8) Linear Transformers

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

As shown in Fig.8.1, the coil that is directly connected to the voltage source is called ***the primary winding.*** The coil connected to the load is called ***the secondary winding.*** The resistances R_1 and R_2 are included to account for the losses (power dissipation) in the coils.

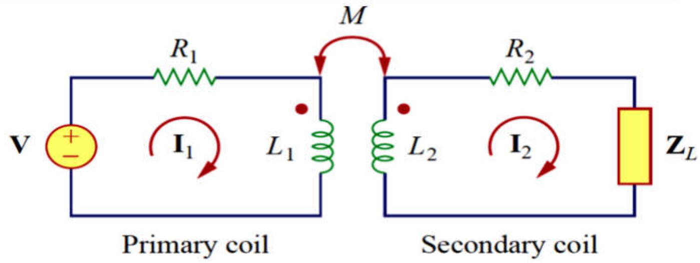


Fig.8.1 A linear transformer

The transformer is said to be ***linear if the coils are wound on a magnetically linear material(a material for which the magnetic permeability is constant).*** Such materials include air, plastic, Bakelite, and wood. In fact, most materials are magnetically linear. Linear transformers are sometimes called ***air-core transformers***, although not all of them are necessarily air-core. They are used in radio and TV sets.

The input impedance ($\boldsymbol{Z_{in}} = \frac{\boldsymbol{V}}{\mathbf{I_1}}$) as seen from the source governs the behavior of the primary circuit.

Applying KVL to the two meshes in Fig.8.1 gives

$$V = (R_1 + j\omega L_1)I_1 - j\omega M I_2 \qquad \ldots(8.1)$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2 \qquad \ldots(8.2)$$

From Eq.(8.2)

$$I_2 = \frac{j\omega M}{(R_2 + j\omega L_2 + Z_L)} I_1 \qquad \ldots(8.3)$$

Substitute Eq. (8.3) in (8.1) we get,

$$V = \left((R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \right) I_1 \qquad \ldots(8.4)$$

From Eq. (8.4) we get,

$$\boxed{\boldsymbol{Z_{in}} = \frac{\boldsymbol{V}}{\mathbf{I_1}} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}} \qquad \ldots(8.5)$$

Notice that the input impedance comprises two terms. The first term, $(R_1 + j\omega L_1)$, is the primary impedance. The second term is due to the coupling between the primary and secondary windings. It is as though this impedance is reflected to the primary. Thus, it is known as the ***reflected impedance*** $\boldsymbol{Z_R}$ and

$$\boxed{Z_R = \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}} \qquad \ldots(8.6)$$

It should be noted that the Eq. (8.5) or (8.6) is not affected by the location of the dots on the transformer, because the same result is produced when M is replaced by $-M$.

9) Conductively Coupled Equivalent Circuits

It is convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling. Now to replace the linear transformer in Fig.8.2 by an equivalent **T** or **Π** circuit, a circuit that would have no mutual inductance.

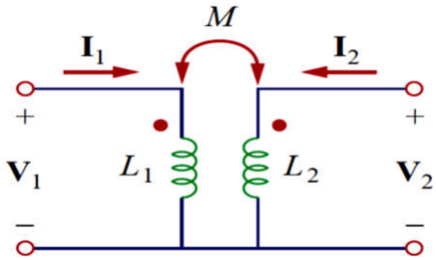


Fig.8.2

The voltage-current relationships for the primary and secondary coils give the matrix equation

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \ldots(9.1)$$

By matrix inversion, this can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{L_2}{j\omega(L_1L_2 - M^2)} & \frac{-M}{j\omega(L_1L_2 - M^2)} \\ \frac{-M}{j\omega(L_1L_2 - M^2)} & \frac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \ldots(9.2)$$

1) For the T (or Y) network of Fig.8.3, mesh analysis provides the terminal equations as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \ldots(9.3)$$

If the circuits in Fig.8.2 and Fig.8.3 are equivalents, Eqs. (9.1) and (9.3) must be identical. Equating terms in the impedance matrices of Eqs. (9.1) and (9.3) leads to

$$L_a = L_1 - M, \quad L_b = L_2 - M, \quad L_c = M \qquad \ldots(9.4)$$

2) For the **Π** (or Δ) network in Fig.8.4, nodal analysis gives the terminal equations as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \ldots(9.5)$$

