وزارة التعليم العالي والبحث العلمي

جامعة الانبار

كلية الصيدلة

المرحلة الأولى

المحاضرة الثانية

مادة الرياضيات

Math & Biostatistics

أستاذة المادة

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" Function "

Function: is any rule that assigns to each element in A, B are two sets, $F: A \to B$ (two set element)

$\therefore B = F(A)$

$f(x) \to y$

$y = f(x)$

For example: $f(x) = x^2 + 2$

- We must keep two restrictions in mind when we define function

1- We never divide by zero
2- We will deal with real-valued functions only

Ex1: $g(x) = x^2 - 3x + 7$ Find $g(r^2)$

$g(r^2) = r^4 - 3r^2 + 7$

Ex2: $F(x) = x^2 + 3$ Find domain

Domain $f(x)$ to $x^2 + 3$ it is All real number

Ex3: $g(x) = \frac{1}{x-2}$ Find domain

Domain $g(x)$ is $\{x \in R : x \neq 2\}$

H.W: $f(x) = \sqrt{1 + 5x}$ Find the domain

Algebra Application with the Functions

Let the $f(x)$ and $g(x)$ it is function there is:

1- $(F \mp g)(x) = F(x) \mp g(x)$
2- $(f.g)(x) = f(x).g(x)$
3- $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
4- $kf(x) = kf(x)$

Example: if the

1- $f(x) = x^2, \quad g(x) = x$
Find:
1) $(f + g)(x)$
2) $(f.g)(x)$

Sol:

1- $(f + g)(x) = f(x) + g(x) \Longrightarrow x^2 + x$
2- $(f.g)(x) = f(x).g(x) \Longrightarrow x^2.x = x^3$

Ex2: $f(x) = x^2, \quad g(x) = x^2 + 1$ Find $\left(\frac{f}{g}\right)(x)$

Sol:

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2}{x^2+1}$$

Ex3: $f(x) = x^2 + 2x - 3$

find $x = -2$, $x = 0$, $x = (x + 2)$

1- $f(-2) = (-2)^2 + 2(-2) - 3 \Rightarrow 4 - 4 - 3 = -3$

2- $f(0) = (0)^2 + 2(0) - 3 \Rightarrow -3$

3- $f(x+2) = (x+2)^2 + 2(x+2) - 3 = x^2 + 6x + 5$

Inverse Function

Suppose that the outputs of a function can by used as inputs of a function g. we can then hook f and g together to from a new function whose inputs are the inputs of f and whose output are the number:

1- $f\big(g(y)\big) = y$
2- $g\big(f(x)\big) = x$

Ex3: let $(gof)(x) = x$ and $f(x) = \frac{1}{x}$ find $g(x)$

Sol: $(gof)(x) = g\left(\frac{1}{x}\right) = x$

$\Longrightarrow g(x) = \frac{1}{x}$

Example1: if the f, g it is function

$f(x) = 3x$

$g(y) = \frac{1}{3}y$, prove the function $g(y)$ it is invers function

Sol:

$f\big(g(y)\big) = f\left(\frac{1}{3}(y)\right) = 3\left(\frac{1}{3}y\right) = y$

$g(f(x) = g(3x) = \frac{1}{3}(3x) = x$

Ex2: $f(x) = 2x + 1$

Sol:

$$f(x) = y \Longrightarrow 2x + 1 = y$$

$$2x = y - 1$$

$$\therefore x = \frac{1}{2}(y - 1) = g(y)$$

$$f(g(y)) = f\left[\frac{1}{2}(y - 1)\right]$$

$$= 2\left[\frac{1}{2}(y - 1)\right] + 1 = y - 1 + 1 = y$$

The limits

$\lim_{x \to a} f(x) = L$ or $f(x) \to L$ when $x \to a$

$\lim_{x \to a} f(x) = L1$, $\lim_{x \to a} g(x) = L_2$

1- $\lim_{x \to a} f(x).g(x) = \lim_{x - a} f(x).\lim_{x \to a} g(x)$

2- $\lim_{x \to a} f(x) \mp g(x) = \lim_{x \to a} f(x) \mp \lim_{x \to a} g(x)$

3- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

4- $\lim_{x \to a} k = k$
$\lim k\, f(x) = k \lim_{x \to a} f(x)$

5- $\lim_{x \to \infty} \frac{1}{x} = 0$ $\lim x_{x \to \infty} = \infty$

There are the number of solution method to solve the limits which are:

Example:

1- $\lim_{x\to 2}\frac{5x^3+4}{x-3}=\frac{\lim_{x\to 2}(5x^3+4)}{\lim_{x\to 2}x-3}=\frac{5(2)^3+4}{(2)-3}=\frac{44}{-1}=-44$

