Experiment No. 6
Delta – Star connection

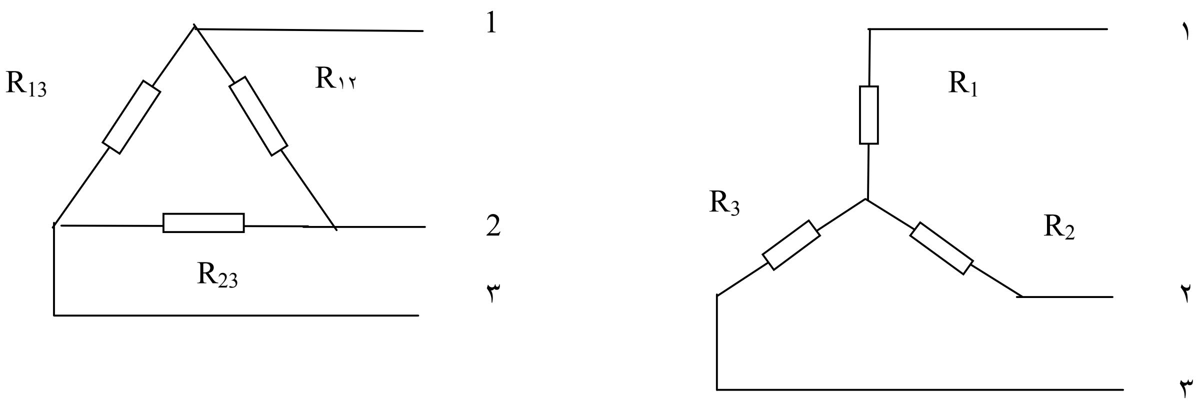
Aim of experiment: To study the properties of delta-star connection.

Apparatus

1. DC circuit training system
2. Set of wires.
3. DC Power supply
4. Digital A.V.O. meter

Theory

In solving networks (having considerable number of branches) by the application of Kirchhoff's Laws, one sometimes experiences great difficulty due to a large number of simultaneous equation that have to be solve. However, such complicated networks can simplify by successively replacing delta meshes by equivalent star systems and *vice versa*.



Delta-connection (b) Star-connection (a)

Fig (1)

Suppose we are given three resistance R_{12},R_{23} and R_{13} connected in delta fashion between terminals 1,2 and 3 as in Fig.(1-a). So far as the respective terminals are concerned, these given three resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in Fig.(1-b). These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both arrangements.

1. To convert from ***delta connection*** to ***star connection***

$$R_1 = \frac{R_{12} \times R_{13}}{R_{12} + R_{23} + R_{13}}$$

$$R_2 = \frac{R_{12} \times R_{23}}{R_{12} + R_{23} + R_{13}}$$

$$R_3 = \frac{R_{23} \times R_{13}}{R_{12} + R_{23} + R_{13}}$$

2. To convert from ***star connection*** to ***delta connection***

$$R_{12} = R_1 + R_2 + \frac{R_1 \times R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_3 \times R_2}{R_1}$$

$$R_{13} = R_1 + R_3 + \frac{R_3 \times R_1}{R_2}$$

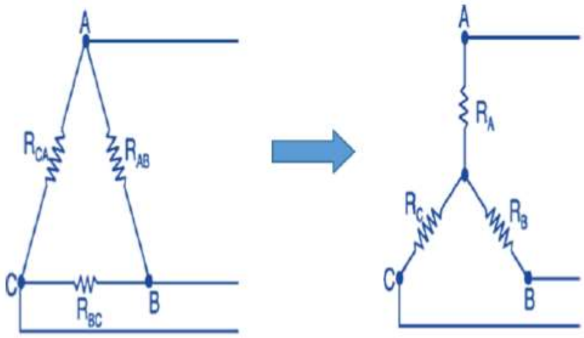
Procedure

1. Using the DC circuit trainer, connect the circuit shown below.
2. Measure "$\mathbf{I_1}$, $\mathbf{I_2}$, $\mathbf{I_3}$ " practically.
3. Record your results in the table below
4. By using delta-star conversion, find the star resistance R_1, R_2, R_3 theoretically.

