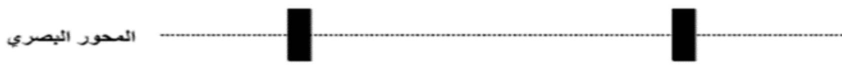
# LASER BASIC

# Basic Concepts

**LASER** is an acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation.

**A photon** is the amount of light energy in electromagnetic waves. They possess specific properties such as wavelength (π) and frequency (ν).

**optical axis** is an imaginary straight line that connects two optical elements, as shown in the figure.




**Phase:** This is the state of an electromagnetic wave at a specific point in space and at a particular moment in time.

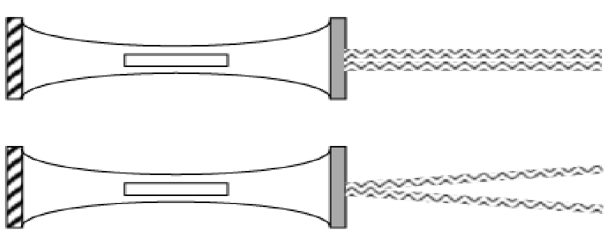
**Interference:** This is the condition of having two or more electromagnetic waves on the same optical path, with or without a phase difference between the waves.

**Constructive Interference:** This is the interference where there is no phase difference between the waves, and the resultant amplitude is the sum of the amplitudes of the individual waves. This is a condition for multilayer laser mirrors

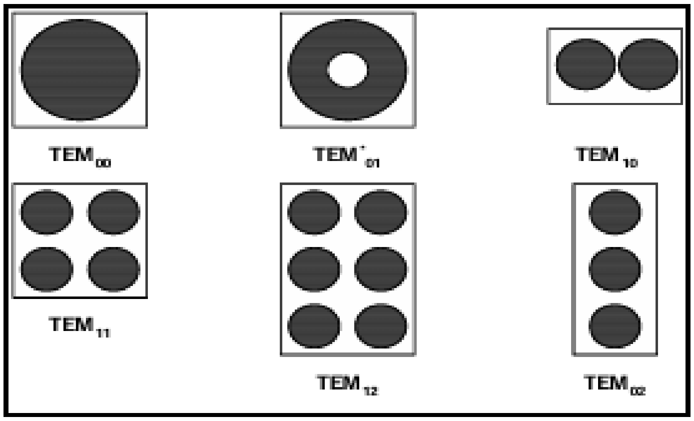
**Destructive Interference:** This is the interference where there is a phase difference between the waves, causing one wave to weaken or completely cancel out the other wave. This type of interference is harmful to lasers.

**A resonator** is an array of two mirrors aligned on a common optical axis. Photons of laser light travel back and forth between the two mirrors in order to be amplified. One of these mirrors is 100% fully reflective (the rear mirror), while the other is partially reflective (the front mirror).

**Divergence:** This is the amount of deviation of a laser beam from the optical axis as it travels out of the resonator. As shown in the following figure, the first beam has no divergence, while the second beam has a clear divergence from the optical axis.




perpendicular to the optical axis of the resonator, as shown in the following figure:




The mode (TEM00) is called the fundamental mode or Gaussian mode. This mode contains approximately 85% of the output laser beam energy and is circular and uniform. It is considered the most preferred mode for operation of laser systems in general. The mode (TEM01) is called the donut mode. This situation results from the presence of impurities or particles on the surface of the mirror or within the active medium. The pattern with a higher order is more open than the pattern with a lower order. Question:

**How do we calculate the divergence angle ($\theta_{div}$) of the laser beam?**

Answer: For the fundamental mode ($TEM_{00}$), it is calculated as follows

$$\theta_{div} = \lambda / \pi w_0$$

Where $\lambda$ is the wavelength of the light and $w_0$ is the minimum beam radius inside the resonator.

**Coherence:** This is a state where two or more electromagnetic waves (photons) have the same spatial or temporal phase. It is a characteristic of laser light.

**population:** Population number (N) refers to the number of atoms or molecules of a substance present in a specific state or energy level.

**Energy difference (ΔE):** The energy difference (ΔE) is the difference in energy between the highest energy level (E2) and the lowest energy level (E1) of the system. It is calculated as follows:

$$\Delta E = E_2 - E_1$$

**Work function($\Phi_m$):** The work function ($\Phi_m$) is the energy required to liberate one electron from the surface of a material if one photon of light falls on it.

Unit conversions:

**1 electron volt (eV) = $1.06 \times 10^{-19}$ joule (J)**

**1 joule (J) = $9.434 \times 10^{18}$ electron volts (eV)**

**Frequency (Frequency):** The number of electromagnetic wave oscillations per second. It is denoted by (ν) and calculated as follows:

$$\nu = E/h = c/\lambda$$

Where E is the energy of the electromagnetic wave, h is Planck's constant ($h = 6.63 \times 10^{-34}$ J.s), and c is the speed of light in a vacuum ($c = 3 \times 10^8$ m/s), and λ is the wavelength of the electromagnetic wave.

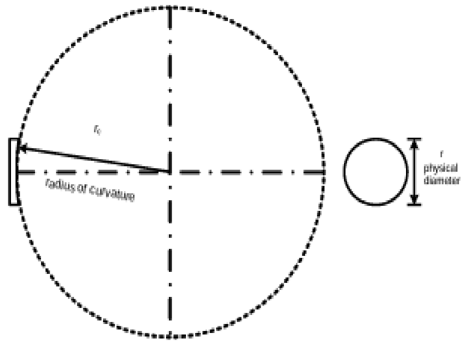
**The electromagnetic spectrum (Electromagnetic Spectrum):** The range of wavelengths of electromagnetic radiation, extending from radio waves (Radiowaves) to gamma rays (Gamma Rays), as shown in the following table

| Name | الترددات | الأطوال الموجية | المنطقة الطيفية |
|---|---|---|---|
| RF (Radiowaves) | 30Hz-3GHz | 3m upward | الموجات الراديوية |
| MW(Microwaves) | 3GHz-30GHz | 3cm-3m | الموجات المايكروية |
| IR (Infrared) | 30GHz-30THz | $0.7\mu m$-3cm | تحت الحمراء |
| VIS (Visible) | 30THz-3PHz | $0.4\mu m$-$0.7\mu m$ | المنطقة المرئية |
| UV (Ultraviolet) | 3PHz upward | 3nm-$0.4\mu m$ | فوق البنفسجية |
| X-Ray | 100PHz-10000PHz | 0.03nm-3nm | الأشعة السينية |
| $\gamma$-Ray (Gamma Rays) | $10^4$PHz-$10^5$PHz | 3pm-0.03nm | أشعة كاما |
| Cosmic Rays | $10^5$PHz upward | 3pm downward | الأشعة الكونية |

**Operation Mode:** This refers to the way a laser system operates. There are two main types: pulsed and continuous wave (CW). CW lasers emit a continuous beam of light, while pulsed lasers emit short bursts of light. There is also a type called quasi-CW, which is a pulsed laser with a high repetition rate that mimics a continuous wave.

