

زكاة التبع العجايب والبعث العليمي



جامعة الانبار كلية علوم الحاسوب وتكنولوجيا المعلومات قسم علوم الحاسبات

Department	علوم الحاسبات	القسم:
Subject Name:	Logic Design	أسم المادة :
Year of Study:	2025-2024	السنة الدراسية:
Course:	الكورس الاول	الكورس:
Title and No of lecture:	Lecture 2:Digital Number System I	عنوان ورقم المحاضرة:
Instructor Name:	د. مصطفى معد حمدي	أسم التدريسي:

السنة : 2025-2024

اسم المادة: التصميم المنطقي

أستاذ المادة: د. مصطفى معد حمدي

LECTURE TWO

DIGITAL NUMBER SYSTEMS I

Objectives:

- 1. Understanding decimal, binary, octal and hexadecimal numbers.**
- 2. Counting in decimal, binary, octal and hexadecimal systems.**
- 3. Convert a number from one number system to another system.**
- 4. Advantage of octal and hexadecimal systems.**

1. Understanding decimal, binary, octal and hexadecimal numbers

Decimal number systems:

- ✓ Decimal numbers are made of decimal digits:
(0,1,2,3,4,5,6,7,8,9 -----10-base system)
- ✓ The decimal system is a "*positional-value system*" in which the value of a digit depends on its position.

Examples:

- ❖ 453 → 4 hundreds, 5 tens and 3 units.
 - ✓ 4 is the most weight called "**most significant digit**" MSD.
 - ✓ 3 carries the last weight called "**least significant digit**" LSD.
- ❖ number of items that a decimal number represent:
 $9261 = (9 \times 10^3) + (2 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$
- ❖ The decimal fractions:
 $3267.317 = (3 \times 10^3) + (2 \times 10^2) + (6 \times 10^1) + (7 \times 10^0) + (3 \times 10^{-1}) + (6 \times 10^{-2}) + (1 \times 10^{-3})$

- ✓ *Decimal point* used to separate the integer and fractional part of the number.
- ✓ *Formal notation* → $(3267.317)_{10}$.
- ✓ Decimal position values of powers of (10).

Positional values "weights"

10^4	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	10^{-4}
↑	↑	↑	↑	↑		↑	↑	↑	↑
2	7	7	8	3	.	2	3	4	5
MSD									LSD

Binary numbers:

- **Base-2** system (**0 or 1**).
- We can represent any quantity that can be represented in decimal or other number systems using *binary numbers*.
- Binary number is also *positional-value system* (power of **2**).

Example: 1101.011

2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}
↑	↑	↑	↑		↑	↑	↑
1	1	0	1	.	0	1	1
MSD						LSD	

Notes:

- To find the equivalent of binary numbers in decimal system, we simply take the *sum of products of each digit value (0,1) and its positional value*:

Example: $(1011.101)_2$

$$= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$

$$= 8 + 0 + 2 + 1 + \frac{1}{2} + 0 + \frac{1}{8} = 11.625_{10}$$

In general, any number (decimal, binary, octal and hexadecimal) is simply the sum of products of each digit value and its positional value.

- In binary system, the term binary digit is often called *bit*.

- Binary values at the output of digital system must be converted to decimal values for presentation to the outside world.
- Decimal values must be converted into the digital system.
- Group of 8 bits are called a *byte*.

Octal Number System

- octal number system has a **base of 8** : (0,1,2,3,4,5,6,7)

Examples:

• $(1101.011)_8$

8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
↑	↑	↑	↑		↑	↑	↑
1	1	0	1	.	0	1	1
MSD					LSD		

• $(4327)_8$

$$= (4 \times 8^3) + (3 \times 8^2) + (2 \times 8^1) + (7 \times 8^0)$$

- 372.36_8

$$= (3 \times 8^2) + (7 \times 8^1) + (2 \times 8^0) + (3 \times 8^{-1}) + (6 \times 8^{-2})$$

Note: octal number don't use digits 8 or 9

Hexadecimal number system (16-base)

- ✓ Hexadecimal numbers are made of 16 digits, it uses the digits **0** through **9** plus the letters **A, B, C, D, E, F**.

Examples:

- $(A29)_{16}$

$$= (10 \times 16^2) + (2 \times 16^1) + (9 \times 16^0) = (2601)_{10}$$

- $(2c7.38)_{16}$

$$= (2 \times 16^2) + (12 \times 16^1) + (7 \times 16^0) + (7 \times 16^0) + (3 \times 16^{-1}) + (8 \times 16^{-2})$$

Note:

- ✓ For hex numbers the digits **10, 11, 12, 13, 14, 15** are represented by **a, b, c, d, e, f** as shown in the following table:

Number Systems			
Decimal	Binary	Octal	Hex
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

2. Counting in decimal ,binary, octal and hexadecimal systems

Decimal counting:

- Start with 0 in the units position and take each digit in progression until reach 9.
- Add 1 to the next higher position and start over 0 in the first position.
- Continue process until the count 99.
- Add 1 to the third position and start over with 0 in the first position.

Note: the largest number that can be represented using 8 bits is

$$2^n - 1 = 2^8 - 1 = 255_{10} = 11111111_2$$

Counting in hexadecimal:

- ✓ For **n** hex digit positions, we can count for decimal **0** to **16ⁿ-1**, for a total of **16ⁿ** different values.
- ✓ The *general representation* for a number in the form:

$$a_4a_3a_2a_1a_0 \cdot a_{-1}a_{-2}a_{-3}$$

Using **r-base/radix** number system, in which the number of *radix* *r* can be written as

$$n_r = \dots + a_4 \cdot r^4 + a_3 \cdot r^3 + a_2 \cdot r^2 + a_1 \cdot r^1 \\ + a_0 \cdot r^0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \dots$$

Numbering System	Radix
Decimal	r=10
Binary	r=2
Octal	r=8
Hex	r=16

Counting in binary system: (counting range)

- ✓ Using **n** bits, we can represent decimal numbers ranging from **0** to **2ⁿ - 1**, a total of **2ⁿ** different numbers.

Examples:

- for $n=4$ bits

We can count from **0000** to **1111**₂ (see table above) which is 0_{10} to 15_{10} (16 different numbers).

- How many bits are needed to represent decimal values ranging from 0 to 12500?

Answer:

- With **13** bits , we can count from **0** to $2^{13}-1 = \mathbf{8191}$ (*not enough*)
- With **14** bits , we can count from **0** to $2^{14}-1 = \mathbf{16.383}$ (*okay*)
 - What is the total range of decimal values that can be represented in 8 bits?

Answer:

For **N=8**, we can represent form **0** to $2^8-1 = \mathbf{255}$.