- Similarly for resistance between two terminals B-C and C-A,

$$\Rightarrow R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad (3)$$

$$\Rightarrow R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad (4)$$

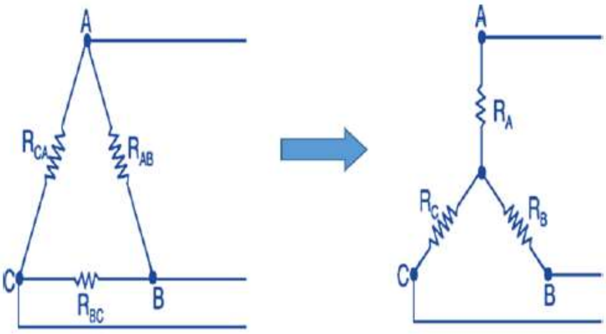


- The objective is to find R_A, R_B *and* R_C in terms of R_{AB}, R_{BC} *and* R_{CA} .
- Subtracting (3) from (2) and adding to (4) we obtain,

$$\Rightarrow R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad (5)$$

$$\Rightarrow R_B = \frac{R_{BC}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \quad (6)$$

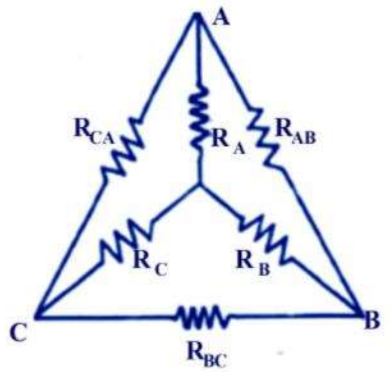
$$\Rightarrow R_C = \frac{R_{CA}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad (7)$$



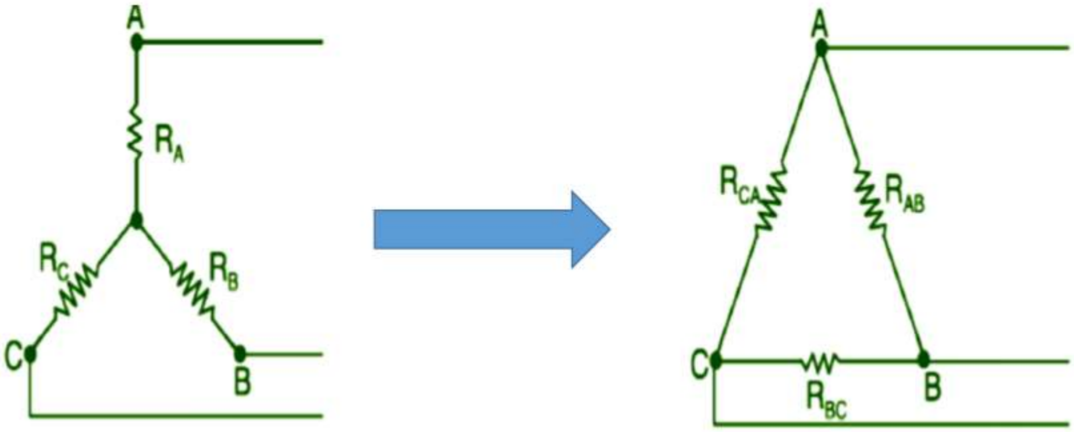
DELTA TO STAR TRANSFORMATION

- Easy way to remember delta to star transformation is,

$$\textbf{Any arm of star connection} = \frac{\textbf{Product of two adjacent arms of } \Delta}{\textbf{Sum of arms of } \Delta}$$



Three resistors R_A, R_B *and* R_C connected in star formation and its equivalent delta connection is shown below