**Physical Diameter (D):** This is the distance between any two points on the circumference of a mirror.

**Radius of Curvature (r):** This is the radius of the circle of which the mirror is a part.




## Laser Generation: The process of converting energy into coherent light.

**Basic Components of a Laser:**

1. Active Medium: This is the material used for amplifying light. It can be a gas (like helium-neon), liquid, or solid (such as ruby).
2. Energy Source: Used to pump energy into the active medium, exciting electrons to higher energy levels. This source can be a flash lamp, an electric arc, or even another laser.
3. Optical Cavity: Consists of two mirrors, one fully reflective and the other partially reflective. The cavity amplifies light and produces a coherent laser beam.

**Laser Generation Process:**

1. Energy Pumping: Energy is directed from the source to the necessary medium, which excites the electrons to higher energy levels.
2. Spontaneous Emission: The excited electrons spontaneously return to lower energy levels, resulting in the emission of light photons.
3. Stimulated Emission: When a photon collides with an excited electron, it stimulates it to return to the lower energy level and emit another photon identical to the first in phase, frequency, and direction. This is the fundamental process that leads to light amplification.
4. Amplification in the Optical Cavity: The output light reflects back and forth between the cavity mirrors, increasing the number of identical photons and amplifying the optical signal.
5. Laser Beam Emission: A portion of the emitted light passes through the partially reflective mirror, forming a coherent laser beam.

**. Conditions for Laser Operation:**

1. **Presence of an active medium:** This is the fundamental basis for laser operation, a system with a large number of atoms, molecules, or ions that emit a spectrum, part of which falls within the visible range of electromagnetic radiation.
2. **Achieving reverse qualification:** This is the process of making the number of atoms in the upper level greater than their number in the ground state. This is achieved through one of the pumping methods.

3. **Optical feedback:** This is a necessary condition for the emitted radiation to take its correct oscillation and thus obtain a beam of radiation with a high degree of directionality and coherence.
4. **Achieving the threshold condition:** This is a condition that must be met in order to start the amplification process in the active medium and then the oscillation process in the resonator.

## Laser properties

Monochromaticity: This property signifies that laser light is monochromatic, meaning it consists of a single wavelength. This distinguishes it from ordinary light, such as sunlight or light from a bulb, which is composed of a spectrum of wavelengths.

Directionality: Laser light exhibits a high degree of directionality, traveling in a nearly parallel beam with minimal divergence. In contrast, light from a conventional light source spreads out in all directions.

Coherence: Laser light possesses a property known as coherence, which means that the light waves maintain a constant phase relationship. This can be mathematically represented by the equation:

$y=A\cos(wt+f)$

where A is the amplitude, w is the angular frequency, and f is the initial phase. Coherence can be categorized into temporal coherence and spatial coherence.

Brightness: The brightness of an electromagnetic wave refers to the power emitted per unit surface area per unit solid angle

**Coherence**

Laser light exhibits a high degree of coherence, meaning it has both high temporal and spatial coherence. In contrast, traditional light sources are considered incoherent.

Imagine laser light as waves with a frequency of approximately $10^{14}$ Hertz (corresponding to a wavelength of a few micrometers). For such waves to be coherent, two conditions must be met:

first, they must have a nearly single frequency, meaning the spread of frequencies around this value (the spectral linewidth) must be narrow. When this condition is met, the light is said to have high temporal coherence. Second, the wavefront must maintain its shape over time. When this condition is met, the wave is said to have spatial coherence. For a light source to be fully coherent, it must have both perfect temporal coherence and perfect spatial coherence

## What is the difference between laser and maser?

**(Maser) Microwave Amplification by Stimulated Emission of Radiation**

**1.** The difference between energy levels is large

The difference between energy levels is small

**2.** Spontaneous emission is dominant

Stimulated emission is dominant

**3.** The beam does not have coherence and monochromaticity properties

The beam has coherence and monochromaticity properties

**4.** Occurs in the microwave frequency range

Occurs in the visible light frequency range and other frequencies

**Pumping:**

This is the process of transferring energy from an external source to the active medium of a laser.

The type of material used to generate the laser (the active medium) determines the pumping method for the system.

Pumping depends on:

- **The type of active medium:** solid, liquid, or gas.
- **The operating mode:** continuous or pulsed.
- **The nature of the pumping element:** electrical, chemical, optical, or thermal.
- **The width of the absorption spectrum of the pumping radiation, as the radiation must be absorbed well.**

**Question: Why can't a CO2 laser be pumped using a flash lamp?**

**Answer:** Because the absorption spectrum of $CO_2$ gas is very narrow

**Three main factors affect the efficiency of pumping:**

1. **The optical pumping:** In this method, a high-power light source is used to excite the active medium, causing its atoms, ions, or molecules to absorb this energy and transition to a higher energy level. This method is commonly used in solid-state and liquid lasers.

The efficiency of pumping using a flash lamp depends on four factors:

- **The efficiency of transferring the energy available in the lamp to the active material,** represented by the quantity (nt), which is the efficiency of converting the electrical energy supplied to the lamp into light.
- **The radiative efficiency of the lamp (nr).**
- **The absorption efficiency (na).**
- **The quantum efficiency of pumping (ng).**

The pumping efficiency (ηp) is given by the equation: $\eta p = (\eta t).(\eta r).(\eta a).(\eta g)$

2. **Electrical pumping:** This method is used in gas lasers through electrical discharge, and in semiconductor lasers by applying an electric voltage whose field injects charge carriers (current carriers) into the junction region

3. **Chemical pumping:** In this method, you do not need an external energy source, as it is naturally available within the material used. The products of the chemical reaction between the components of the chosen material constitute the active medium of the laser process, while the energy released from the reaction itself excites this material and achieves its reverse qualification.

