Kirchhoff's Circuit Laws

Kirchhoff's Laws

What are Kirchhoff's Laws?

- ❖ Kirchhoff's laws govern the conservation of charge and energy in electrical circuits.

•Kirchhoff's Laws

1.The junction rule

2.The closed loop rule

Kirchhoff's currents Laws

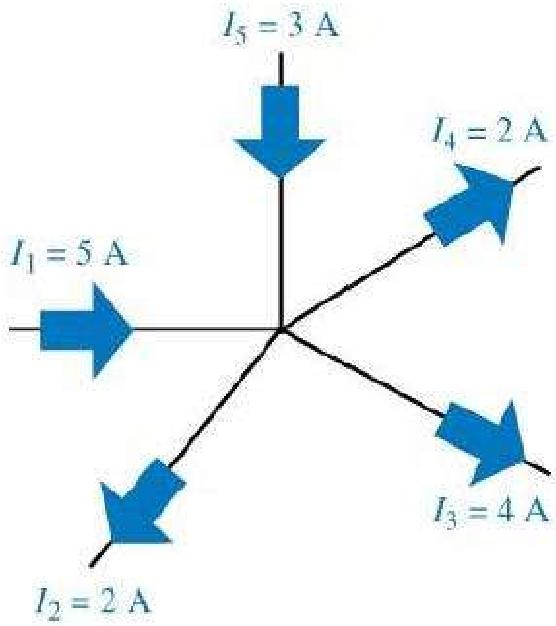
• "At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node, or: The algebraic sum of currents in a network of conductors meeting at a point is zero".

•The sum of currents entering the junction are thus equal to the sum of currents leaving. This implies that the current is conserved (no loss of current).

Kirchhoff's Current Law (KCL) Kirchhoff's Current Law states that the algebraic sum of the currents entering and leaving a node is equal to zero

$$\sum I = 0$$

By convention, currents entering the node are positive, and those leaving a node are negative. For the picture at the right:

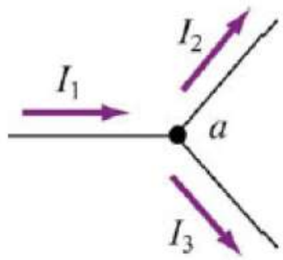


$$\sum_{n=1}^{N} I_n = I_1 + (-I_2) + (-I_3) + (-I_4) + I_5 = 0$$

KCL can also be expressed as “The sum of the currents entering a node is equal to the sum of the currents leaving a node

$$\sum I_{in} = \sum I_{out} \qquad I_1 + I_5 = I_2 + I_3 + I_4$$

- Example 2: If the currents exiting from junction "a" are to be of 2 amps each, what is the value for the current entering the junction?



Recall the junction rule for this case:

$$I_1 = I_2 + I_3$$

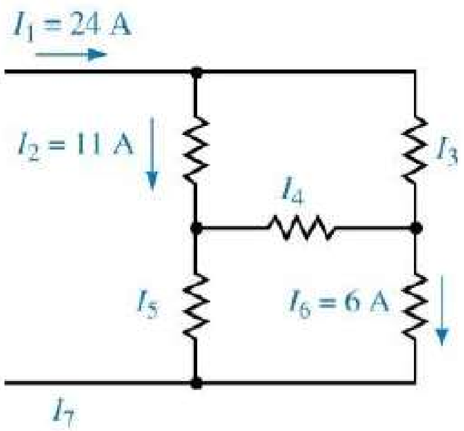
We know the following values:

$$I_2 = I_3 = 2\ amps$$

Then, we can solve for current entering the junction:

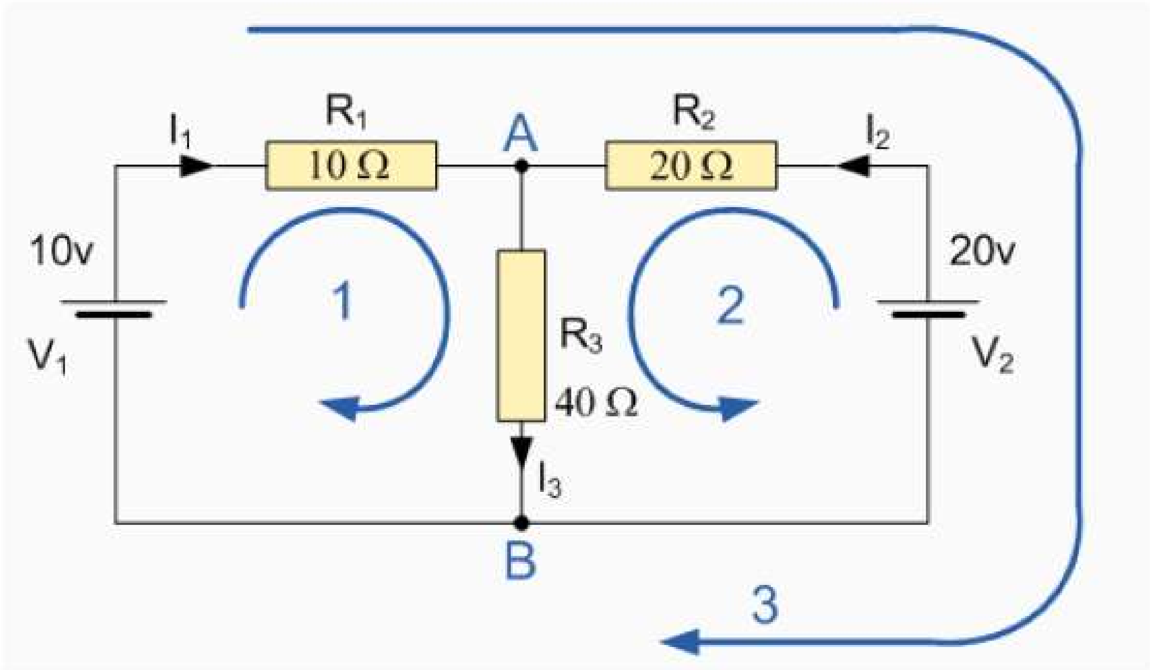
$$I_1 = 2 + 2 = 4\ amps$$

<u>Example</u>: Determine the unknown currents in the circuit shown below.

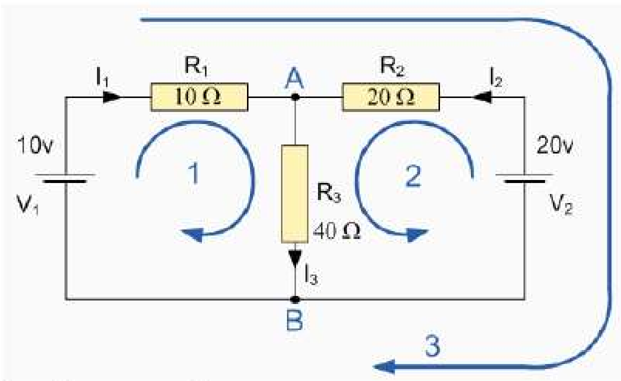


<u>Solution</u>:

Example 3: Determine the values of the the current flowing through each of the resistors.



- Example 3 (cont'd)



The circuit has two nodes (at A and B). We have the choice of choosing only two of the three loops shown (blue). This is because only two of the loops are independent.

