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المصادر والمراجع	Belmont, USA. ،2008 ،Calculus: Early Transcendentals, James Stewart
	Boston, USA. ،2013 ،Multivariable Calculus, Ron Larson and Bruce Edwards
	Thomas' Calculus, 11th Edition

Alternating series

If $\langle u_n \rangle$ is a sequence of positive term. Then, $\sum_{n=1}^{\infty}(-1)^{n+1}u_n$ is called an alternating series as $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$.

Absolutely convergent

A series $\sum_{n=1}^{\infty}u_n$ is said to be absolutely convergent if and only if $\sum_{n=1}^{\infty}|u_n|$ converges. For example $\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$.

Conditionally convergent

If a series $\sum_{n=1}^{\infty}u_n$ satisfies each of the following three conditions, then $\sum_{n=1}^{\infty}u_n$ conditionally convergent

- $\sum_{n=1}^{\infty}u_n$ is an alternating series.
- $\lim_{n\to\infty}u_n = 0$.
- $|u_{n+1}| \leq |u_n|$

Example: Does $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is converge or diverge??

Sol

u_n=$|an| \Rightarrow un = \left|\frac{(-1)^n}{n}\right| \Rightarrow un = \frac{1}{n}$

$$\lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0$$

$u_n \geq u_{n+1} \Rightarrow \frac{1}{n} \geq \frac{1}{(n+1)}$ convergent

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n} \; by\ H.S$$

$n = 1$ divergent

This series is conditionally convergent

For example: Does $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}$ converge or diverge?

Solution: We have

- $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1} = \frac{-1}{2} + \frac{2}{5} - \frac{3}{10} + \cdots$ which is an alternating series.
- $\lim_{n\to\infty}(-1)^n\frac{n}{n^2+1} = 0$.
- It is clearly $|u_{n+1}| \leq |u_n|$.

Thus, the series is a conditionally convergent.

Example: Does $\sum_{n=1}^{\infty} \frac{(-1^n)}{lnn}$ converge or diverge?

Sol/

U_n=$|an|$ $\rightarrow U_n = \sum_{n=1}^{\infty} \frac{(-1^n)}{lnn} = U_n = \frac{1}{lnn}$

$$\lim_{n\to\infty} \frac{1}{lnn} = \frac{1}{\infty} = 0$$

$$U_n \geq U_{n+1} \rightarrow \frac{1}{lnn} \geq \frac{1}{lnn+1}$$

this series is convergent

Absolutely and conditionally convergent

The conditionally convergent

If the series is convergent by satisfy the condition of alternating series and it's called the conditionally convergent

The absolute convergent

f the series $\sum_{n=1}^{\infty} an$ be finite series and$\sum_{n=1}^{\infty} |an|$is converged then$\sum_{n=1}^{\infty} an$ is convergent

Example : Determine each of the following series convergent or divergent

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

Sol/

U_n=$|an|$ $\rightarrow U_n == \left|\frac{(-1)^{n+1}}{n^2}\right|$=$un$=$\frac{1}{n}$

Now we apply the terms

$$\lim_{n\to\infty} = \frac{1}{n^2} = \frac{1}{\infty} = 0$$

U_n=$\geq U_{n+1} \rightarrow \frac{1}{n^2} \geq \frac{1}{(n+1)^2}$

this series is convergent

2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Sol/

$U_n=|an| \to U_n = \left|\frac{(-1)^n}{n}\right|$=un=$\frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n}=\frac{1}{\infty} = 0$

$U_n \geq U_{n+1} \to \frac{1}{n} \geq \frac{1}{(n+1)}$ convergent

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \; by\ H.S$$

$n = 1$ divergent

This series is conditionally convergent