For example:

$F + H_2 = HF + H$

$H + F_2 = HF + F$

The HF* molecule forms the active medium in the above-mentioned chemical laser, and has the ability to produce stimulated emission

## Absorption and Emission

Absorption: This is a process of transition (gaining energy) from the surrounding medium to the matter. As a result, the energy of the atoms or molecules of the matter increases and they move to higher energy levels (E2) from their initial state (E1) before absorbing the energy.

Emission: This is a process of transition (losing energy) from the matter to the surrounding medium. As a result, the energy of the atoms or molecules of the matter decreases and they move to lower energy levels (E1) from their initial state (E2) before emitting the energy.

**1-3. Absorption:**

Suppose we have two energy levels of an atom, E1 and E2, where E2 is the higher energy level. The atom is initially in the ground state (E1).

When the atom is exposed to electromagnetic radiation with a frequency (ν) such that the energy of the photon (hv) is equal to the energy difference between the two levels (E2-E1), the atom has a probability of being excited to the higher energy level E2. This process is called absorption.

**Note:** hv represents the energy of a photon, where h is Planck's constant and ν is the frequency of the radiation.

The rate of absorption can be expressed by the equation:

$$\mathbf{(dN1/dt)_{abs} = -W_{12}N_1 \ \ldots\ldots\ldots\ldots.. (5)}$$

Where N1 is the number of atoms in the lower energy level (E1) per unit volume, and W12 is the probability of absorption, which depends on the intensity of the incident radiation:

$$\mathbf{W_{12} \propto \rho\nu_{12} \ \ldots\ldots\ldots\ldots.. (6)}$$

The intensity of the incident radiation is given by:

$$\mathbf{\rho\nu_{12} = \sigma_{12} \cdot F}$$

Calculate the ratio between spontaneous emission and stimulated emission for a tungsten lamp operating at a temperature of T=1727°C, where the emitted light is visible?

"Where $\sigma_{12}$the cross-sectional area **F** is called photon flux density, when the material is exposed to electromagnetic radiation with intensity, $\rho\nu_{12}$ the material's atom at level 1 absorbs this radiation and jumps to level 2 with aprobability $W_{12}$ of one atom per second

$$\mathbf{W_{12} = B_{12}\ \rho\nu_{12} \ \ldots\ldots\ldots\ldots..(7)}$$

The quantity $B_{12}$ is called the Einstein absorption coefficient, and the number of these transitions from 1 to 2 per second per cubic meter is equal to $W_{12}N_1$

**$(dN1/dt)_{abs} = B_{12}\ \rho v_{12} N_1$ …………..(8)**

**2.3 Spontaneous Emission:**

If there are two atomic energy levels (E2 > E1) and the atom is initially in the higher energy level (E2), the atom will naturally tend to decay to the lower energy level (E1). In doing so, it will release energy equal to the difference between the two levels (E2 - E1) in the form of electromagnetic waves. This phenomenon is called spontaneous emission. The frequency of the emitted wave can be expressed using Planck's law as:

**$v = (E_2 - E_1) / h$ ..... (9)**

Where h is Planck's constant

To describe this emission, we assume that a unit volume of the material contains a certain **$N_2$** number of atoms in level 2 at a given time(t). Therefore, the decay rate of the material's atoms due to spontaneous emission is expressed by the relationship.

**$(dN_2/dt)_{sp} = -AN_2$** ..... (10)

Where N2 is the number of atoms in the specific energy state, t is time, and A is Einstein's coefficient of spontaneous emission, which represents the probability of spontaneous emission.

The emission, which is the average time of the atom in the excited state to the average lifetime of spontaneous emission (τ), is equal to the reciprocal of the quantity (A).

$$\tau_{sp} = 1/A$$

**3-3Stimulated Emission**

Let's now consider an atom that is initially in energy level (2), but in the presence of electromagnetic radiation in the medium with a frequency ν such that (hν) is exactly equal to the energy difference between levels (1) and (2). There is a probability that this radiation will stimulate the atom in level (2) and force it to transition to the lower level (1). In this case, the energy difference (E2-E1) of the transitioning atom is released as electromagnetic waves that are added to the incident wave and take on its characteristics. In other words, the wave emitted from an atom stimulated in level (2) is in phase with the incident wave.

$$(dN_2/dt)_{st} = -W_{21}N_2 \text{ ------------------ } (11)$$

Where the term $(dN2/dt)_{st}$ represents the rate of change in the number of atoms in the second energy level (2) with respect to time due to stimulated emission.

W21: is the probability of stimulated emission and its unit is (sec)-1. It depends on the radiation density.

$$W_{21} \propto \rho\nu \text{ ------------------ } (12)$$

$$\rho\nu = \sigma_{21} F \text{ ------------------ } (13)$$

Where $\sigma_{21}$ is the stimulated emission cross-section and F is the photon flux of the incident wave.

When a substance is exposed to electromagnetic radiation with a spectral density (ρν) and the atom is in the upper level, it is stimulated and transitions to the lower level with a probability of $W_{21}$ atoms per second, where:

$$W_{21} = B_{21}\rho\nu_{21} \text{ ------------------ } (14)$$

The quantity ($B_{21}$) is called Einstein's coefficient of stimulated emission, and the number of these transitions per second per cubic meter is equal to ($N_2W_{21}$).

$$(dN_2/dt)_{st} = B_{21}\rho\nu_{21}N_1 \text{ ------------------ } (15)$$

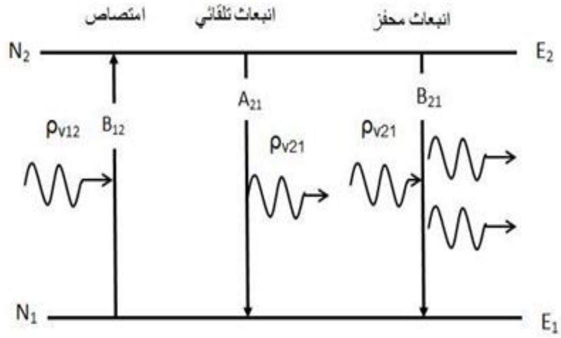
4- Einstein's calculations for probability coefficients:

Einstein studied the relationship between the transition probability for the three processes of absorption, spontaneous emission, and stimulated emission. If we assume that a substance is placed in an electromagnetic radiation cavity at a temperature (T), then when thermal equilibrium is reached, the radiation is distributed within the cavity and this radiation has a spectral distribution whose density (ρν) is given by the relation:

$$\rho\nu = (8\pi h\nu^3/c^3) * (1 / (e^{(h\nu/kT)} - 1)) \text{ ------------------ } (16)$$

This is the equation for the spectral distribution of blackbody radiation as found by Planck.