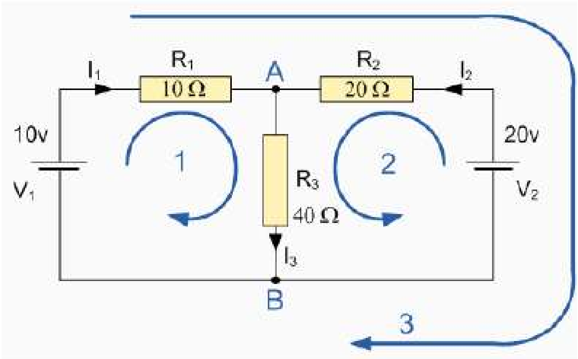
Node A $I_1 + I_2 = I_3$

Node B $I_3 = I_1 + I_2$

Loop 1 $10 - I_1R_1 - I_3R_3 = 0$

Loop 2 $20 - I_2R_2 - I_3R_3 = 0$

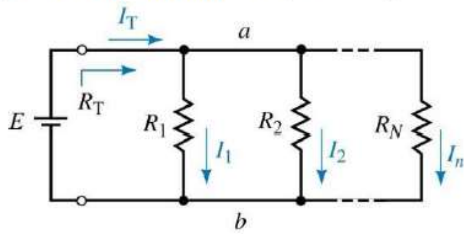
$$I_1 + I_2 = I_3$$
$$I_3 = I_1 + I_2$$
$$10 - I_1R_1 - I_3R_3 = 0$$
$$20 - I_2R_2 - I_3R_3 = 0$$



- By substitution, the answer can be shown to be I1=-0.143amps, and I2=0.429amps.

Resistors in Parallel Consider a circuit with 3 resistors in parallel (such as the circuit below, if $N = 3$).

$$I_T = I_1 + I_2 + I_3 \Rightarrow \frac{E}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$



Since the voltages across all the parallel elements in a circuit are the same ($E = V_1 = V_2 = V_3$), we have:

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \Rightarrow \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This result can be generalized to provide the total resistance of any number of resistors in parallel:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Special Case: Two Resistors in Parallel For only two resistors connected in parallel, the equivalent resistance may be found by the product of the two values divided by the sum:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

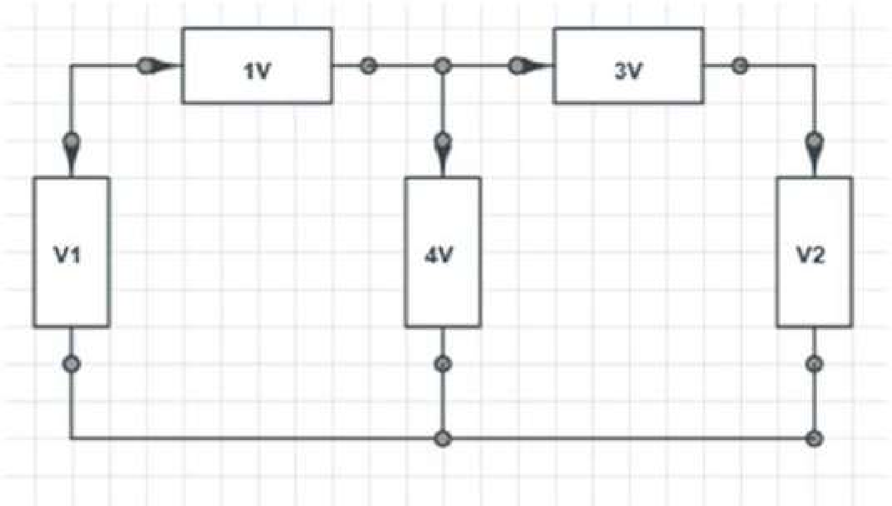
If you want to be cool, you should refer to this as the "product over the sum" formula. Your EE friends will really admire this.

