10) Ideal Transformers

An ideal transformer is one with perfect coupling (k =1). It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling. Consider the circuit in Fig.10.1.

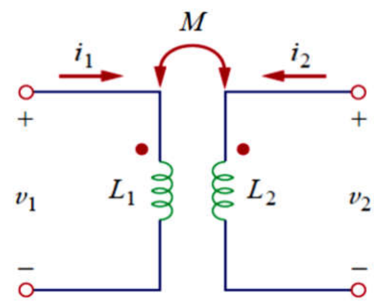


Fig.10.1

In the frequency domain,

$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \ \rightarrow\ I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1} \quad \ldots(10.1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \quad \ldots(10.2)$$

Substituting Eq. (10.1) in (10.2) gives

$$V_2 = j\omega M\left(\frac{V_1 - j\omega M I_2}{j\omega L_1}\right) + j\omega L_2 I_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2 I_2}{L_1} \quad \ldots(10.3)$$

But $M = \sqrt{L_1 L_2}$ for perfect coupling (k =1). Hence,

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}\, V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\, V_1 = nV_1 \quad \ldots(10.4)$$

where $n = \sqrt{\frac{L_2}{L_1}}$ and is called the ***turns ratio***.

As $L_1, L_2, M \rightarrow \infty$ such that n remains the same, the coupled coils become an ideal transformer.

A transformer is said to be ideal if it has the following properties:

1. Coils have very large reactances ($L_1, L_2, M \rightarrow \infty$).
2. Coupling coefficient is equal to unity (k =1).
3. Primary and secondary coils are lossless ($R_1 = 0, R_2 = 0$)

> An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

Iron-core transformers are close approximations to ideal transformers. These are used in power systems and electronics.

When a sinusoidal voltage is applied to the primary winding of ideal transformer as shown in Fig.10.2. and according to Faraday's law, the voltage across its windings are,

$$v_1 = N_1 \frac{d\emptyset}{dt} \quad \ldots(10.5)$$

$$v_2 = N_2 \frac{d\emptyset}{dt} \quad \ldots(10.6)$$

Divide Eq.(10.6) by (10.5), we get

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n \quad \ldots(10.7)$$

where n is, again, the ***turns ratio or transformation ratio***. We can use the phasor voltages and rather than the instantaneous values and Thus, Eq. (10.7) may be written as

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad \ldots(10.8)$$

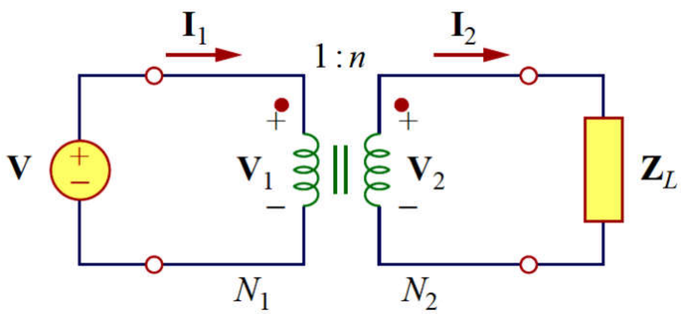


Fig.10.2

For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer. This implies that

$$v_1 i_1 = v_2 i_2 \qquad \ldots(10.9)$$

or

$$\frac{i_1}{i_2} = \frac{v_2}{v_1} = n \qquad \ldots(10.10)$$

In phasor form,

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = n \qquad \ldots(10.11)$$

$$\therefore \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n} \qquad \ldots(10.12)$$

1) If n > 1 (i.e. $N_2 > N_1$& $V_2 > V_1$), then the transformer is called ***'step-up transformer'***.
2) If n < 1 (i.e. $N_2 < N_1$& $V_2 < V_1$), then the transformer is called ***'step-down transformer'***.
3) If n = 1 (i.e. $N_2 = N_1$& $V_2 = V_1$),then the transformer is called ***'isolation transformer'*** or ***1:1 transformer***.

