

Example 9: An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

$$(a) n = \frac{v_2}{v_1} = \frac{120}{2400} = 0.05$$

$$(b) n = \frac{N_2}{N_1} \rightarrow 0.05 = \frac{50}{N_1} \rightarrow N_1 = 1000 \text{ turns}$$

$$(c) S_1 = V_1 I_1 = V_2 I_2 = 9.6 \text{ kVA}$$

$$I_1 = \frac{9600 \text{ VA}}{V_1} = \frac{9600 \text{ VA}}{2400} = 4 \text{ A}$$

$$I_2 = \frac{9600 \text{ VA}}{V_2} = \frac{9600 \text{ VA}}{120} = 80 \text{ A} \quad \text{or} \quad I_2 = \frac{I_1}{n} = \frac{4 \text{ A}}{0.05} = 80 \text{ A}$$

H.W.7: The primary current to an ideal transformer rated at 3300/110 is 5 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

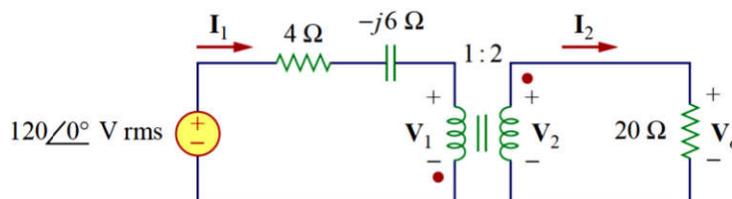
Answer: (a) 1/30, (b) 16.5 kVA, (c) 150 A.

Example 10: For the ideal transformer circuit of Fig. shown, find: (a) the source current I_1 (b) the output voltage V_o and (c) the complex power supplied by the source.

Solution:

(a) The 20Ω impedance can be reflected to the primary side and we get

$$Z_R = \frac{20}{n^2} = \frac{20}{2^2} = 5\Omega$$



$$\therefore Z_{in} = 4 - j6 + Z_R = 9 - j6 = 10.82\angle -33.69^\circ \Omega$$

$$I_1 = \frac{120\angle -0^\circ}{Z_{in}} = \frac{120\angle -0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ \text{ A}$$

(b) since both I_1 & I_2 leave the dotted terminals,

$$I_2 = \left(-\frac{1}{n}\right) I_1 = -5.545\angle 33.69^\circ \text{ A}$$

$$V_o = 20 I_2 = 110.9\angle 213.69^\circ \text{ V}$$

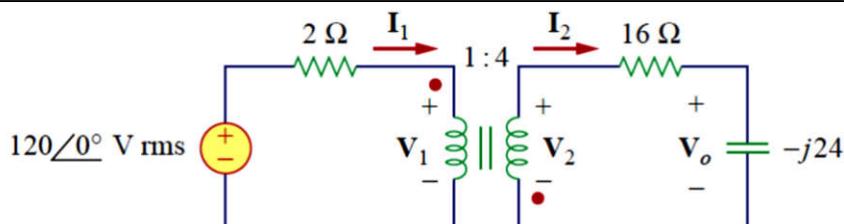
$$(c) S_1 = V_s I_1^* = (120\angle 0^\circ)(11.09\angle -33.69^\circ) = 1330.8\angle -33.69^\circ \text{ VA}$$

H.W.8: In the ideal transformer circuit of Fig. shown, find V_o and the complex power supplied by the source.

Answer:

$$214.7\angle 116.56^\circ \text{ V,}$$

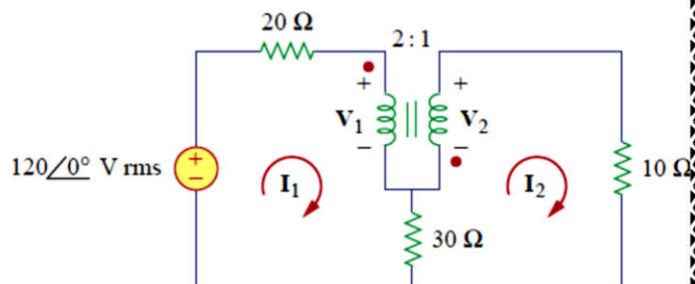
$$4.293\angle -26.56^\circ \text{ kVA}$$



Example 11: Calculate the power supplied to the 10- Ω resistor in the ideal transformer circuit of Fig. shown

Solution:

Reflection to the secondary or primary side cannot be done with this circuit: there is direct connection between the primary and secondary sides due to the 30- resistor. We apply mesh analysis.