Assuming the existence of two energy levels for an atom of the substance, and assuming that N1 and N2 are the populations of these two levels, respectively, in the equilibrium state, then there is a probability of occurrence of each of the three processes between these two levels and as follows.




شكل (6): توضيح لمعاملات الاحتمالية لاينشتاين

Since the substance is in a state of thermal equilibrium, the number of downward transitions (emission) must equal the number of upward transitions (absorption), i.e.:

Radiation interaction with matter

$$B_{12} N_1 P_{V12} = A_{21} N_2 + B_{21} N_2 PV_{12} \text{....(17)}$$

$$P_{V12} = A_{21} N_2 / (B_{12} N_1 - B_{21} N_2) + B_{21} N_2$$

By arranging the terms, we get the following form:

$$P_{V12} = (A_{21} / B_{21}) / (B_{12} N_1 / B_{21} N_2 - 1) \qquad \text{....(18)}$$

And using Boltzmann statistics, the state of thermal equilibrium and the distribution of the atoms of the substance on the energy levels has the following equation (4):

$$N_2/N_1 = e\text{^}(-(E_2-E_1)/kT) = e^{(-hv/kT)} \text{.....(19)}$$

And substituting equation (19) in equation (18), we get:

$$\rho_{v12} = (A_{21}/B_{21}) / (B12/B_{21} * e^{(hv/kT)} - 1) \text{.....(20)}$$

And since we assumed that the atomic system is in a state of thermal equilibrium, this gives radiation identical to blackbody radiation equation (16).

By comparing equations (16) & (20), we get:

$$B_{12} = B_{21} = B \text{.....(21)}$$

$$A_{21}/B_{21} = 8\pi hv^3/c^3 \text{.....(22)}$$

Equations (21) & (22) are referred to as Einstein's equations. Equation (21) indicates that the probability of the absorption process is equivalent to the probability of the stimulated emission process. While equation (22) enables us to calculate the ratio between the spontaneous emission rate and the stimulated emission rate for two energy levels. By substitute equation (22) into (16).

**$\rho_v = (A_{21}/B_{21}) / (e^{(hv/kT)} - 1)$ .....(23)**

**$R = e^{(hv/kT)} - 1$.....(24)**

**Thus, $\rho_v = (A_{21}/B^{21}) * (1/R)$**

**Note that R represents the ratio between the rate of spontaneous emission to the stimulated emission.**

**$R = A_{21}/ \rho_v B_{21}$.....(25)**

**Example: A tungsten lamp operates at a temperature of 2000K. If the emission frequency is equal to (5x10^14 Hz), calculate the ratio of spontaneous emission to stimulated emission.**

Example: Find the wavelength at which the spontaneous emission rate equals the stimulated emission rate in a tungsten lamp at room temperature under thermal equilibrium condition؟

**Example:** If the energy difference between the levels E2 and E1 equals KT (0.025eV) at room temperature, calculate the number of electrons N2 in terms of E1?

solution:

**$\Delta E = E_2 - E_1 = kT \Rightarrow N2/N1 = e^1 \Rightarrow N2 = 0.37\ N1$**

This means that in the normal state, the number of atoms in energy level E1 is greater than the number of atoms in energy level E2 (N1 > N2). Therefore, it is impossible to receive the laser due to the lack of population reflectance

**Example:** Explain mathematically why laser generation does not occur when the thermal energy is equal to the photon energy?

solution:

E2-E1 = High voltage = kT

N2/N1 = Exp(-hv/KT) = Exp(-1)

N2 = 0.37N1

n2 < n1

therefore, laser generation does not occur.

**Exp.(1):**

Find the relative population of the two states in a ruby laser that produces a light beam of wavelength $\mathbf{6943A^o}$ at $\mathbf{300\ K}$ and $\mathbf{500K}$.

**Solution:** The population ratio is given by:

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} = \frac{12400}{6943\text{A}^\circ} = 1.79eV$$

a- $$\frac{N_2}{N_1} = \exp\left[\frac{-1.79eV}{(8.61*10^{-5}eV/K)*300K}\right] = e^{-69.3} = 8*10^{-31}$$

b- $$\frac{N_2}{N_1} = \exp\left[\frac{-1.79eV}{(8.61*10^{-5}eV/K)*500K}\right] = e^{-41.58} = 8.7*10^{-19}$$

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**Exp. (2)** :Find the ratio of population of the two states in a He-Ne laser that produces light of wavelength $\mathbf{6628A^o}$ at $\mathbf{27^o\ C}$.

**Solution:**

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} = \frac{12400}{6328\text{A}^\circ} = 1.96eV$$

$$\frac{N_2}{N_1} = \exp\left[\frac{-1.96eV}{(8.61*10^{-5}eV/K)*300K}\right] = e^{-75.88} = 1.1*10^{-33}$$

**Exp. :**

The wavelength of emission is $\mathbf{6000\ A^o}$ and the lifetime $\tau_{sp}$ is $\mathbf{10^{-6}s}$. Determine the coefficient for the stimulated emission.

**Solution :**

The coefficient for stimulated emission is given by

$$B_{21} = \frac{c^3 A_{21}}{8\pi\,\mu^3\ \text{h}\nu^3}$$

we get $\mu = 1 \Longrightarrow \nu^3 = \frac{c^3}{\lambda^3}$ taking and $\lambda^3 = \frac{c^3}{\nu^3}$ But $A_{21} = \frac{1}{\tau_{sp}}$

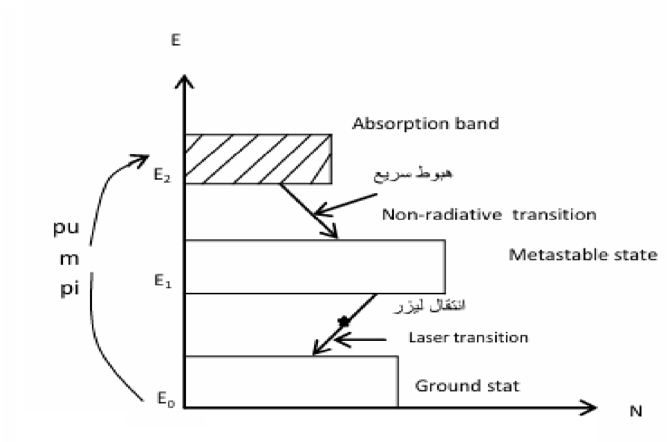
$$B_{21} = \frac{\lambda^3}{8\,\pi\ \text{h}\,\tau_{sp}} = \frac{(6000 * 10^{-10})^3 m^3}{8 * 3.14 * 6.626 * 10^{-34}\ Js\ 10^{-6}\ s}$$

$$= \frac{216 * 10^{-21} m^3}{166.6\ * 10^{-40} J\ s^2} = 1.3 *\ 10^{19}\boldsymbol{m/kg}$$

