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1	رقم المحاضرة
Calculus: Early Transcendentals, James Stewart ،2008 ،Belmont, USA. Multivariable Calculus, Ron Larson and Bruce Edwards ،2013 ،Boston, USA. Thomas' Calculus, 11th Edition	المصادر والمراجع

Infinite sequences

The term sequence in mathematics is used to describe an unending succession of numbers. The numbers in a sequence are called the terms of the sequence. For example

$$\langle 1,3,5,\dots \rangle$$

$$\left\langle 1,\frac{1}{2},\frac{1}{4},\dots \right\rangle$$

Definition: An infinite sequence is a function whose domain is the set of positive integers.
Let $U_n =< U_1, U_2, U_3, \dots >$ be infinite sequence and by definition above $U_n: \mathbb{N} \longrightarrow \mathbb{R}$ (U_n is called the n^{th} term of the sequence).

For example,

$U_n = 1,-1,1,\dots,(-1)^n + 1,\dots$

$U_n = 2,4,6,\dots,2n,\dots$

$U_n = 0,\frac{1}{2},\frac{2}{3},\frac{3}{4},\dots,\frac{n-1}{n},\dots$

Examples Find a formula for n^{th} term of the infinite sequence?

$$1. \langle 1,-4,9,-16,\dots \rangle$$

$$2. \langle 1,\frac{1}{2},\frac{1}{6},\frac{1}{24},\dots \rangle$$

Solution :

1) $.< 1,-4,9,-16,\dots\dots >,\ u_n = (-1)^{n+1}n^2$
2) $< 1,\frac{1}{2},\frac{1}{6},\frac{1}{24},\dots >, u_n = \frac{1}{n!}$

1.1 Graphs of Sequences

The graph of the sequence U_n is the graph of the equation

$$f(n) = U_n, n = 1,2,3,\dots$$

For example the graph of sequence $U_n = 1,\frac{1}{2},\frac{1}{3},\dots$ We need to find the formula of sequence $U_n =$

Definition: The sequence U_n **converges** to the number L if for every positive number there corresponds an integer N such that for all $n > N$

$$|U_n - L| < \epsilon$$

Definition: The sequence U_n **diverges to infinity** if for every number M there is an integer N such that for all n larger than$N$, $U_n > M$. If this condition holds, we write

$$\lim_{n\to\infty} U_n = \infty,$$

In this case, we write $\lim_{n\to\infty} U_n = L$.

1- if $\lim_{n\to\infty} U_n = L$, U_n converges.

2- if $\lim_{n\to\infty} U_n = \infty$, U_n diverges.

Examples: Do these sequences converge or diverge: $U_n = \frac{2n^2+1}{2n^2+n+1}$

$$an = \frac{2n^2+1}{2n^2+n+1} \Rightarrow \lim_{n\to\infty} \frac{\frac{2n^2}{n^2}+\frac{1}{n^2}}{\frac{2n^2}{n^2}+\frac{n}{n^2}+\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{2+\frac{1}{n^2}}{2+\frac{1}{n}+\frac{1}{n^2}} = \frac{2+0}{2+0} = 1$$

The sequence $U_n$is convergent

Examples Do these sequences converge or diverge?

1) $U_n = \frac{1-2n}{1+2n}$

2) $U_n = \frac{\ln(n)}{n}$

3) $U_n = \frac{n^2-2n+1}{n-1}$

Solution :

1) $\lim_{n\to\infty} \frac{1-2n}{1+2n} = \lim_{n\to\infty} \frac{\frac{1}{n}-2}{\frac{1}{n}+2} = \lim_{n\to\infty} \frac{-2}{2} = -1$, thus it converges.

2) $\lim_{n\to\infty} \frac{n^2-2n+1}{n-1} = \lim_{n\to\infty} \frac{(n-1)(n-1)}{n-1} = \lim_{n\to\infty} n-1 = \infty$, thus it diverges.

3) $\lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{1/n}{1}$, (by using L'Hopital's rule)

$\Rightarrow \lim_{n\to\infty} \frac{1}{n} = 0$, thus it converges.

Theorem: The Sandwich Theorem for Sequences

Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be sequences of real numbers. If $a_n \leq b_n \leq c_n$ holds for all n beyond some index N, and if $\lim_{n\to\infty} = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$ also.
Applying the sandwich theorem.

Example Is the sequence $U_n = \frac{\sin\, n}{n}$ converge or diverge?
Solution:

$$\cos\, n \quad \Rightarrow -1 \leq \sin\, n \leq 1$$
$$\Rightarrow \frac{-1}{n} \leq \frac{\sin\, n}{n} \leq \frac{1}{n}$$
$$\Rightarrow 0 \leq \frac{\sin n}{n} \leq 0, \text{since } \lim_{n\to\infty} \frac{-1}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

Thus, the sequence $\frac{\sin\, n}{n} \Rightarrow 0$ converges.

Note that a sequence is called increasing if $u_n < u_{n+1}$ for all n as $< \frac{2^n-1}{2^n} >$. Similarly,

a sequence is decreasing if $u_n > u_{n+1}$ for all n as$\langle \frac{n+1}{n} \rangle$.

Theorem: The following sequences converge to the limits listed below:

1) $\lim_{n\to\infty} \frac{\ln\, n}{n} = 0$ 2) $\lim_{n\to\infty} \sqrt[n]{n} = 1$ 3) $\lim_{n\to\infty} x^{\frac{1}{n}} = 1, \quad (x > 0)$

4) $\lim_{n\to\infty} x^n = 0, \quad (|x| < 1)$ 5) $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$, (for any x)