It is important that we know how to get the proper polarity of the voltages and the direction of the currents for the transformer in Fig.10.2. If the polarity of V_1 or V_2 or the direction of I_1 or I_2 is changed, n in Eqs. (10.7) to (10.12) may need to be replaced by $(-n)$. The two simple rules to follow are:

1) If V_1 and V_2 are ***both positive*** or ***both negative*** at the dotted terminals, use $(+n)$ in Eq.(10.8). Otherwise, use $(-n)$

2) If I_1 and I_2 ***both enter into*** or ***both leave the*** dotted terminals, use $(-n)$ in Eq.(10.12). Otherwise, use $(+n)$

The rules are demonstrated with the four circuits in Fig.10.3.

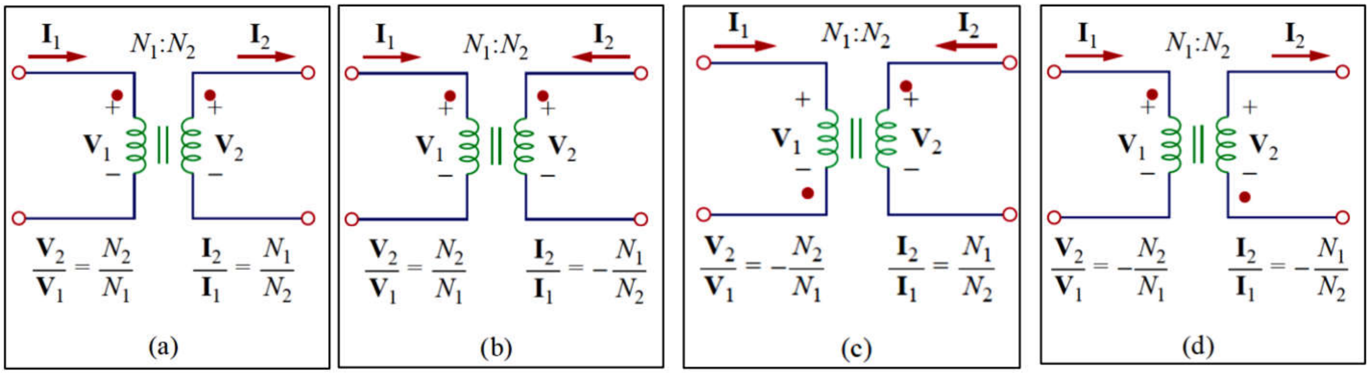


Fig.10.3

$$V_1 = \frac{V_2}{n} \quad or \quad V_2 = nV_1 \qquad \ldots(10.13)$$

$$I_1 = nI_2 \quad or \quad I_2 = \frac{I_1}{n} \qquad \ldots(10.14)$$

The complex power in the primary winding is

$$S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2 \qquad \ldots(10.15)$$

The ***input impedance*** is found as,

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{n^2}\frac{V_2}{I_2} \quad \& \quad Z_L = \frac{V_2}{I_2}$$

$$\therefore Z_{in} = \frac{Z_L}{n^2} \qquad \ldots(10.16)$$

The input impedance is also called the ***reflected impedance***, since it appears as if the load impedance is reflected to the primary side. This ability of the transformer to transform a given impedance into another impedance provides us a means of *impedance matching* to ensure maximum power transfer.

In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other. In the circuit of Fig.10.4, suppose we want to reflect the secondary side of the circuit to the primary side. We find the Thevenin equivalent of the circuit to the right of the terminals We obtain V_{TH} as the open-circuit voltage at terminals *a-b*, as shown in Fig.10.5(a).