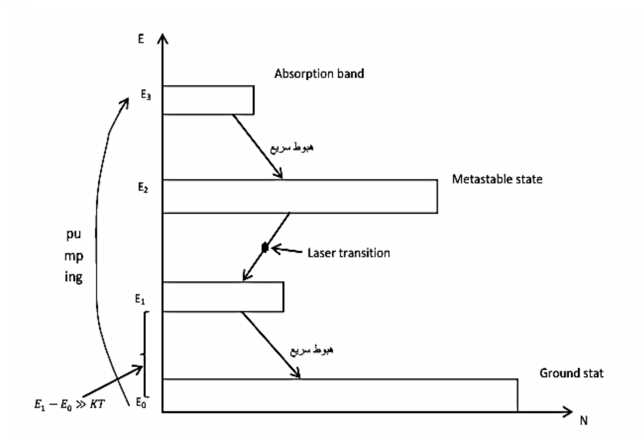
**Example: What is the temperature required for laser action to occur and laser generation?**

**Compare between the three-level and four-level systems in a tabular form. Which system is better? Prove it using equations.**

| Three-Level System | Four-Level System |
|---|---|
| 1. Consists of three energy levels | 1. Consists of four energy levels |
| 2. Requires pumping a large number of atoms from the ground state to the excited state to achieve population inversion | 2. Requires pumping a smaller number of atoms from the ground state to the excited state to achieve population inversion |
| 3. The ground state is the same as the lower laser level | 3. The ground state is not the laser level |
| 4. Laser action occurs between $E_2$ and E1 | 4. Laser action occurs between $E_3$ and $E_2$ |
| 5. Requires a high-energy pumping source | 5. Does not require a high-energy pumping source |
| 6. The lifetime of level $E_2$ is relatively long | 6. The lifetime of level $E_3$ is very short |
| 7. The lifetime of level E3 is very short | 7. The lifetime of level $E_2$ is relatively long |
| 8. A fast transition occurs between E3 and $E_2$ | 8. A fast transition occurs between $E_3$ and $E_2$ |
| 9. The output laser power equation is: $P_1=hv(P_2-W_{21}AN_C)$ | 9. The output laser power equation is: $P=hv(w_PBN_1-A_{21}N_2)$ |
| 10. Lower efficiency than the four-level system | 10. Higher efficiency than the three-level system |
| 11. The laser power depends on: 1. Laser beam frequency (v) 2. Stimulated emission probability (w) 3. Effective pumping rate ($P_2$) 4. The difference in the number of atoms in the two levels N1and $N_2$ ($\Delta N_C$) | 11. The laser power depends on: 1. Laser beam frequency (v) 2. Stimulated emission probability (w) 3. Efficiency of level B (E2) 4. Spontaneous emission probability A21 5. The number of atoms in levels N2 and N1 |
| 12. Under thermal equilibrium, $N_1$ and $N_2$ are very small and can be neglected | 12. Under thermal equilibrium, $N_2$ is very small and can be neglected |
| 13. There is a transition between level $E_1$ and the ground state | 13. There is no transition between level $E_2$ and the ground state |




Qualification Energy levels for the three-level system




Qualification Energy levels for the four-level system

## saturation

In this case, we find that atoms in a medium are subjected to a high-intensity electromagnetic field with a suitable frequency. When there are two energy levels for the atoms of a substance in an electromagnetic field with intensity (1) and frequency ωo, the effect of the field on the atoms of the medium is to make the energy levels equal. Usually, the lower energy level is more populated than the higher energy level N2. And trying to make the energy level of the atom equal under the influence of a high-intensity electromagnetic field is what we call (saturation).

We assume that the atom has only two energy levels: that is, the total number of atoms of the substance present in a unit volume N is equal to

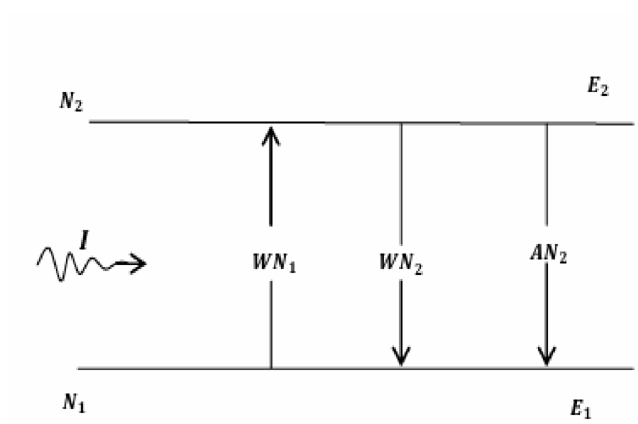
$$N_t = N_1 + N_2 \ (1)$$

$$N_1 = N_t - N_2 \ (1)$$

$$dN_2/dt = WN_1 - (WN_2 + AN_2) = 0$$

$$WN_1 = WN_2 + A\,N_2 \ (2)$$

$$\Rightarrow N_1 = N_2(W+A)/W \ (2)$$




**By equating equations (1) and (2), we get:**

$$N_t - N_2 = N_2(W+A)/W$$

=> $N_t = N_2 + N_2(W+A)/W$ **Two energy levels under the influence of a magnetic field with intensity I**

=> $N_t = N_2(2W+A)/W$

$N_2 = (N_1W) / (2W + A)$ **...(3)**

**Show that equation (1) simplifies to:**

$N_1 = (N_1W(W+A)) / (W(2W+A))$ **...(4)**

$\therefore \Delta N = N_1 - N_2 = [(N_1(W+A)) / (2W+A)] - [N_1W / (2W+A)] = (N_1A) / (2W+A)$

$$\tau_{sp} = 1/A$$

$$\Delta N = \frac{N_t}{1+\frac{2W}{A}} = \frac{N_t}{1+2W\mathcal{T}} \text{-----------------} \quad (5)$$

**Equation 5 shows that the population reversal between two energy levels depends on the atomic properties of the substance**

**W: spectral energy density of incident radiation.**

**$\tau$: average lifetime of the upper energy level.**

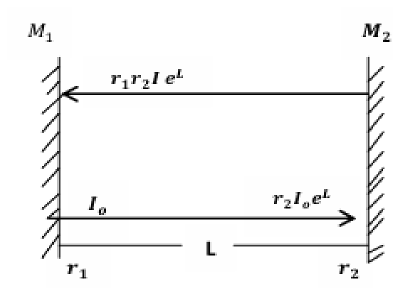
**A: Einstein coefficient A related to spontaneous emission.**

## Threshold condition

As the light bounces back and forth in the optical resonator , it undergoes amplification as well as it suffers various losses . The losses occur mainly due to transmission at the output mirror and due to the scattering and diffraction of light within the active medium .