For mesh 1,
 $50I_1 - 30I_2 + V_1 = 120 \quad \dots(1)$

For mesh 2,
 $-30I_1 + 40I_2 - V_2 = 0 \quad \dots(2)$

At the transformer terminals,

$V_2 = -\frac{1}{2} V_1 \quad \dots(3)$

$I_2 = -2 I_1 \quad \dots(4)$

Solve Eq.s(1), (2), (3) and (4), we get

$I_2 = -0.7272 \text{ A}$

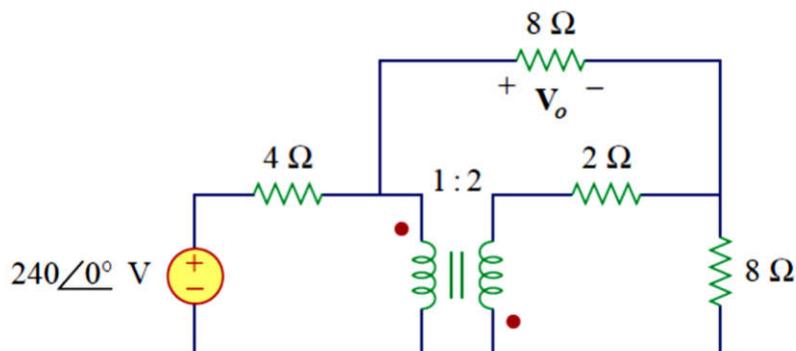
The power absorbed by 10- Ω resistor is,

$p = (-0.7272)^2(10) = 5.3 \text{ W}$

H.W.9: Find V_o in the circuit of Fig. shown.

Answer:

96 V



11) Ideal Autotransformers

Unlike the conventional two-winding transformer we have considered so far, an *autotransformer* has a single continuous winding with a connection point called a *tap* between the primary and secondary sides. The tap is often adjustable so as to provide the desired turns ratio for stepping up or stepping down the voltage. This way, a variable voltage is provided to the load connected to the autotransformer.

An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding.

Fig.11.1, the autotransformer can operate in the *step-down* or *stepup* mode. The autotransformer is a type of power transformer. Its major advantage over the two-winding transformer is its ability to transfer larger apparent power. **Example 12** will demonstrate this. Another advantage is that an autotransformer is smaller and lighter than an equivalent two-winding transformer. However, since both the primary and secondary windings are one winding, *electrical isolation* (no direct electrical connection) is lost. The lack of electrical isolation between the primary and secondary windings is a major disadvantage of the autotransformer.

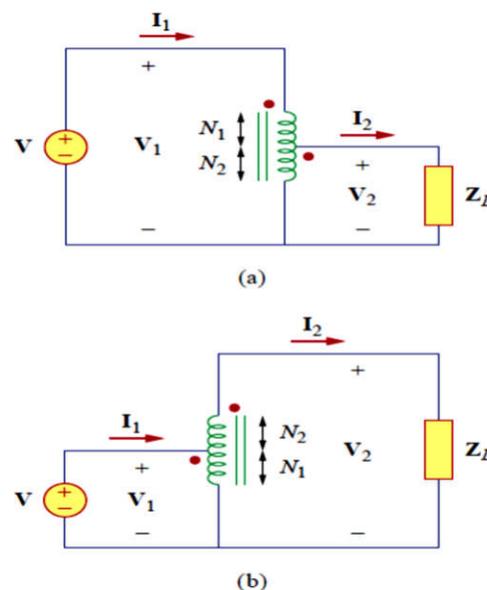


Fig.11.1

For the step-down autotransformer circuit of Fig.11.1(a),

$$\frac{V_2}{V_1} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2} \quad \dots(11.1)$$

$$S_1 = V_1 I_1^* = V_2 I_2^* = S_2 \quad \dots(11.2)$$

$$V_1 I_1 = V_2 I_2 \quad \dots(11.3)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1 + N_2} \quad \dots(11.4)$$

For the step-up autotransformer circuit of Fig.11.1(b),

$$\frac{V_2}{V_1} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1} \quad \dots(11.5)$$

$$S_1 = V_1 I_1^* = V_2 I_2^* = S_2 \quad \dots(11.6)$$

$$V_1 I_1 = V_2 I_2 \quad \dots(11.7)$$

$$\frac{I_1}{I_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1} \quad \dots(11.7)$$

A major difference between conventional transformers and autotransformers is that the primary and secondary sides of the autotransformer are not only coupled magnetically but also coupled conductively. The autotransformer can be used in place of a conventional transformer when electrical isolation is not required.