Star and its Equivalent Delta

- Dividing (5) by (6) we obtain,

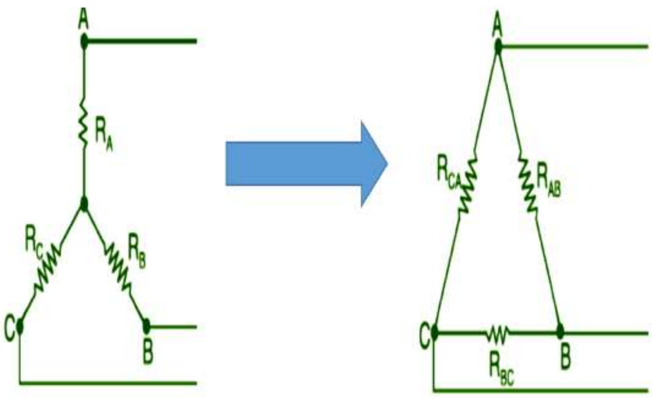
$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}} \qquad (8)$$

$$\Rightarrow R_{CA} = \frac{R_A R_{BC}}{R_B} \qquad (9)$$

- Dividing (5) by (7) we obtain,

$$\frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}} \qquad (10)$$

$$\Rightarrow R_{AB} = \frac{R_A R_{BC}}{R_C} \qquad (11)$$



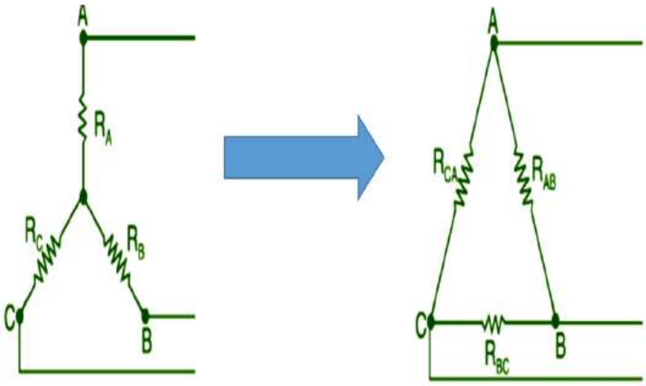
- Substituting (9) and (11) into (5),

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \qquad (12)$$

- Similarly,

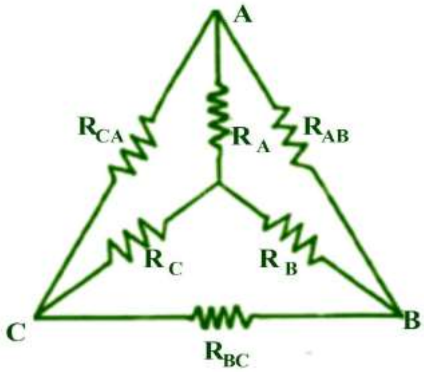
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \qquad (13)$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B} \qquad (14)$$



- Easy way to remember star to delta transformation is,

Resistance between two terminals of Δ =
Sum of star resistances connected to those terminals + product of same two resistances divided by the third

