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أستاذة المادة

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Derivative of trigonometry function

Rules of the Derivative of trigonometry function

1) $y = \sin(g(x)) \Rightarrow y' = \cos(g(x)).g'(x)$
2) $y = \cos(g(x)) \Rightarrow y' = -\sin(g(x)).g'(x)$
3) $y = \tan(g(x)) \Rightarrow y' = \sec^2(g(x)).g'(x)$
4) $y = \cot(g(x)) \Rightarrow y' = -\csc^2(g(x)).g'(x)$
5) $y = \sec(g(x)) \Rightarrow y' = \tan(g(x)).\sec(g(x)).g'(x)$
6) $y = \csc(g(x)) \Rightarrow y' = -\cot(g(x)).\csc(g(x)).g'(x)$

Example: Find the derivative of the following function:

1) $f(x) = \sin(x^2) + \cot(x^4 - 1)$
$f'(x) = \cos x^2 . 2x - \csc^2(x^4 - 1) . 4x\text{^}3$

2) $f(x) = \frac{2}{\cos(3t)}$
$f'(x) = \frac{(\cos(3t).0 - 2.(-\sin(3t)).3}{\cos^2(3t)} = \frac{6\sin(3t)}{\cos^2(3t)}$

3) $f(x) = \sin(3x) \Rightarrow f'(x) = 3\cos 3x$

4) $f(x) = \sin(\sqrt{3x}) \Rightarrow f'(x) = \cos(\sqrt{3x}).\left(\frac{1}{2\sqrt{3x}}.3\right)$

$$\Rightarrow \frac{3}{2\sqrt{3x}}\cos\sqrt{3x}$$

5) $y = x^2 \tan 3x$
$y' = x^2.\sec^2(3x).3 + \tan 3x.2x$
$= 3x^2\sec^2(3x) + 2x\tan 3x$

6) $y = \cos(x^2 + 1)$
$y' = -\sin(x^2 + 1).2x$
$= -2x\sin(x^2 + 1)$

7) $y = \sec^2(\sqrt{x})$

$y' = 2\sec(\sqrt{x}).\sec(\sqrt{x}).\tan(\sqrt{x}).\frac{1}{2\sqrt{x}}.1$

$= \frac{\sec^2(\sqrt{x}).tan\sqrt{x}}{\sqrt{x}}$

8) $y = \frac{\cos x}{x}$ [H.W]

9) $y = \tan^2(3x-2)$ [H.W]

10) Find $\frac{dy}{dt}$ if you know $y = \tan(x)$ and $x = 4t^2 + t$

Sol:

$\frac{dy}{dx} = \sec^2(x), \frac{dx}{dt} = 8t \quad + 1$

$\therefore \frac{dy}{dt} = \sec^2(x).(8t \quad + 1)$

11) Find y' of the function $x.\sin(2y) = y.\cos(2x)$

Sol:

$x.\cos(2y).2y' + \sin(2y).1 = y.(-\sin(2x)).2 + \cos(2x).y'$

$x.\cos(2y).2y' - \cos(2x).y' = y.(-\sin(2x)).2 - \sin(2y)$

$2x\cos(2y)\,y' - \cos(2x).y' = -2y\sin(2x) - \sin(2y)$

$y'(2xcos(2y) - \cos(2x)) = -2y\sin(2x) - \sin(2y)$

$\therefore y' = \frac{-2y\sin(2x) - \sin(2y)}{(2x\cos(2y) - \cos(2x))}$

12) $\cot(xy) + xy = 0$ [H.W]

13) $\sqrt{xy} + \csc(-xy) = y$ [H.W]

The Derivative of Inverse Trigonometric function

Let u be a function of x, the

1) $\frac{d}{du}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}$
2) $\frac{d}{du}\cos^{-1}(u) = -\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}$
3) $\frac{d}{du}\tan^{-1}(u) = \frac{1}{1+u^2}.\frac{du}{dx}$
4) $\frac{d}{du}\cot^{-1}(u) = -\frac{1}{1+u^2}.\frac{du}{dx}$
5) $\frac{d}{du}\sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}}.\frac{du}{dx}$
6) $\frac{d}{du}css^{-1}(u) = -\frac{1}{|u|\sqrt{u^2-1}}.\frac{du}{dx}$

Example: Find the derivative of the following functions:

1) $f(x) = \sin^{-1}(x^2)$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}}.2x$$

2) $f(t)\cos^{-1}(\sqrt{t}) \quad \Rightarrow f'(t) = -\frac{1}{\sqrt{1-(\sqrt{t})^2}}.\frac{1}{2}t^{-\frac{1}{2}}$