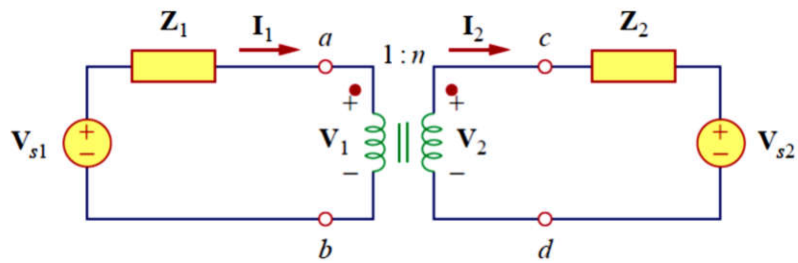


Fig.10.4

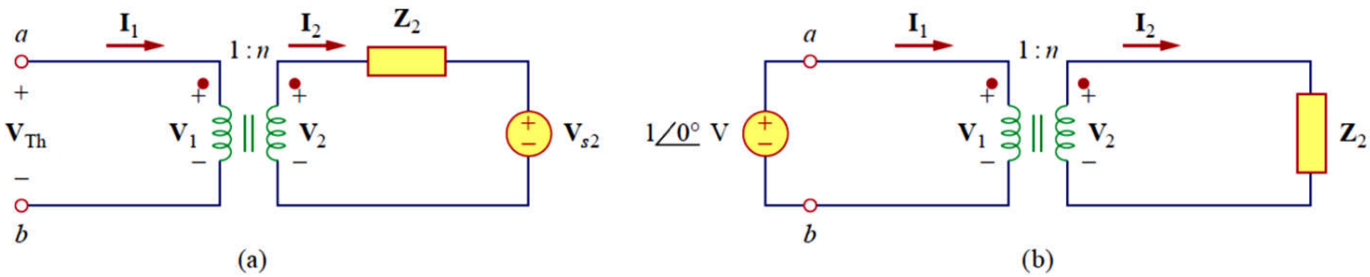


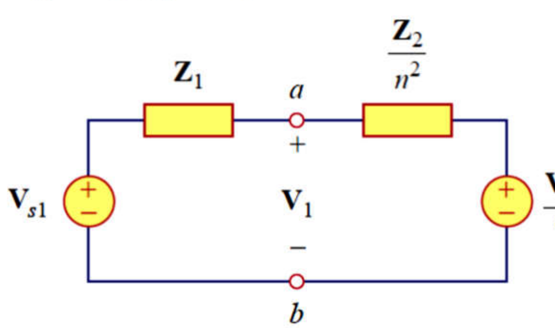
Fig.10.5(a) Obtaining $\mathbf{V}_{Th}$ for Fig. 10.4, (b) obtaining $\mathbf{Z}_{Th}$ for Fig. 10.4.

Since terminals *a-b* are open, $I_1 = 0 = I_2$ so that $V_1 = V_2$ Hence, From Eq. (10.13),

$$V_{TH} = V_1 = \frac{V_2}{n} = \frac{V_{s2}}{n} \qquad \ldots(10.17)$$

To get $\mathbf{Z}_{Th}$ we remove the voltage source in the secondary winding and insert a unit source at terminals *a-b* as in Fig. 10.5(b). From Eqs. (10.13) and (10.13),

$$Z_{TH} = \frac{V_1}{I_1} = \frac{V_2}{n}\frac{1}{nI_2} = \frac{Z_2}{n^2}, \quad V_2 = Z_2 I_2 \qquad \ldots(10.18)$$



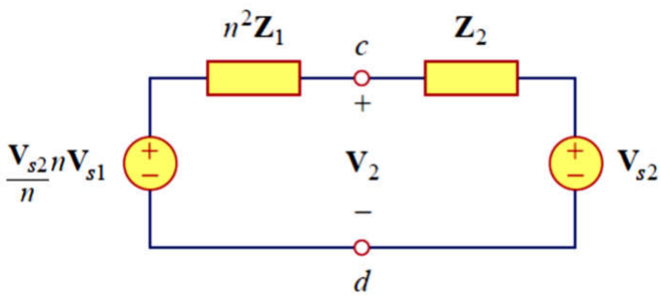


Fig. 10.6 Equivalent circuit for Fig.10.4 obtained by reflecting the secondary circuit to the primary side

Fig. 10.7 Equivalent circuit for Fig.10.4 obtained by reflecting the primary circuit to the secondary side

1) *The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side is: divide the secondary impedance by n^2, divide the secondary voltage by n, and multiply the secondary current by n.*

2) *The rule for eliminating the transformer and reflecting the primary circuit to the secondary side is: multiply the primary impedance by n^2, multiply the primary voltage by n, and divide the primary current by n.*