For the proper buildup of oscillation , it is essential that the amplification between two consecutive reflection of light from rear end mirror can balance the losses .We can determine thethreshold gain by considering the change in intensity of a beam of light undergoing round tripe within the resonator .

Let us assume that the laser medium fills the space between the mirrors $M_1$ to $M_2$ , which have reflectivity $r_1$**to** $r_2$ . Let the mirrors be separated by a distance $L$ . Further , let the intensity of the light beam be $I_o$. Then , in traveling from mirror $M_1$ tomirror $M_2$ , the beamintensity increases from $I_o$ to $I(L)$ , which given by ,




$$(L) = I_o e^{(\gamma - \alpha_s)L} \text{ ---------------------- (1)}$$

After reflection at $M_2$ , the beam intensity will be $r_2 I_o e^{(\gamma - \alpha_s)L}$ and after a complete trip the final intensity will be

$$e^{2(\gamma - \alpha_s)L} \geq \frac{1}{r_1 r_2}$$

Taking logarithms on both the sides , we get

$$2L\, r_1 r_2\, (\gamma - \alpha_s) \geq -ln\, r_1 r_2$$

$$\gamma - \alpha_s \geq -\frac{1}{2L} ln\, r_1 r_2$$

$$\gamma \geq \alpha_s - \frac{1}{2L}\, ln\, r_1 r_2$$

$$\gamma \geq \alpha_s + \frac{1}{2L}\, ln\, \frac{1}{r_1 r_2} \text{---------------- (3)}$$

Eq. (3) is known as the condition for lasing. It shown that the initial gain must exceed the sum of the losses in the cavity. This condition is used to determine the threshold value of pumping energy necessary for lasing action

the amplification of the laser will be dependent on how hard the laser medium is pumped . As the pump power is slowly increased , a value of called threshold value will be reached and the laser starts oscillating .

$\gamma_{th}$ the amplification of the laser will be dependent on how hard the laser medium is pumped .

As the pump power is slowly increased , a value of $\gamma_{th}$ called threshold value will be reached and the laser starts oscillating .

The threshold value of $\gamma_{th}$ is given by

$$\gamma_{th} = \alpha_s + \frac{1}{2L} \ln \frac{1}{r_1 r_2} \text{----------------} \quad (4)$$

Therefore , for the laser to oscillate , $\gamma \geq \gamma_{th}$ . Eq. (4) is known as the threshold condition for lasing and state the criterion when the net gain would be able to counteract the effect of losses in the cavity . The value of $\gamma$ must be at least $\gamma_{th}$ for laser oscillation to commence .

If $\gamma \geq \gamma_{th}$ , the wave grow and the amplifier reaches .

saturation . It lowers the value of $\gamma$ in turn and eventually an equilibrium value is attained at $\gamma_{th}$ .

We define a quantity $N_{th}$ as $N_{th} = (N_2 - N_1)$

$N_{th}$ is called the critical population inversion or threshold population inversion density . It denotes the minimum population inversiondensity required to start lasing action and then to sustain .

from eq. :

$$G = \gamma = (N_2 - N_1)\frac{B\, h\nu}{\mathrm{v}}.\ g(\nu)$$

$$B = \frac{c^3 .\ A}{8\,\pi\, \mathrm{h}\nu^3.\ n^3} = \frac{c^3}{8\,\pi\, \mathrm{h}\nu^3.\ n^3\, \tau_{sp}} = \frac{\mathrm{v}^3}{8\,\pi\, \mathrm{h}\nu^3.\ \tau_{sp}}, A = \frac{1}{\tau_{sp}}, \mathrm{v}^3 = \frac{c^3}{n^3}$$

$$\therefore\ \gamma_{th} = (N_2 - N_1)\frac{\mathrm{v}^3}{8\,\pi\, \mathrm{h}\nu^3.\ \tau_{sp}}.\frac{h\nu}{\mathrm{v}}.g(\nu)$$

$$\gamma_{th} = (N_2 - N_1)\ .\ \frac{\mathrm{v}^2.\ g(\nu)}{8\,\pi\, \nu^2.\ \tau_{sp}}$$

$$\therefore\ N_{th} = \frac{8\,\pi\, \nu^2.\tau_{sp}\,\gamma_{th}}{\mathrm{v}^2.\ g(\nu)} = \frac{8\,\pi\, \nu_o{}^2.\tau_{sp}\,\gamma_{th}\,\Delta\nu}{\mathrm{v}^2}\text{------------- (5)},\ g(\nu) = \frac{1}{\Delta\nu}$$

we replaced $g(\nu)$by $\frac{1}{\Delta\mathrm{v}}$in the above equation using the relation $[\gamma_{th} = \alpha_s - \frac{1}{2L}\ ln\ (\ r_1 r_2\ )]$

we note that the lasing threshold will be achieved readily at the central frequency $\nu_o$ .

Thus , we write the threshold population inversion :

$$\therefore\ N_{th} = \frac{8\,\pi\, \nu_o{}^2\,\tau_{sp}\,\Delta\nu}{\mathrm{v}^2}.\ [\alpha_s - \frac{1}{2L}\ ln\ (\ r_1 r_2\ )]\text{------------- (6)}$$

OR

$$\therefore\ N_{th} = \frac{8\,\pi\, \nu_o{}^2\,\tau_{sp}\,\Delta\nu}{\mathrm{v}^2}.\ [\alpha_s + \frac{1}{2L}\ln(\frac{1}{r_1 r_2})]\text{------------- (6)}$$

**Example** For a neodymium YAG (Nd:YAG) laser, calculate the inverse qualification required to achieve a gain factor of 1 $cm^{-1}$

$\tau_2$=230μsec, λ=1.06μm, $\Delta\nu=3\times10^{12}$Hz, n=1.82.

Calculate the population inversion to obtain on gain coefficient $G = 1m^{-1}$ for **Nd : YAG** Laser , if the lifetime $\tau_2 = 230\ \mu s$ , wavelength $= 1.06\ \mu m$ , width frequency $\Delta\nu = 3 * 10^{12} Hz$ and refractive index is $n = 1.82$ .