Equating terms in admittance matrices of Eqs. (9.2) and (9.5), we obtain

$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \quad L_B = \frac{L_1L_2 - M^2}{L_1 - M}, \quad L_C = \frac{L_1L_2 - M^2}{M} \qquad \ldots(9.6)$$

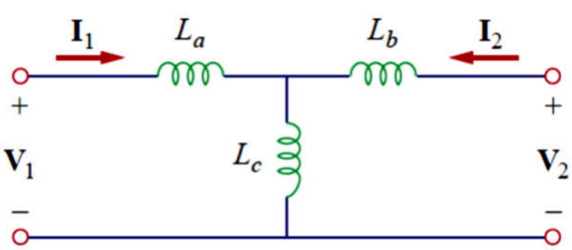


Fig.8.3 An equvalient **T** circuit

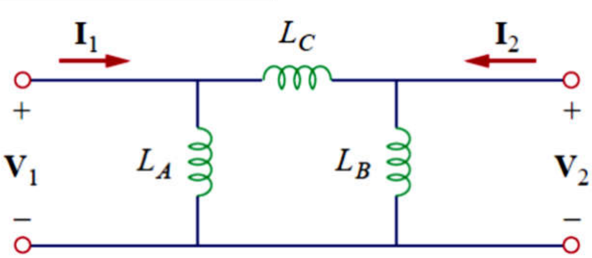
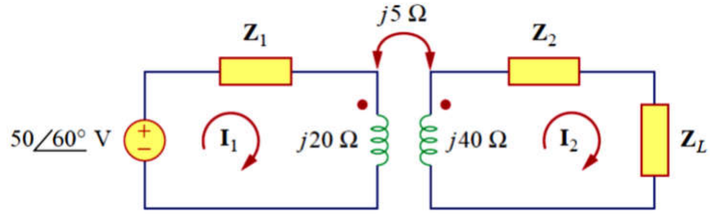


Fig.8.4 An equvalient **Π** circuit

Example 6: In the circuit shown, calculate the input impedance and current $\mathbf{I}_1$. Take $Z_1 = 60 - j100\ \Omega, Z_2 = 30 + j40\Omega$ and $Z_L = 80 + j60\Omega$

Solution: From Eq.(8.22)

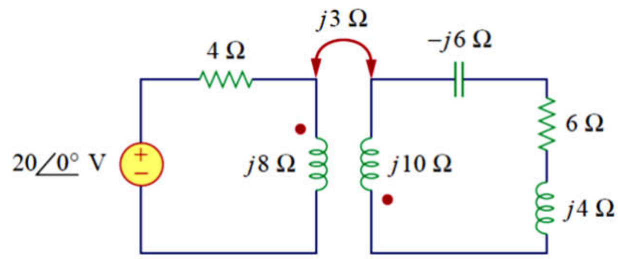


$$Z_{in} = (Z_1 + j20) + \frac{5^2}{(j40 + Z_2 + Z_L)}$$
$$= 60 - j100 + j20 + \frac{25}{110 + j140}$$
$$= 60.09 - j80.11 = 100.14\angle -53.1^o\Omega$$
$$I_1 = \frac{V}{Z_{in}} = \frac{50\angle 60^o}{100.14\angle -53.1^o} = 0.5\angle 113.1^o \text{ A}$$

H.W.4: Find the input impedance of the circuit and the current from the voltage source.

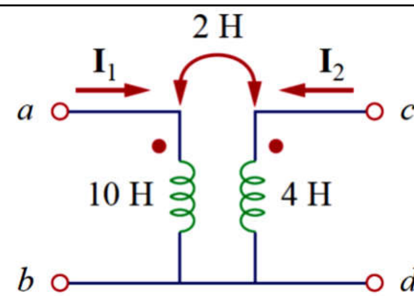
Answer:

$8.58\angle 58.05^o\Omega, 2.331\angle -58.05^o$ A



Example 7: Determine the T-equivalent circuit of the linear transformer shown.

