وزارة التعليم العالي والبحث العلمي

جامعة الانبار

كلية الصيدلة

المرحلة الأولى

المحاضرة الثالثة

مادة الرياضيات

Math & Biostatistics

أستاذة المادة

م.م. كوثر عبدالمجيد احمد العاني

1444هـ

2023م

Differentiation

- If the function $f(x)$ and the point (x, y), Lies on the line, the slope of the tan gent pass through the function in this point it is:

$$\frac{dy}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Where the $f'(x)$ it is represent to derivative of a function that can be used to find the equation of the tangent line.

Example: Find by using the definition of the derivative of the following function?

1) $f(x) = \frac{1}{x}, where\ x \neq 0$

Solution: $f'(x) = \lim_{\Delta x \to 0} = \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x} \Rightarrow \lim_{\Delta x \to 0} \frac{-\Delta x}{x \Delta x (x+\Delta x)} = \frac{-1}{x^2}$$

2) $f(x) = x^2 + 2$

Sol :

$$f'(x) = \lim_{\Delta x \to} \frac{(x+\Delta x)^2 + 2 - \left(x^2 + 2\right)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 2 - x^2 - 2}{\Delta x} \Rightarrow \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$\therefore f'(x) = 2x$$

3) $f(x) = \sqrt{x}\ , x > 0$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x \sqrt{x+\Delta x} + \sqrt{x}} \Longrightarrow \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Rules of derivatives:- Let c and n are constants,

1) $\frac{d}{dx}[c] = 0$
2) $\frac{d}{dx}[x^n] = nx^{n-1}$
3) $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$
4) $\frac{d}{dx}[f(x) \mp g(x)] = \frac{d}{dx}f(x) \mp \frac{d}{dx}g(x)$
5) $\frac{d}{dx}[f(x).g(x)] = f(x).\frac{d}{dx}[g(x)] + g(x).\frac{d}{dx}[f(x)]$
6) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).\frac{d}{dx}[f(x)] - f(x).\frac{d}{dx}[g(x)]}{[g(x)]^2}$

- **Find $\frac{dy}{dx}$ for the following functions:**

1) $y = (x^2+1)^5$

$y' = 5(x^2+1)^4 . 2x \Longrightarrow 10 \times (x^2+1)^4$

2) $y = \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4}$

$y = 12x^{-1} - 4x^{-3} + 3x^{-4}$

$\therefore y' = -12x^{-2} + 12x^{-4} - 12x^{-5}$ $\qquad \therefore y' = \frac{-12}{x^2} + \frac{12}{x^4} - \frac{12}{x^5}$

3) $y = 3x^8 - 2x^5 + 6x + 1$ $\qquad \therefore y' = 24x^7 - 10x^4 + 6$

Derivatives (Higher derivatives)

- The derivative of the function $y = f(x)$ is the function $y' = f'(x)$ whose value at each x is define by rule and can found $y'' = f''(x) \Longrightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx} or\ y'\right]$

Example: Find y''

1) $y = x^6 + 3x^4 - 2x^2 + 9$

Sol

$y' = 6x^5 + 12x^3 - 4x$

$y'' = 30x^4 + 36x^2 - 4$

2) Find y'' of the function $y = 3x^4 - 2x^3 + x^2 - 4x + 2$

Sol:

$y' = 12x^3 - 6x^2 + 2x - 4$

$y'' = 36x$^2 – 12x+2

Chain rule

- Let $y = f(u), u = g(x)$ where is $f(u), g(x)$ it is two function are different able Suen that

$$y'' = f'[g(x). g'(x)]$$

$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} \neq$ First low of the chain rule

Example: use the chain rule to express the $\frac{dy}{dx}$ in terms of x and y:

1) $y = u^2$ and $u = x^2 + 3$

Sol:

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} \Longrightarrow \frac{dy}{du} = 2u, \frac{du}{dx} = 2x$$

$$\therefore \frac{dy}{dx} = (2u).(2x) = 4ux \quad \Longrightarrow 4x(x^2 + 3) = 4x^3 + 12x$$

2) $y = \frac{t^2}{t^2+1}$ and $t = \sqrt{2x + 1}$

$$\frac{dy}{dt} = \frac{(t^2+1).2t - t^2.2t}{(t^2+1)^2} = \frac{2t(t^2+1) - 2tt^2}{(t^2+1)^2} = \frac{2t}{(t^2+1)^2}$$

$$t = (2x + 1)^{\frac{1}{2}} \Longrightarrow \frac{dt}{dx} = \frac{1}{2}.(2x + 1)^{-\frac{1}{2}}.2 = \frac{1}{\sqrt{2x+1}}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{2t}{(t^2+1)^2}.\frac{1}{\sqrt{2x+1}}$$

$$= \frac{2\sqrt{2x+1}}{((2x+1)+1)^2}.\frac{1}{\sqrt{2x+1}}$$

$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ second low of the chain rule

1) By used the chain rule find $\frac{dy}{dx}$ if you know the value of $y = 2t^3 - 6t$ and $x = t^2 + 2t$

Solution:

$$\frac{dy}{dt} = 6t^2 - 6 \Rightarrow 6(t^2 - 1)$$

$$\frac{dx}{dt} = 2t + 2 \Rightarrow 2(t + 1)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 6(t^2 - 1) \times \frac{1}{2(t+1)} \Rightarrow \frac{6(t^2-1)}{2(t+1)}$$

Problem's/ by use the chain rule find $\frac{dy}{dx}$

1) $y = 1 - \frac{1}{t}$ and $t = \frac{1}{1-x}$

2) $y = \frac{1}{t^2+1}$ and $x = \sqrt{4t + 1}$

3) Find the third derivative of the following function $y = \frac{1}{x} + \sqrt{x}$

4) Find the second derivative for the following Furet $y = \left(x + \frac{1}{x}\right)^3$

Implicit Differentiation

- If the formula for f is an algebraic combination of *powe* of x and y
- To calculate the derivatives of the simplicity defined function, we simply differentiate both sides of the defining equation will respect to x

$x^2 + y^2 = 1 \Longrightarrow y = \mp\sqrt{(1-x)^2}$

$x^3 + y^3 = 3xy$

Example: Find $\frac{dy}{dx}$ for the following functions:

1) $x^2 . y^2 = x^2 + y^2$

$$x^2 . \left(2y \frac{dy}{dx}\right) + y^2 . (2x) = 2x + 2y \frac{dy}{dx}$$

$$2x^2 y \frac{dy}{dx} + 2xy^2 = 2x + 2y \frac{dy}{dx} \Longrightarrow \frac{dy}{dx} = \frac{x - xy^2}{x^2 y - y}$$

2) Find $\frac{d^2y}{dx^2}$ to the function $y^2 = 2x^3$

Sol:

$$2y\left(\frac{dy}{dx}\right) = 6x^2 \Longrightarrow \frac{dy}{dx} = \frac{6x^2}{2y} = \frac{3x^2}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y.(6x) - 3x^2.\left(\frac{dy}{dx}\right)}{y^2} = \frac{6xy - 3x^2.\left(\frac{3x^2}{y}\right)}{y^2} = \frac{6xy - 9x^4}{y^3}$$

3) $x^2 + 3y^2 = 3$

$$2x + 6yy' = 0 \Longrightarrow 2x = -6yy'$$

$$\therefore y' = -\frac{2x}{6y} = -\frac{x}{3y}$$

4) $xy + 2x - 5y = 2$ [H.W]