3) $f(x) = x^3\cot^{-1}\left(\frac{x}{3}\right)$

$$f'(x) = x^3.\left[-\frac{\frac{1}{3}}{1+\frac{x^2}{9}}\right] + 3x^2.\cot^{-1}\left(\frac{x}{3}\right)$$

$$\Rightarrow f'(x) = \frac{-3x^2}{9+x^2} + 3x^2\cot^{-1}\left(\frac{x}{3}\right)$$

4) $y = \sin^{-1}(cosx)$

$$y' = \frac{-sinx}{\sqrt{1-\cos^2 x}} \Rightarrow \frac{-\sin x}{\sqrt{\sin^2 x}}$$

5) $y = \left(\tan^{-1}\sqrt{x}\right)^3$

$$y' = 3\left(\tan^{-1}\sqrt{x}\right)^2 . \frac{1}{2\sqrt{x}\,(1+x)}$$

6) $f(x) = x\sec^{-1}\left(\frac{1}{x}\right) + \sqrt{1+x^2}$ [H.W]

7) $y = \frac{\cos^{-1}(3\theta)}{1+\theta^2}$ [H.W]

8) $y = \sin^{-1}\frac{x-1}{x+1}$ [H.W]

Exponential Logarithmic Functions.

$f(x) = q^x$ الدالة الاسية

$a: real\ number$

$x: rational\ number\ \left(x = \frac{p}{q}\right)$

$g(x) = \log_a x$

$Df(x) = R^+$

$Rf(x) = R$

$f(x) = a^x$

$y = a^x$

$\log_a y = x$

$x \log_a y \Longrightarrow a^x \leftrightarrow \log_a y$

Properties of the logarithm function

$\log x = \log_{10} x$

1) $\log_a x = \frac{\ln x}{\ln a}$
2) $\log_a a^x = x$
3) $\log_a (u.v) = \log_a u + \log_a v$
4) $\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$
5) $\log_a u^n = n \log_a u$

Example:

1) $5 \log 2 + \log 3 - \log 8$

$\log 2^5 + \log 3 - \log 8 \Longrightarrow \log 32 + \log 3 - \log 8$

$= \log \frac{(32)(2)}{8} \Longrightarrow \frac{96}{8} \Longrightarrow \log 12 \cong 1.079$

2) $\log \frac{xy^5}{\sqrt{3}} \Longrightarrow \log x + \log y^5 - \log 3^{\frac{1}{2}}$

$\log x + 5 \log y - \frac{1}{2} \log 3$

3) $\log_2 5 \Longrightarrow \frac{\ln 5}{\ln 2} \cong 2.3219$

Properties of the logarithm (ln)

$\log_e x = \frac{\ln x}{\ln e} \Rightarrow \ln e = 1$ $\quad\quad$ $\therefore \log_e x = \ln x$

1) $\ln ax = \ln a + \ln x$
2) $\ln \frac{a}{x} = \ln a - \ln x$
3) $\ln x^n = n \ln x$
4) $\ln \frac{1}{x} = -\ln x$

* $\ln 1 = 0$

Properties of Exponetial function

1) $e^{x_1+x_2} = e^{x_1} e^{x_2}$
2) $e^{x_1-x_2} = \frac{e^{x_1}}{e^{x_2}}$
3) $e^{ax} = (e^x)^9$
4) $e^{-x} = \frac{1}{e^x}$

* $e^0 = 1$

Derivative of the (e^x and ln an log)

1) $\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a} \cdot \frac{du}{dx}$

2) $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$

3) $\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$

4) $\frac{d}{dx}[e^u] = \frac{du}{dx}$

Example: Find y' of the following function

1) $y = \ln(x^2 + 2x)$

$y' = \frac{2x+2}{x^2+2x}$

2) $y = \ln(x^2 + 1)$

$y' = \frac{1}{x^2+1} . 2x \Rightarrow \frac{2x}{x^2+1}$

3) $y = e^{x^3} \Rightarrow y' = 3x^2 e^{x^3}$

4) $y = e^{\tan^{-1} x} \Rightarrow y' = e^{\tan^{-1} x} . \frac{1}{1+x^2}$

5) $y = 2^{\sin x} \Rightarrow y' = 2^{\sin x} \ln 2 \cos x = \ln 2 \cos x \ 2^{\sin x}$

6) $y = \ln(\sin x) \Rightarrow y = \frac{1}{\sin x} . \cos x = \frac{\cos x}{\sin x} = \cot x$

7) $\ln(\tan x + \sec x)$ [H.W]