Example 12: Compare the power ratings of the two-winding transformer in Fig. (a) and the autotransformer in Fig. (b).

Solution:

For the two winding transformer, the power rating is,

$$S_1 = 0.2 \times 240 = 48 \text{ VA}$$

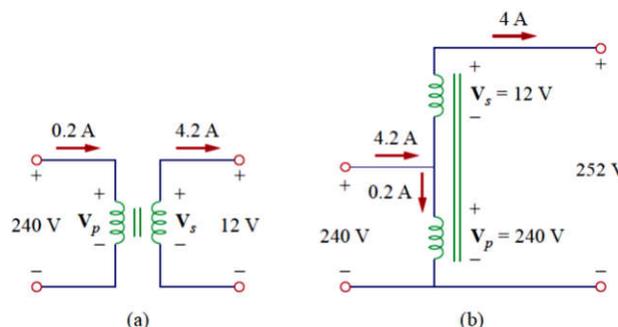
or $S_2 = 4 \times 12 = 48 \text{ VA}$

For the autotransformer, the power rating is,

$$S_1 = 4.2 \times 240 = 1008 \text{ VA}$$

or $S_2 = 4 \times 252 = 1008 \text{ VA}$

which is 21 times the power rating of the two-winding transformer.



H.W.10: Refer to **Example 12**. If the two-winding transformer is a 30-VA, 120/10V transformer, what is the power rating of the autotransformer?

Answer: 390 VA

Example 13: For the autotransformer circuit in Fig. shown. Calculate: (a) I_1 , I_2 and I_o if $Z_L = 8 + j6 \Omega$, and (b) the complex power supplied to the load.

Solution:

(a) This is a step-up autotransformer with,

$$N_1 = 80, N_2 = 120, V_1 = 120 \angle 30^\circ$$

$$V_2 = \frac{N_1 + N_2}{N_1} V_1 = \frac{200}{80} V_1$$

$$\therefore V_2 = \frac{200}{80} V_1 = \frac{200}{80} (120 \angle 30^\circ) = 300 \angle 30^\circ \text{ V}$$

$$I_2 = \frac{V_2}{Z_L} = \frac{300 \angle 30^\circ}{8 + j6} = 30 \angle -6.87^\circ \text{ A}$$

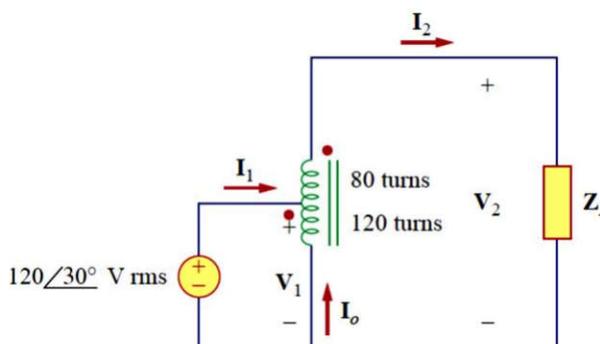
$$\frac{I_1}{I_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80} \rightarrow I_1 = \frac{200}{80} I_2 = \frac{200}{80} (30 \angle -6.87^\circ) = 75 \angle -6.87^\circ \text{ A}$$

$$\text{At the tap KCL gives}$$

$I_1 + I_o = I_2 \rightarrow I_o = I_2 - I_1 = 30 \angle -6.87^\circ - 75 \angle -6.87^\circ = 45 \angle 173.13^\circ \text{ A}$

$$(b) \text{ The complex power supplied to the load is}$$

$$S_2 = V_2 I_2^* = |I_2|^2 Z_L = (30)^2 (10 \angle 36.87^\circ) = 9 \angle 36.87^\circ \text{ kVA}$$



H.W.11: In the autotransformer circuit of Fig. shown. Find currents I_1 , I_2 and I_o . Take $V_1 = 1250 \text{ V}$, $V_2 = 500 \text{ V}$.

Answer: 12.8 A, 32 A, 19.2 A

