7) Coupling coefficient (k)

We will now establish an upper limit for the inductance M. mutual The energy stored in the circuit cannot be negative because the circuit is passive. This means that the quantity ($\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2$) must be greater than or equal to zero:

$$\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2 \geq 0 \qquad \ldots(7.1)$$

To complete the square, we both add and subtract the term $i_1i_2\sqrt{L_1L_2}$ on the right-hand side of Eq. (6.11) and obtain

$$\frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + i_1i_2\left(\sqrt{L_1L_2} - M\right) \geq 0 \qquad \ldots(7.2)$$

The squared term is never negative; at its least it is zero. Therefore, the second term on the right-hand side of Eq. (6.12) must be greater than zero; that is,

$$\sqrt{L_1L_2} - M \geq 0$$

or

$$M \leq \sqrt{L_1L_2} \qquad \ldots(7.3)$$

Thus, the mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils. The extent to which the mutual inductance M approaches the upper limit is specified by the ***coefficient of coupling k***, given by

$$k = \frac{M}{\sqrt{L_1L_2}} \qquad \ldots(7.4)$$

$$\boxed{M = k\sqrt{L_1L_2}} \qquad \ldots(7.5)$$

Where $0 \leq k \leq 1$ or equivalently $0 \leq M \leq \sqrt{L_1L_2}$.

The coupling coefficient is the fraction of the total flux emanating from one coil that links the other coil. For example, in Fig.6.2(a),

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11}+\phi_{12}} \qquad \ldots(6.6)$$

and in Fig.6.2(b),

$$k = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{21}}{\phi_{21}+\phi_{22}} \qquad \ldots(6.7)$$

1) For $k = 1$ (the entire flux produced by one coil links another coil, 100% coupling) the coils are said to be ***perfectly coupled.***
2) For $k < 0.5$ coils are said to be ***loosely coupled;***
3) For $k > 0.5$, coils are said to be ***tightly coupled.***

> The coupling coefficient *k* is a measure of the magnetic coupling between two coils; $0 \leq k \leq 1$

coupling coefficient k depend on the closeness of the two coils, their core, their orientation, and their windings.

The air-core transformers used in radio frequency circuits are loosely coupled, whereas iron-core transformers used in power systems are tightly coupled. The linear transformers are mostly air-core; the ideal transformers are principally iron-core.

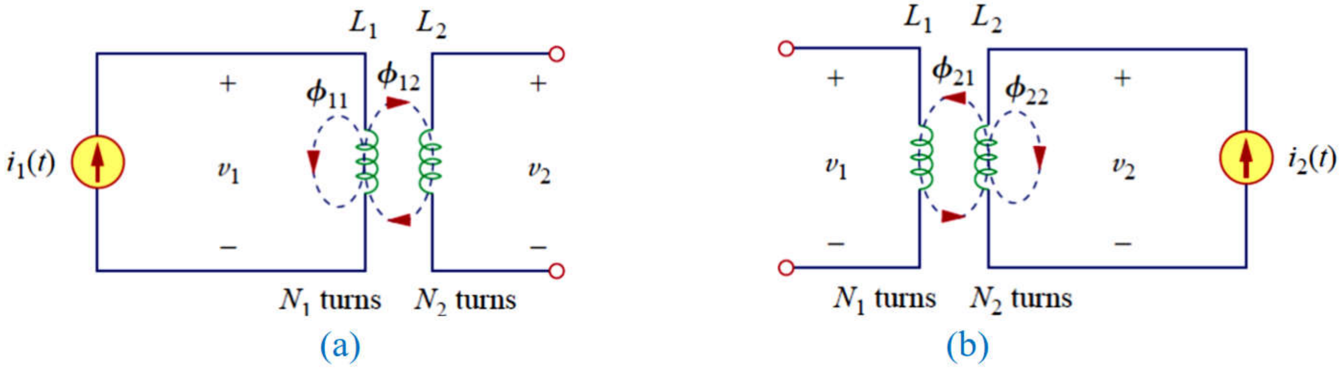
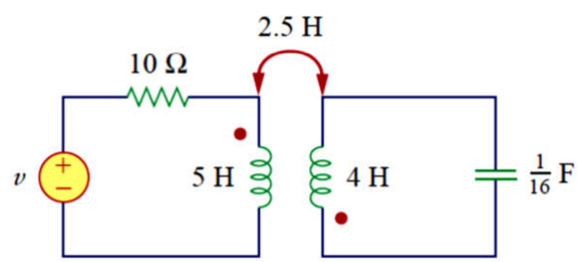


Fig.6.2

Example 5: Determine the coupling coefficient and calculate the energy stored in the coupled inductors at time t=1 s if $v = 60\cos(4t + 30^o)$ V.

Solution:



$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{5\times4}} = 0.56 \quad \text{(tightly coupled)}$$

To find the energy stored, we need to calculate the current. To find the current, we need to obtain the frequency-domain equivalent of the circuit.

$$60\cos(4t + 30^\circ) \Rightarrow 60\angle 30^\circ, \quad \omega = 4 \text{ rad/s}$$

$$5\text{ H} \Rightarrow j\omega L_1 = j20\ \Omega$$

$$2.5\text{ H} \Rightarrow j\omega M = j10\ \Omega$$

$$4\text{ H} \Rightarrow j\omega L_2 = j16\ \Omega$$

$$\frac{1}{16}\text{F} \Rightarrow \frac{1}{j\omega C} = -j4\ \Omega$$

The frequency-domain equivalent is shown in Fig.. Now apply mesh analysis.

For mesh 1,

$$(10 + j20)I_1 + j10I_2 = 60\angle 30^0 \qquad \ldots(1)$$

For mesh 1,

$$j10I_1 + (j16 - j4)I_2 = 0 \qquad \ldots(2)$$

Solve Eq.s (1) and (2) we get,

$I_1 = 3.905\angle{-19.4^o}$ A & $I_2 = 3.254\angle 160.6^o$ A

In time domain.

$i_1 = 3.905\cos(4t - 19.4^o)$ & $i_2 = 3.254\cos(4t + 160.6^o)$

At t=1s, $4t$ = 4 rad = 229.2°

$i_1 = 3.905\cos(229.2^o - 19.4^o) = -3.389$ A