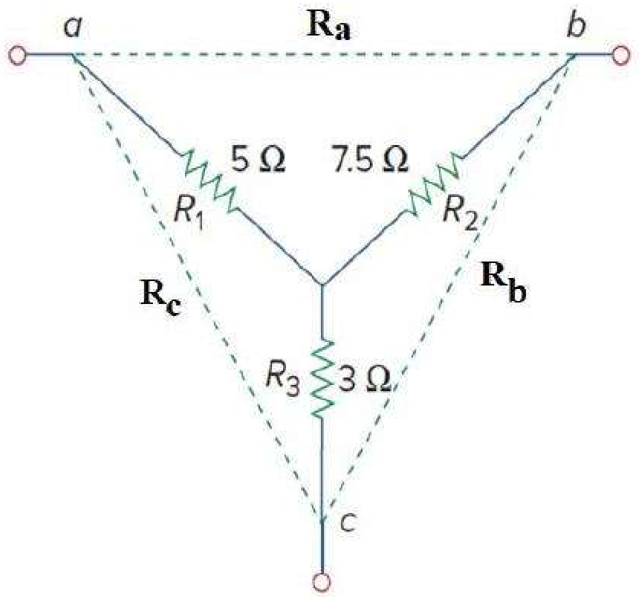


- If a star network has all resistances equal to R, its equivalent delta has all resistances equal to ?

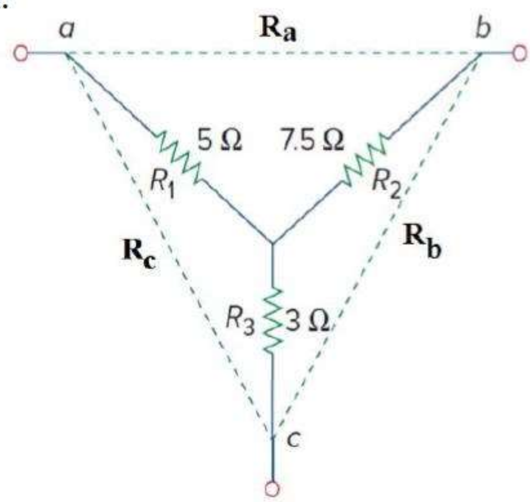
- If a delta network has all resistances equal to R, its equivalent star has all resistances equal to ?

Example :-

Q. Convert the Y network to an equivalent Δ network.



Soln:



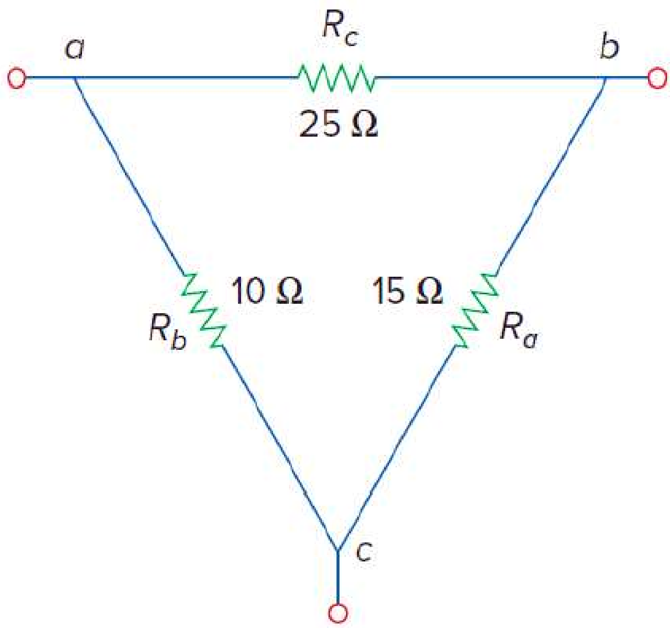
$$R_a = 7.5 + 5 + \frac{7.5 \times 5}{3} = 25 \ ohms$$

$$R_b = 7.5 + 3 + \frac{7.5 \times 3}{5} = 15 \ ohms$$

$$R_c = 5 + 3 + \frac{5 \times 3}{7.5} = 10 \ ohms$$

Example

Q. Convert the Δ network to an equivalent Y network.

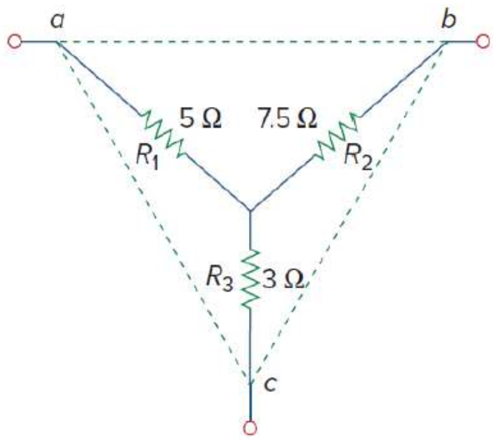


Soln:

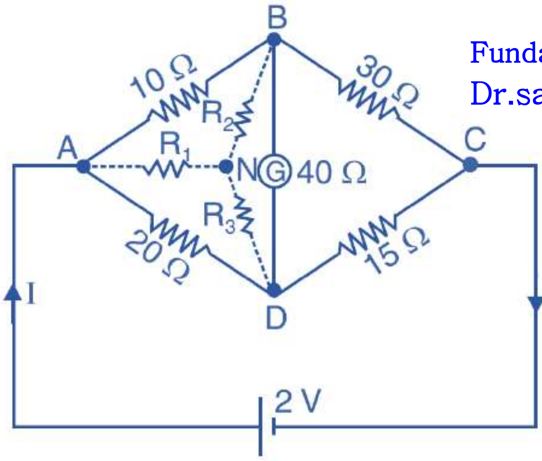
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10\times 25}{15+10+25} = 5\ ohms$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25\times 15}{50} = 7.5\ ohms$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15\times 10}{50} = 3 ohms$$



Q. Using delta/star transformation, find equivalent resistance across AC.



Soln: Delta can be replaced by equivalent star-connected resistances,

$$R_1=\frac{R_{AB}R_{DA}}{R_{AB}+R_{DA}+R_{BD}}=\frac{10\times 20}{10+40+20}=2.86\ ohms$$

$$R_2=\frac{R_{AB}R_{BD}}{R_{AB}+R_{DA}+R_{BD}}=\frac{10\times 40}{10+40+20}=5.72\ ohms$$

$$R_3=\frac{R_{DA}R_{BD}}{R_{AB}+R_{DA}+R_{BD}}=\frac{10\times 40}{10+40+20}=11.4\ ohms$$

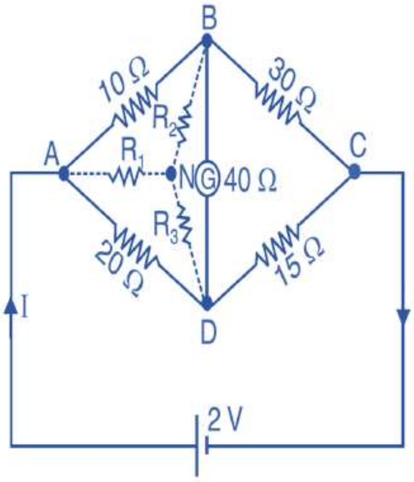
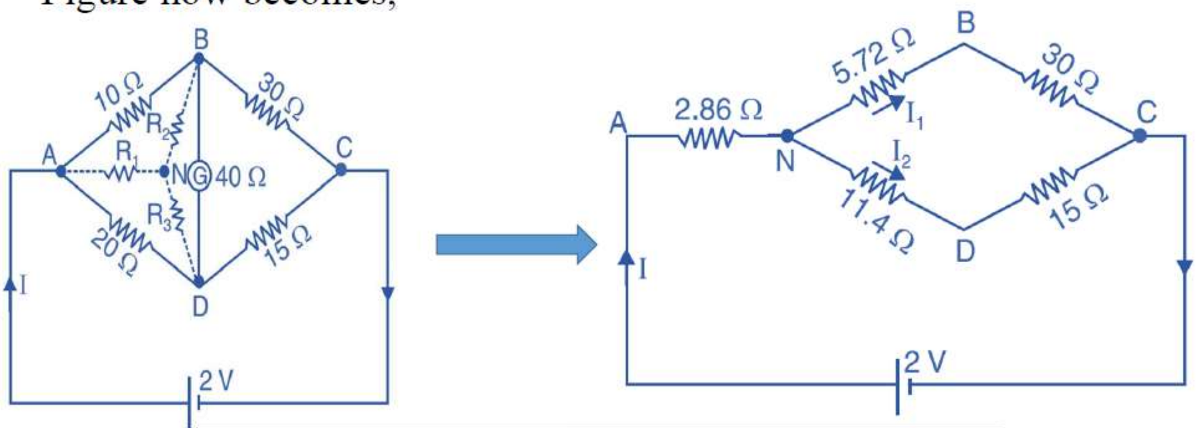
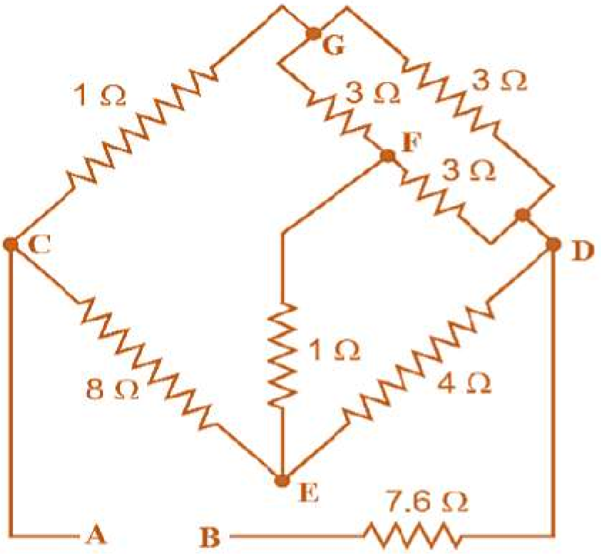


Figure now becomes,

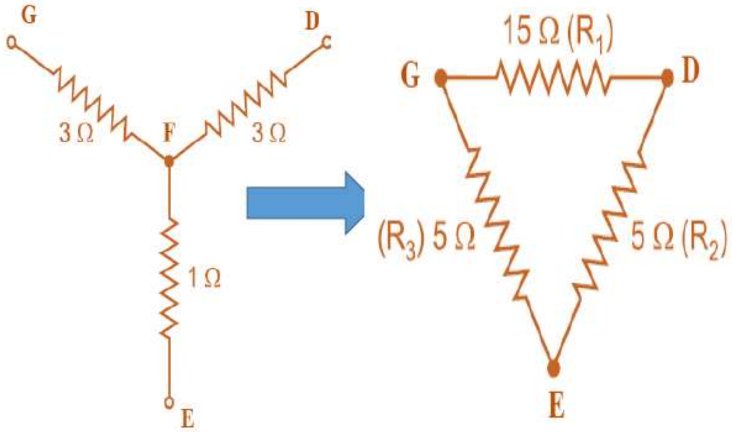


$$R_{AC}=2.86+\frac{(30+5.72)\times(15+11.4)}{(30+5.72)+(15+11.4)}=18.04\ ohms$$

Q. Calculate equivalent resistance across terminals A and B.



Soln: Converting inner STAR (3 ohms, 3 ohms and 1 ohms) into Delta.