2- $\lim_{x\to 2}\frac{x^2-4}{x-2}$

$\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}\Rightarrow \lim_{x\to 2}(x+2)=4$

3- $\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}$

$\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}\cdot\frac{\sqrt{x+3}+\sqrt{3}}{\sqrt{x+3}+\sqrt{3}}\Rightarrow \lim_{x\to 0}\frac{x+3-3}{x\sqrt{x+3}+\sqrt{3}}$

$=\lim_{x\to 0}\frac{1}{\sqrt{x+3}+\sqrt{3}}\Rightarrow \lim_{x\to 0}\frac{1}{\sqrt{x+3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}$

4- $\lim_{x\to\infty}\frac{2x^2-x}{2x^3-5}$

$\text{Lim}_{x\to\infty}\frac{2\frac{x^2}{x^3}-\frac{x}{x^3}}{2\frac{x^3}{x^3}-\frac{5}{x^3}}\Rightarrow \lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^2}}{2-\frac{5}{x^3}}=\frac{0-0}{2-0}=0$

5- $\lim_{x\to 1}\frac{x^2+1}{2x-3}=-2$

$\frac{(1)^2+1}{2\times 1-3}=\frac{2}{-1}=-2$

6- $\lim_{x\to 0} \frac{(a+x)^3-a^3}{x}$

$$\lim_{x\to 0}[a^3+b^3] \Longrightarrow (a-b)[a^2+ab+b^2]$$

$$\lim_{x\to 0} \frac{[(a+x)-a][(a+x)^2+a(a+x)+a^2]}{x}$$

$$\text{Lim}_{x\to 0} \frac{x(a^2+2ax+x^2-a^2+ax+a^2)}{x}$$

$$\text{Lim}_{x\to 0}\, 3a^2+3ax+x^2=3a^2$$

- **Same important limits**

1- $\lim_{x\to 0} sinx = 0$

2- $\lim_{x\to 0} cosx = 1$

3- $\lim_{x\to 0} \frac{sinx}{x} = 1$

4- $\lim_{x\to 0} \frac{tanx}{x} = 1$

5- $\lim_{x\to 0} e^{x} = 1$

6- $\lim_{x\to 0} \frac{1-cosx}{x} = 0$

7- $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

8- $\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

9- $\lim_{x\to 0} \frac{1-cosx}{x}$

$$\lim_{x\to 0} \frac{1-cosx}{x} \cdot \frac{1+cosx}{1+cosx}$$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x(1+cosx)}$$

$[\sin^2 x + \cos^2 x = 1]$ → $1 - \cos^2 x = \sin^2 x$

→ $\lim_{x\to 0} \frac{\sin^2 x}{x(1+cosx)}$

$$= \lim_{x\to 0}\frac{sinx}{x}.\lim_{x\to 0}\frac{sinx}{1+cosx}$$

$$= \lim_{x\to 0}\frac{sinx}{1+cosx} = \frac{0}{1+1} = \frac{0}{2} = 0$$

10- $\lim_{x\to 0}\frac{sin2x}{x}$

$$\lim_{x\to 0}\frac{sin2x}{x}.\frac{2}{2} \Longrightarrow 2\lim_{x\to 0}\frac{sin2x}{2x} = 2$$

11- $\lim_{x\to -1}\frac{x^2-1}{x+1}$

$$\lim_{x\to -1}\frac{(x-1)(x+1)}{x+1} \Longrightarrow \lim_{x\to -1}(x-1) \Longrightarrow (-1-1) = -2$$

12- $\lim_{x\to\infty}\frac{3x^4+3x^2+2x}{x^4-x+1} \Longrightarrow \lim_{x\to\infty}\frac{3\frac{x^4}{x^4}+3\frac{x^2}{x^4}+2\frac{x}{x^4}}{\frac{x^4}{x^4}-\frac{x}{x^4}+\frac{1}{x^4}}$

$$\lim_{x\to\infty}\frac{3+\frac{3}{x^2}+\frac{2}{x^3}}{1-\frac{1}{x^3}+\frac{1}{x^4}} \Longrightarrow \frac{3+\frac{3}{\infty^2}+\frac{2}{\infty^3}}{1-\frac{1}{\infty^3}+\frac{1}{\infty^4}} = 3$$

Exercise:

Evaluate the following limits:

1- $\lim_{x\to\infty} \frac{3x+7}{x^2-2}$

2- $\lim_{x\to 0} \cos\left[1 - \frac{\sin x}{x}\right]$

3- $\lim_{x\to 0} \frac{\tan 2x}{3x}$