$G = 1m^{-1}$

$\tau_2 = 230\ \mu s = 230 * 10^{-6} sec$

$\lambda = 1.06\ \mu m = 1.06 * 10^{-6} m$

$\Delta\nu = 3 * 10^{12} Hz$

$n = 1.82$

**Solution:**

$$N = \frac{G\ c}{B_{21} h\nu n\ g(\nu)} = \frac{G\ \lambda\ \Delta\nu}{B_{21} h n}$$

Where : $g(\nu) = \frac{1}{\Delta\nu}$ , $\nu = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{\nu}$

$$\frac{A_{21}}{B_{21}} = \frac{8\,\pi\,h\nu^3}{c^3} \Rightarrow B_{21} = \frac{A_{21} c^3}{8\,\pi\,h\nu^3} = \frac{\lambda^3}{8\,\pi\,h\,\tau_2}$$

$$\therefore\ B = \frac{(1.06 * 10^{-6})^3}{8 * 3.14 * 230 * 10^{-6} * 6.6 * 10^{-34}} = \frac{1.19 * 10^{-18}}{38132.1 * 10^{-40}} = 0.000031 * 10^{22}$$

$B = \frac{m^3}{J.s.s} = \frac{m^3}{w.s.s.s}$ $B = m^3.w^{-1}.s^{-3}$

$$\therefore\ B = 3.1 * 10^{17} m^3.w^{-1}.s^{-3}$$

$$\therefore\ N = \frac{G\ \lambda\,\Delta\nu}{Bhn}$$

$$\therefore\ N = \frac{1 * 1.06 * 10^{-6} * 3 * 10^{12}}{3.1 * 10^{17} * 6.6 * 10^{-34} * 1.82} = \frac{3.18 * 10^{6}}{37.23 * 10^{-17}} = 0.0854 * 10^{23} \approx 8.5 * 10^{21} m^{-3}$$

$\tau_2$=230μsec, $\lambda$=1.06μm, $\Delta\nu = 3\times10^{12}$Hz, n=1.82.

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \dots\dots\dots\dots (39)$$

$$A_{21} = 1/\tau_2 = 4347.826$$

$$\nu = c/\lambda = 2.83\times10^{14}\ Hz$$

$$B_{21} = 3.113\times10^{17} m^3 W^{-1} s^{-1}$$

$$N = \frac{G_{th}\,c}{B_{21} h\nu n g_{(\nu)}}$$

$$N = 8\times10^{22}\ m^{-3}$$

Thelength of a laser tube is **150*mm*** and the gain factor of the laser material is **0.0005/*cm*** . If one of the cavity mirrors reflects **100%** light than is incident on it , what the required reflectance of the other cavity mirror ?

**Solution:** $\gamma \geq \alpha_s + \frac{1}{2L} \ln \frac{1}{r_1 r_2}$, when $\alpha_s = 0, \gamma = 0.0005/cm$ , $L = 150mm = 15cm$

$$\therefore 2\,L\,\gamma = \ln \frac{1}{r_1 r_2}$$

$$\frac{1}{r_1 r_2} = e^{2L\gamma}$$

$$r_2 = \frac{1}{r_1 e^{2L\gamma}} = \frac{1}{1 * e^{2*15\,cm*\,0.0005/cm}} = \frac{1}{e^{0.015}} = 0.985 = 98.5\,\%$$

Given that the life time of the upper Level of the ( **632.8 *nm*** ) transition in the He-Ne Laser is (**10⁻⁷*s***) . Calculate the degree ofpopulation inversion required to give a gain coefficient of (**0.07m⁻¹**), [ ignoring line broadening effect ] .

**Solution:**

$$G = (N_2 - N_1) = \frac{B\,n\,h\nu\,g(\Delta\nu)}{c}$$

$$B = \frac{A\ c^3}{8\,\pi\,\text{h}\nu^3} \quad , \ \boldsymbol{\lambda} = \frac{\boldsymbol{c}}{\nu} \ \Longrightarrow \ \boldsymbol{\lambda}^2 = \frac{c^2}{\nu^2} \quad , \ \boldsymbol{A} = \frac{\mathbf{1}}{\boldsymbol{\tau}}$$

$$\therefore G = (N_2 - N_1) * \frac{A\ c^3\,.\ n\,h\nu}{8\,\pi\,\text{h}\nu^3.\,c} = (N_2 - N_1) * \frac{c^2\,.\ n\,\boldsymbol{\lambda}^2}{8\,\pi\ \boldsymbol{\tau} c^2}$$

$$\therefore (N_2 - N_1) = \frac{8\,\pi\ .\ \boldsymbol{G}\,.\boldsymbol{\tau}}{n\ \boldsymbol{\lambda}^2} = \frac{8\ * 3.14 * (0.07m^{-1}) *\ 10^{-7}\,s}{1 * (\,632.8 *\ 10^{-9}\,)^2 m^2}$$

population inversion

$$(N_2 - N_1) = 0.00000439 * 10^{-7}\ * 10^{18}\,m^{-3}\ = 4.39 *\ 10^5\,m^{-3}$$

**Exp.(4) :**

Calculate the mirror reflectance's required to sustain Laser oscillation in a Laser which is (**0.1*m***) longgiven that small signal gain coefficient is(**1*m*⁻¹**) , [ assumethe mirrors have the same value of reflectance ] .

In a ruby Laser (**λ = 694.3 nm**) therubycrystal is (**0.2m**) Long and the mirror reflectance's are(**0.95**) and (**0.9**) **.** Given that the losses are (**10 %**) per round trip, that the spontaneous life time of the upper Laser level is (**$3*10^{-3}$ s**) . that the line width is (**$1.5*10^{11}$ Hz**) and thatrefractive index is (**1.78**) . Calculate :

(1) The threshold gain coefficient .

(2) The population inversion .

