of an **n-digit** number **D** is obtained by:

Subtracting the number from r^n

$$r^n - D$$

- **r**- The base of the system.

- This can be accomplished by *complementing the individual digits of D*, and *adding 1 to the result*.
 - In decimal system, it's called the **10's complement**.
 - For binary numbers, it's called **two's complement**.
 - The *MSB* of a number in this system is used as the *sign bit*.

Example 1:

$$17_{10} = 00010001_2$$

$$\begin{array}{r}
 \phantom{17_{10} = } 00010001_2 \\
 \phantom{17_{10} = } \downarrow \text{Complement bits} \\
 11101110 \\
 + 1 \\
 \hline
 11101111 = -17_{10}
 \end{array}$$

Example 2:

$$-99_{10} = 10011101_2$$

$$\begin{array}{r}
 \phantom{-99_{10} = } 10011101_2 \\
 \phantom{-99_{10} = } \downarrow \text{complement bits} \\
 01100010 \\
 + 1 \\
 \hline
 01100011_2 = 99_{10}
 \end{array}$$

Example 3:

$$0_{10} = 00000000_2$$

$$\begin{array}{r}
 \phantom{0_{10} = } 00000000_2 \\
 \phantom{0_{10} = } \downarrow \\
 11111111 \\
 + 1 \\
 \hline
 X \textcircled{1} 00000000_2 = 0_{10}
 \end{array}$$

Important Note:

The number 0 has one representation using two's complement.

One's complement representation:

- In one's complement representation the complement of an n -digit number D is obtained by: *Subtracting the number from $r^n - 1$*

$$(r^n - 1) - D$$

- This can be accomplished by *complementing the individual digits of D , without adding 1* as in two's complement systems.
 - In decimal system it's called **9's complement**.
 - In binary system it's called **1's complement**.

Example 1

$$17_{10} = 00010001_2$$



$$11101110_2 = 17_{10}$$

Example 2

$$\begin{array}{l} -99_{10} = 10011100_2 \\ \quad \quad \downarrow \\ \quad \quad 01100011_2 = 99_{10} \end{array}$$

in one's complement :

- MSB is used as sign digit.
- The number 0 has **two representations** using one's complement:

00000000 positive zero.

11111111 negative zero.

3) Two's complement addition and subtraction

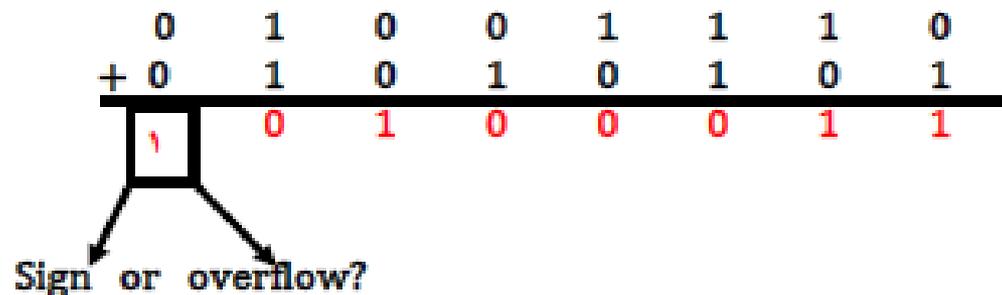
Example 1: *adding two positive numbers: +15 +27*

$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 = (42)_{10} \rightarrow \text{(okay)} \end{array}$$

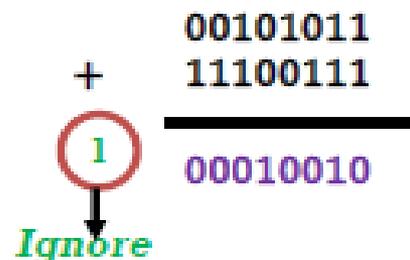
↓
Sign bit

Example 2: adding two positive numbers with overflow: +78 +85

$$+78 + 85 = (163)_{10} \quad (\text{we need 8 bits +1 for sign bit})$$



Example 3 43 -25:



Example 4 25 - 43:

