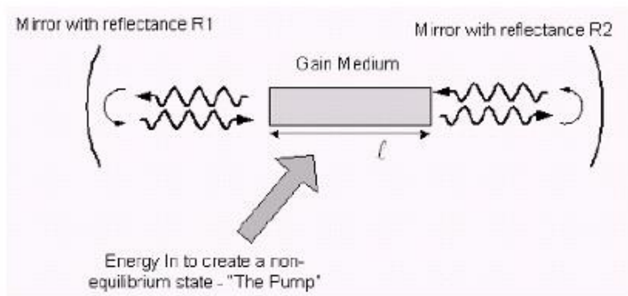
[Date]

Types of resonators

Resonator: It is a system consisting of two mirrors on a common optical axis, where laser photons are transmitted back and forth between the two mirrors for amplification.

It has a reflectivity of 100% and is usually made of aluminum (Al), silver (Ag), or copper (Cu), and sometimes plated with gold (Au). Sometimes, laser mirrors are made of a dielectric material consisting of multiple alternating layers of high and low refractive index



The intensity of reflected light is calculated in the case of a mirror with multiple layers

where n is the refractive index of the substrate material, n_1 is the refractive index of the low-index layer, and n_2 is the refractive index of the high-index layer, and k is the number of layers.

$$R = \frac{n_2^{k+1} - nn_1^{k-1}}{n_2^{k+1} - nn_1^{k+1}}$$

Notes:

1. When the difference in the refractive index values of the two materials (n1-nn) increases and the number of layers (k) increases, the reflectivity increases.

2. The thickness of the layers (t_1, t_2) is not equal due to the difference in refractive indices.

3. The coating material is selected based on the wavelength of the lase

Question: How can we calculate the reflectivity of a metallic mirror? The provided equation calculates the reflectivity based on the electrical resistivity, wavelength of the used laser, and the density of the metal:

$R = 100 - 3.65 * \sqrt{(\rho/\lambda)}$

Where ρ is the density of the metal in g/cm^3 and λ is the wavelength in micrometers (μm).

Question: Calculate the reflectivity of three metals with densities 10 g/cm^3, 15 g/cm^3, and 20 g/cm^3 when using wavelengths of 850 nm, 1064 nm, and 10600 nm.

Optical alignment: It is the process of placing two mirrors exactly in the same alignment so that the optical axis is straight. Visual alignment depends on:

a. The type of active medium b. The size of the system c. The complexity of the system

Question: What are the factors that determine the reflectivity of a dielectric mirror?

Answer:

1. **$n_1t_1 = n_2t_2 = \lambda/4$:** This equation represents the condition for maximum reflection in a multilayer dielectric mirror.
2. **(k):** The number of layers in the mirror.
3. **The type of material used for the coating.**

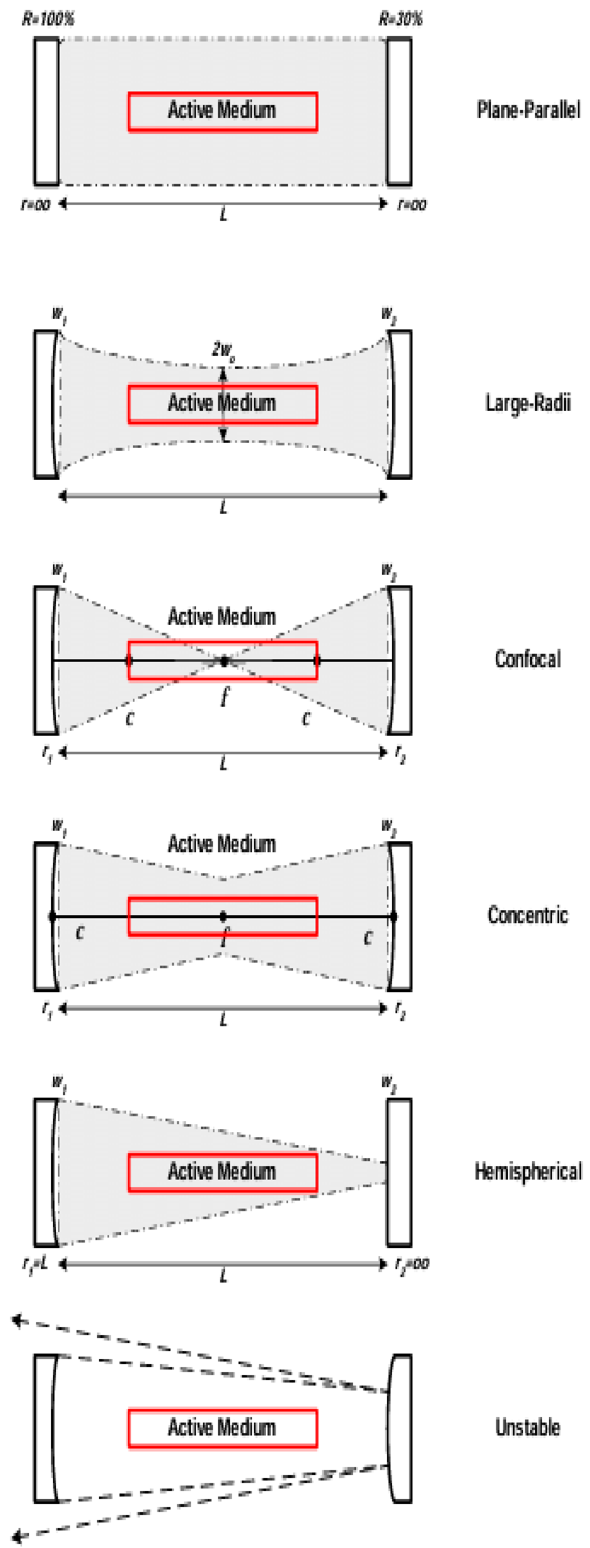
Question: Calculate the reflectivity of a multilayer mirror in an Nd:YAG laser system if the number of layers is 20, the refractive index of the first layer is 1.26, the refractive index of the second layer is 1.2, and the refractive index of the substrate is 1.33.

Why do we use polished metal mirrors as rear mirrors in CO_2 lasers?

Answer:

1. They can withstand high energy before being damaged.
2. They have high resistance to environmental and operational conditions.
3. Metals have high reflectivity for long wavelengths.

Types of laser resonators


R=100%
R=30%
Active Medium
Plane-Parallel
r=00
L
r=00
w1
w2
2w0
Active Medium
Large-Radii
L
w1
w2
Active Medium
c
f
c
Confocal
r1
L
r2
w1
w2
Active Medium
c
f
c
Concentric
r1
L
r2
w1
w2
Active Medium
Hemispherical
r1=L
L
r2=00
Active Medium
Unstable

1. **Plane-Parallel Resonator:** This type of resonator is characterized by large laser beam size and high efficiency in exciting the active medium. Additionally, it has a low probability of mirror damage but makes it difficult to control the type of transverse mode.
2. **Large-Radii Resonator:** In this type, the radii of curvature of the mirrors are much larger than the resonator length. The value of the radius of the output Gaussian mode (R) can be calculated as follows:

$R = X(1 + (\pi^2 w_0^2/X^2\lambda^2))$

This type is characterized by the ease of optical alignment and that the smallest radius of the laser beam is at the waist area. It is calculated as follows: the radius of the beam in the waist area is (W_0).

$$W_o^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{L(r_1 - L)(r_2 - L)(r_1 + r_2 - L)}{(r_1 + r_2 - 2L)^2}$$

The beam radius at the first mirror can be calculated as:

$$W_1^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_1^2 L(r_2 - L)}{(r_1 - L)(r_1 + r_2 - L)}$$

The beam radius at the second mirror can be calculated as:

$$W_2^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_2^2 L(r_1 - L)}{(r_2 - L)(r_1 + r_2 - L)}$$

Where r_1 r_2 is the radius of the first and second mirrors. If $r_1 = r_2$, it becomes

$$w_o^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{L(2r-L)}{4}$$

$$w_1^4 = w_2^4 = w^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r^2 L}{(2r-L)}$$

But if r>>L becomes

$$w^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{rL}{2}$$

Example: Calculate the value of the Gaussian beam radius (R) if w_0=1mm. Given that x=25cm and λ=0.5µm.

Solution:

$R = x(1 + (\pi^2 w_0^2)/(\lambda^2 x^2)) = 0.25(1 + ((3.14)^2(0.001)^2)/((0.5\times10^{-6})^2(0.25)^2)) = 158m$

3. Confocal Resonator:

In this type of resonator, the focus of the first mirror coincides with the focus of the second mirror, and the size of the laser beam becomes smaller under the following condition:

$r_1 = r_2 = L \Rightarrow r_1 + r_2 = 2L$

The radius of the laser beam is calculated as follows:

$w^2 = (r\lambda)/\pi$

$w_a^2 = (r\lambda)/(2\pi)$

$W/w_a = \sqrt{2} \approx 1.4$

The divergence angle of the laser beam is given by:

$$\theta_{confocal} = \sin^{(-1)} (\lambda/\pi W_0)$$

This type is highly dependent on the quality of the alignment, and the loss of higher order (non-Gaussian) transverse modes is significant, with the Gaussian mode being about 25 times larger than the modes of the laser beam itself

4- Concentric Resonator:

This resonator consists of two mirrors such that the sum of the radii of curvature of the two mirrors (r1+r2) equals the distance between them (L). If r1=r2 and also L=2r, the concentric resonator is called a spherical resonator. This type is characterized by the difficulty of optical alignment and the large transverse mode size at the output.

5- Hemispherical Resonator:

This type consists of a plane mirror and a concave mirror. Therefore, the transverse mode size is maximum at the concave mirror and minimum at the plane mirror. The beam diameter can be calculated as follows:

$$w_o^4 = w_1^4 = \frac{\lambda^2}{\pi^2} L(r_1 - L)$$

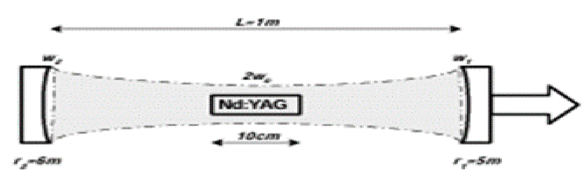
$$w_2^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_1^2 L}{r_1 - L}$$

6-Unstable Resonator

This resonator consists of two mirrors, one concave and the other convex, and is used at high powers of the laser system

Using an Nd:YAG laser resonator consisting of two concave mirrors as shown in the figure.

Calculate:



1. Waist beam size (w_0, w_1, w_2) to position the TEM_{00} at the waist and mirrors.
2. Far-field deviation angle (θdiv).

3. If we replace the front mirror with a plane mirror, what will change in the calculations?

Case 1: Radius of TEM mode laser beam$_{00}$ at waist (w_0)

$$w_0^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{L(r_1 - L)(r_2 - L)(r_1 + r_2 - L)}{(r_1 + r_2 - 2L)^2} = 2.81 \times 10^{-13} m$$

$$w_0 = (w_0^4)^{\frac{1}{4}} = 0.728 mm$$

And the size of the pattern or laser beam at the first woman

$$w_1^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_1^2 L(r_2 - L)}{(r_1 - L)(r_1 + r_2 - L)} = 3.56 \times 10^{-13} m$$

$$w_1 = (w_1^4)^{\frac{1}{4}} = 0.7723 mm$$

As for the size of the pattern or the laser beam for the second woman

$$w_2^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_2^2 L(r_1 - L)}{(r_2 - L)(r_1 + r_2 - L)} = 3.28 \times 10^{-13} m$$

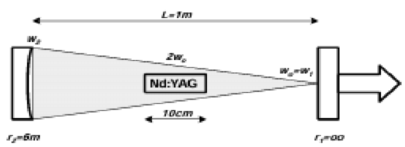
$$w_2 = (w_2^4)^{\frac{1}{4}} = 0.7567 mm$$

2: Now for the size of the pattern or the laser beam at the second mirror

$$\theta_{div} = \frac{\lambda}{\pi w_o} = \frac{1.064 \times 10^{-6} m}{3.14 \times 7.28 \times 10^{-4} m} = 4.634 \times 10^{-4}$$

Because we calculate the divergence angle of the pattern or laser beam as follows

3 Now we replace the mirror r1 =5m Front plane mirror $r_1 = (\infty)$ And we calculate the size or radius of the pattern or laser radiation again, like this



$$w_o^4 = w_1^4 = 8.1968\times10^{-13}\,m$$

$$w_o = w_1 = 8.686\times10^{-4}\,m = 0.8686\,mm$$

$$\theta_{div} = \frac{\lambda}{\pi w_o} = \frac{1.064\times10^{-6}\,m}{3.14\times9.515\times10^{-4}\,m} = 3.884\times10^{-4}$$

Problem:

Consider a laser resonator consisting of two concave mirrors, each with a radius of curvature (r=4m) and separated by a distance (L=1m). Calculate the beam waist size (TEM_{00} mode) at the center of the resonator and at the mirrors, as well as the far-field divergence angle (θdiv). The wavelength of the laser beam is (λ=514.5nm) and it is an argon laser (Ar^3). Then, explain how the beam size and divergence change if one of the mirrors is replaced with a plane mirror and the distance between the mirrors becomes (2L).

Solution:

Case 1: Large-Radii Resonator

Using the formula for the beam waist size, we get:

$$w_o^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{L(2r-L)}{4} = 4.6936 \times 10^{-14}\, m \Rightarrow w_o = 0.465 mm$$

$$w_1^4 = w_2^4 = w^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r^2 L}{(2r-L)} = 6.1310^{-14}\, m \Rightarrow w_1 = w_2 = w = 0.4976 mm$$

$$\theta_{div} = \frac{\lambda}{\pi w_o} = \frac{514.5 \times 10^{-9}\, m}{3.14 \times 23.8 \times 10^{-3}\, m} = 3.522 \times 10^{-4}$$

Case 2: Hemispherical Resonator

$$w_o^4 = w_2^4 = \frac{\lambda^2}{\pi^2} L(r_1 - L) \Rightarrow w_o = w_2 = 0.532 mm$$

$$w_1^4 = \frac{\lambda^2}{\pi^2} \cdot \frac{r_1^2 L}{r_1 - L} \Rightarrow w_1 = 0.615 mm$$

$$\theta_{div} = \frac{\lambda}{\pi w_o} = 3.078 \times 10^{-4}$$

Problem 1:

He-Nelaser with confocal resonator. The diameter of curvature of the mirrors is (r=0.6m). Calculate the radius of the laser beam at the center and in the mirrors, and the angle of divergence.

$$w_o^2 = \frac{r\lambda}{2\pi} = 5.0356\times10^{-8} \Rightarrow w_o = 0.2244mm$$

$$w^2 = \frac{r\lambda}{\pi} = 10^{-7} \Rightarrow w = 0.31735mm$$

$$w = \sqrt{2}\,w_o = 0.31735mm$$

$$\theta_{confocal} = \sin^{-1}(\frac{\lambda}{\pi w_o}) = 0.051433$$

Emission Linewidth

Laser rays are distinguished from ordinary light in that they are single-wavelength, meaning that their bandwidth is the(Δν)

The emission linewidth of a laser beam (Δν) is defined as the response of the active medium atoms to the electromagnetic field, while the quantity (c/2L) is known as the frequency spacing. There are two main groups of reasons for the broadening of the emission linewidth

1- Homogeneous Broadening

2- Inhomogeneous Broadening

Homogeneous Broadening As for the reasons

1- Collision Broadening

2- normal Broadening

Inhomogeneous Broadening As for the reasons

1- Dopant Broadening

2- Doppler Broadening

Example: An argon ion laser beam with a wavelength of 488 nm is incident on a material surface with a work function of 2.2 eV. Calculate the maximum kinetic energy of the emitted electrons.

Axial Modes

The electromagnetic field distribution of the laser wave is parallel along the optical axis of the resonator

The number of longitudinal modes generated within the resonator is calculated as follows:

$$N = \frac{2L}{\lambda} = \frac{2L}{c}\Delta\nu$$

There are procedures taken to reduce the number of longitudinal modes of a laser beam, such as:

1. Cooling the active medium.
2. Shortening the resonator length.
3. Using a quality factor control technique.

Quality Factor (Q)

The quality factor of a resonator is defined as the ratio between the frequency of the emitted laser beam (ν) and the emission line width (Δν). The quality factor equation is:

$$Q = \frac{\nu}{\Delta\nu} = \frac{4\pi\nu L}{c(1 - R_1 R_2)}$$

R_1 and R_2 are the reflectivities of the two mirrors

Example: Calculate the quality factor of a helium-neon laser system with a front mirror reflectivity of 95% and a cavity length of 1m. Then calculate the linewidth (Δν)

Example: A laser resonator has mirrors with reflectivities of R2=99% and R1=99%. The distance between the mirrors is 0.5m. Calculate the linewidth of the laser beam and the number of longitudinal modes.

Example: Calculate the quality factor of a neodymium-glass laser if the linewidth is 3×10^{12} Hz and the cavity length is 0.5m. Then, calculate the number of longitudinal modes