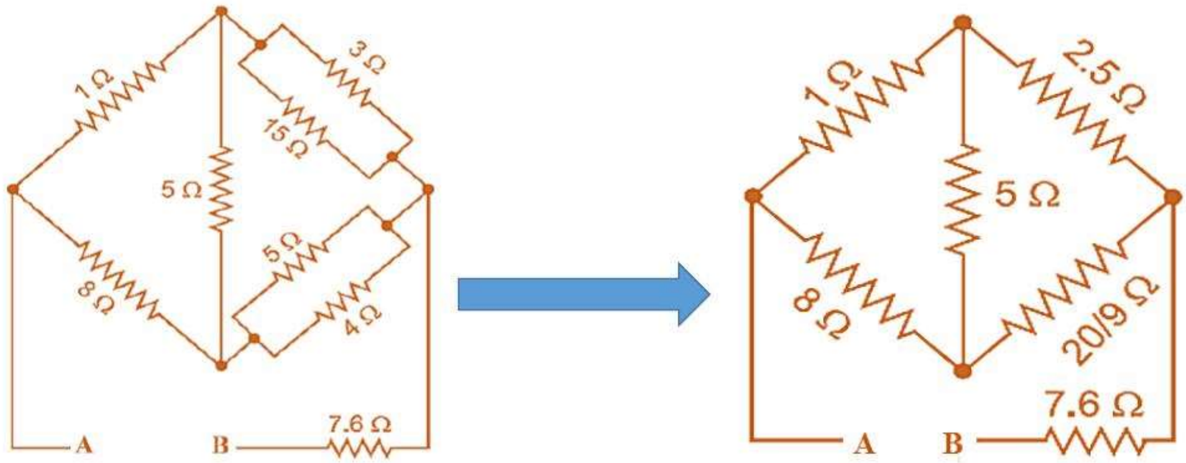


$$R_1 = 3 + 3 + \frac{3 \times 3}{1} = 15 \; ohms$$

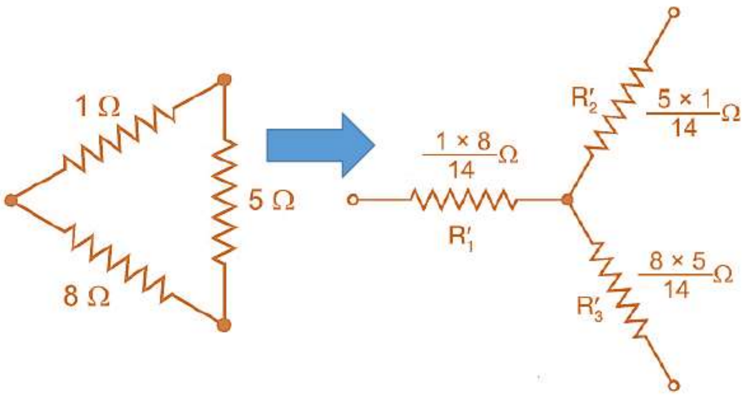
$$R_2 = 3 + 1 + \frac{3 \times 1}{3} = 5 \; ohms$$

$$R_3 = 1 + 3 + \frac{1 \times 3}{3} = 5 \; ohms$$

Circuit now becomes,



Delta-connected resistances 1 Ω, 5 Ω and 8 are converted in star,

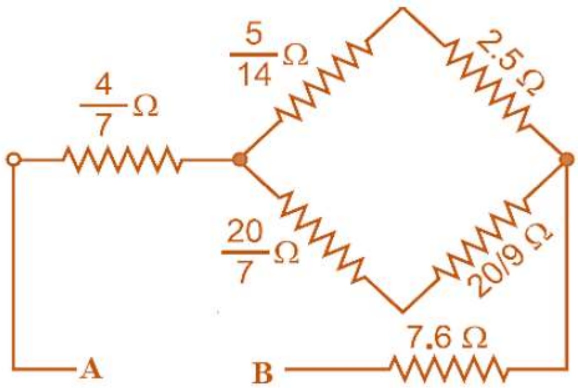


$$R_1' = \frac{1\times 8}{1+5+8} = \frac{4}{7}\, ohms$$

$$R_2' = \frac{5\times 1}{1+5+8} = \frac{5}{14}\, ohms$$

$$R_3' = \frac{8\times 5}{1+5+8} = \frac{20}{7}\, ohms$$

Circuit now becomes,



$$R_{AB} = \frac{4}{7} + \left[\left(\frac{5}{14} + 2.5\right) \parallel \left(\frac{20}{7} + \frac{20}{9}\right)\right] + 7.6 = 10\ ohms$$

Q. Calculate equivalent resistance across terminals A and B.