**Solution:** $\lambda = 694.3nm = 694.3 * 10^{-9}\ m$

$L = 0.1\,m \quad , \quad r_1 = 0.95 \quad , \quad r_2 = 0.9$

Losses $\alpha_s = 10\ \% = 0.1 \quad , \quad \tau_{sp} = 3 * 10^{-3}\, s$

Line width $\Delta v = 1.5 * 10^{11}\, Hz \quad \Longrightarrow \quad g(v) = \frac{1}{\Delta v}\ ,\ v = \frac{c}{\lambda}$

Refractive index $n = 1.78$

1) the threshold gain coefficient $\gamma_{th} = G_{th} = \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{r_1 r_2}\right)$

$$\gamma_{th} = 0.1\,m^{-1} + \frac{1}{2 * 0.1\,m}\ ln\left(\frac{1}{0.95 * 0.9}\right) = 0.1 + \frac{1}{0.2}\ ln\left(\frac{1}{0.855}\right)$$

$$\gamma_{th} = 0.1 + \frac{1}{0.2}\ ln\ (1.16959) = 0.1 + \frac{0.1566}{0.2} = 0.1\ m^{-1} + 0.783\, m^{-1} = 0.883\ m^{-1}$$

2)population inversion $N = N_2 - N_1$

$$N = \frac{\gamma_{th}\ .\ c}{B\ n\ hv\ g(\Delta v)} \quad , \quad B = \frac{c^3\ A}{8\,\pi\ hv^3} = \frac{\lambda^3}{8\,\pi\ h\ \tau_{sp}} = \frac{(694.3 * 10^{-9})^3}{8 * 3.14 * 6.6 * 10^{-34} * 3 * 10^{-3}}$$

$$B = \frac{334{,}689{,}043.8 * 10^{-27}}{497.376 * 10^{-37}} = 672910.11 * 10^{10} = 6.7 * 10^{15}$$

$$N = \frac{\gamma_{th}\ .\ c\ \lambda\ \Delta v}{B\ n\ h\ c} = \frac{0.883 * (694.3 * 10^{-9}) * (1.5 * 10^{11})}{6.71 * 10^{15} * 1.78 * 6.6 * 10^{-34}} = \frac{919.6}{78.711} * \frac{10^2}{10^{-19}}$$

$$N = 11.6 * 10^{21}\ m^{-1} = 1.16 * 10^{22}\ m^{-3}$$

**Question: Show that the ratio R between the spontaneous emission rate to the stimulated emission rate in the following equation is equal to:**

(A)- $R = e^{(E/kT)} - 1$

(b) Calculate this ratio for the wavelength of yellow light, i.e., for a wavelength of 589 nm. Find the ratio between the populations of the energy levels corresponding to this transition.

Solution:

R = e^(hv/kT) - 1

= e^((6.6x10^-34 * 5.04x10^14) / (1.38x10^-23 * 300)) - 1

$= e^{81.3} - 1$

(c) At what temperature is the stimulated emission rate equal to the spontaneous emission rate?

Solution:

$R = e^{(hv/kT)} - 1$

$1 = e^{\wedge(hv/kT)} - 1$

$e_{\wedge(hv/kT)}) = 2$

$hv/kT = ln_2$

$T = hv / (k\ ln_2)$

## Prove that the following relationships are correct

1- $\sigma = \frac{B\, n\, h\, \nu\, g(\Delta\nu)}{c}$

$$\sigma = \frac{W}{F} \dots\dots\dots.(1)$$

$$W = \frac{\pi}{3\, n^2 \varepsilon_o \hbar^2}\ |\mu|^2\ \rho\ g\ (\ \Delta\omega\ ) \dots\dots\dots.(2)$$

$$W = B\ \rho = B\ \rho\ g\ (\ \Delta\omega\ ) \dots\dots\dots.(3)$$

$$F = \frac{I}{\hbar\ \omega} = \frac{I}{\frac{h}{2\pi}.\,2\,\pi\,\nu} = \frac{I}{h\,\nu} \dots\dots\dots.(4)$$

بمساواة المعادلتين (2) و (3) نحصل على قيمة W ثم نعود لتعويضها في (1)

$$\sigma = \frac{\rho\ B\ h\ \nu\ g\ (\Delta\nu)}{I} \dots\dots.(5)$$

$$F = \frac{\rho\ c}{n} \dots\dots.(6)$$

$$\sigma = \frac{B\ n\ h\ \nu\ g(\Delta\nu)}{c} \dots\dots\dots\dots..(7)$$

**-2** $\alpha = \frac{(N_1 - N_2) B\ n\ h\ \nu\ g(\Delta\nu)}{c}$

**-3**

$$G = (\ N_2 - N_1)\ \frac{B\, n\, h\, \nu\, g(\Delta\nu)}{c}$$

"Question: Calculate the population inversion (necessary to give a gain coefficient equal to 0.5) for a carbon dioxide laser with a cavity length of 0.5 m-1. The frequency of the emitted light is $10.6 \times 10^{13}$ Hz and its wavelength is $10.6 \times 10^{-6}$ m. Knowing that Einstein's coefficient A for the stimulated emission is 200 $s^{-1}$.

**Solution:**

$$G = \sigma ( N_2 - N_1 )$$

$$N_2 - N_1 = \frac{G}{\sigma} \quad \ldots\ldots\ldots \quad (1)$$

$$\sigma = \frac{B\, n\, h\, \nu\, g(\Delta\nu)}{c} \ldots\ldots\ldots (2)$$

:

$$N_2 - N_1 = \frac{8\, \pi\, \Delta\nu\, G}{A\ \lambda^2}$$

$$N_2 - N_1 = \frac{8\ (3.14)(\ 10^{12}\ )\ (0.5)}{200\ x\ (\ 10.6\ x\ 10^{-6}\ )^2}$$

$$N_2 - N_1 = 5.6\, x 10^{30}\ \frac{molecule}{m^3}$$

**Question A:** If the intensity of light passing through a laser medium 0.5 mm long is doubled once, calculate the gain coefficient, assuming that the device has no losses.

**Solution:**

1 = $I_o$ e $^{(\alpha l)}$ 1 = $I_o$ e$^{\wedge(\alpha l)}$ ……(1)

1 = $2I_o$ ……(2)

Comparing (1) and (2) we get:

$2I_o$ = $I_o$ e $^{(\alpha l)}$ ln2 = αl G = α = ln2 / l = ln2 / 0.5 = 1.39 m^-1

(b) If the increase in the intensity of radiation is only 5% for the same path, what is the gain coefficient?

Solution:

$1 = I_o e^{(-\alpha l)}$ $1 = I_o e^{(\alpha l)}$ ……(1)

$I - I_o = 0.05 I_o$ $I = (1 + 0.05) I_o = 1.05 I_o$ ……(2)

Comparing (1) and (2) we get:

$1.05 I_o = I_o e^{(\alpha l)}$ $\ln 1.05 = \alpha l$ $G = \alpha = \ln 1.05 / l = \ln 1.05 / 0.5 = 0.098\ m^{-1}$