Solution:

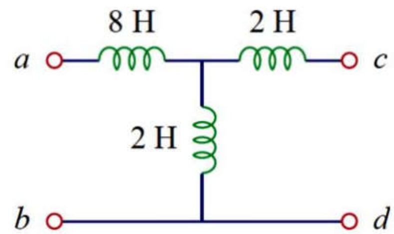


$L_1 = 10, L_2 = 4,$ and $M = 2$

$L_a = L_1 - M = 10 - 2 = 8$ H,

$L_b = L_2 - M = 4 - 2 = 2$ H,

$L_c = M = 2$ H

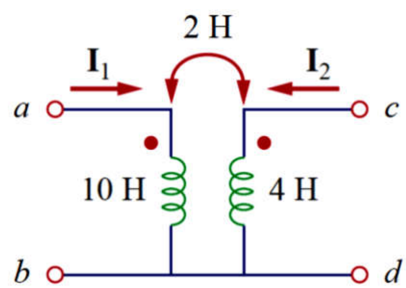


We have assumed that reference directions for currents and voltage polarities in the primary and secondary windings conform to those in Fig.15 . Otherwise, we may need to replace M with $-M$, as **Example 8** illustrates. The T-equivalent circuit is shown in Fig.

H.W.5: For the linear transformer , find the equivalent network.

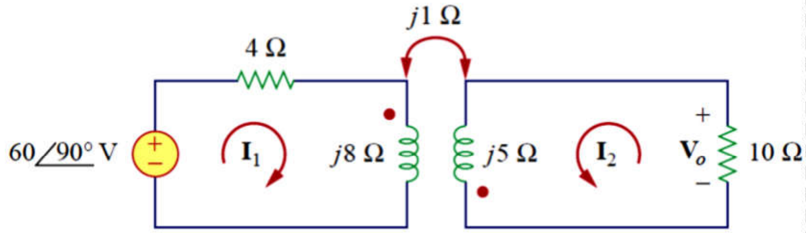
Answer:

$L_A = 18$ H, $L_B = 4.5$ H, $L_C = 18$ H



Example 8: Solve for I_1, I_2 and V_O using the T-equivalent circuit for the linear transformer.

Solution:



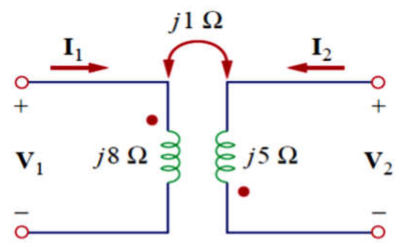
$L_1 = 8, L_2 = 5,$ and $M = -1$

$L_a = L_1 - M = 8 - (-1) = 9$ H,

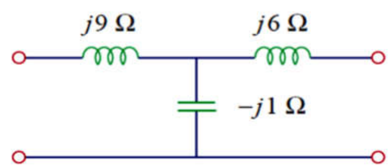
$L_b = L_2 - M = 5 - (-1) = 6$ H,

$L_c = M = -1$ H

To represent the circuit in frequency domain as ω is not specified , we assume ω =1 rad, and the equavalent T circuit in frequency domain is shown in Fig.



(a)



(b)

Apply mesh analysis,

For mesh 1,

$j6 = (4 + j9 - j1)I_1 + (-j1)I_2$ …(1)

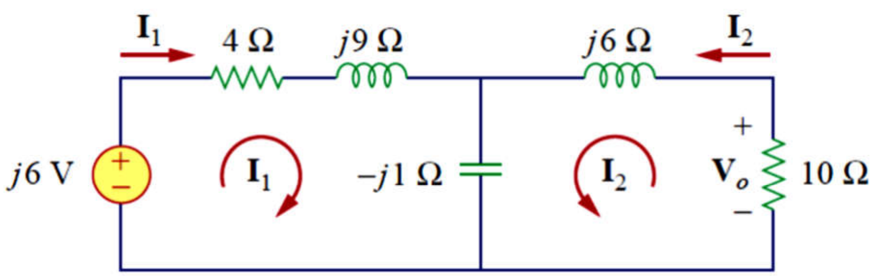
For mesh 2,

$0 = (-j1)I_1 + (10 + j6 - j1)I_2$ …(2)

Solve Eq.s (1) & (2) we get,

$I_1 = 0.6 + j0.3$ A, $I_2 = j0.06$ A & $V_O = -10I_2 = j0.6$ V

The complete equivalent T circuit in frequency domain is shown in Fig below.



H.W.6: Calculate the phasor currents **I**$_1$ and **I**$_2$ in the circuit shown (**in example3**) using the T-equivalent model for the magnetically coupled coils.

Answer:

$13\angle -49.4^o$A, $2.91\angle 14.04^o$ A