Special Case: Equal Resistors in Parallel Total resistance of n equal resistors in parallel is equal to the resistor value divided by the number of resistors (n):

$$R_T = \frac{R}{n}$$

EXAMPLE

Find V_1 and V_2



Loop 1

$$-V_1 + 1 + 4 = 0 = V_1 = 5V$$

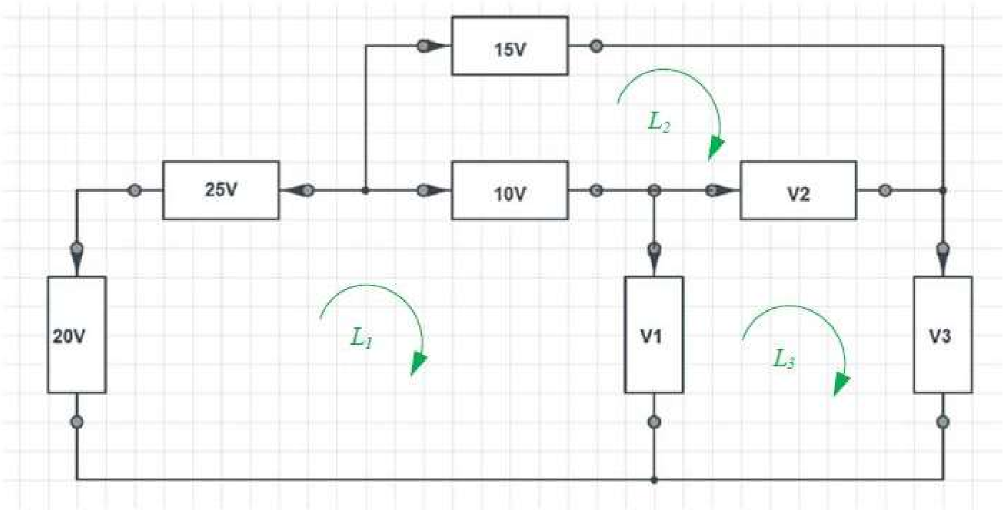
Loop 2

$$-4 + 3 + V_2 = 0 \Rightarrow V_2 = 1V$$

EXAMPLE

Find V_1, V_2, V_3

(note: the arrows are signifying the positive position of the box and the negative is at the end of the box)



Loop 1

$$-20 - 25 + 10 + V_1 = 0 \Rightarrow V_1 = 35V$$

Loop 2

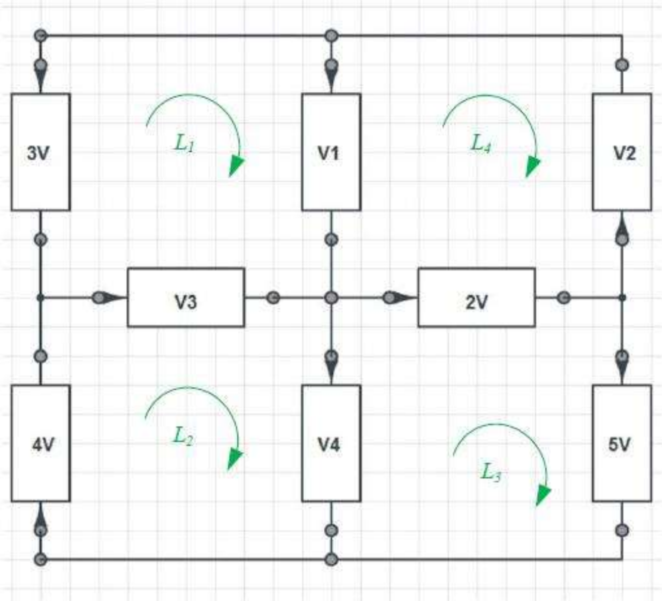
$$-10 + 15 - V_2 = 0 \Rightarrow V_2 = 5$$

Loop 3

$$-V_1 + V_2 + V_3 = 0 \Rightarrow -35 + 5 + V_3 = 0 \Rightarrow V_3 = 30V$$

Find V_1, V_2, V_3, V_4

(note: the arrows are signifying the positive position of the box and the negative is at the end of the box)



Loop 1

$$-V_4 + 2 + 5 = 0$$
$$V_4 = 7V$$

Loop 2

$$4 + V_3 + V_4 = 0$$
$$V_3 = -4 - 7$$
$$= -11V$$

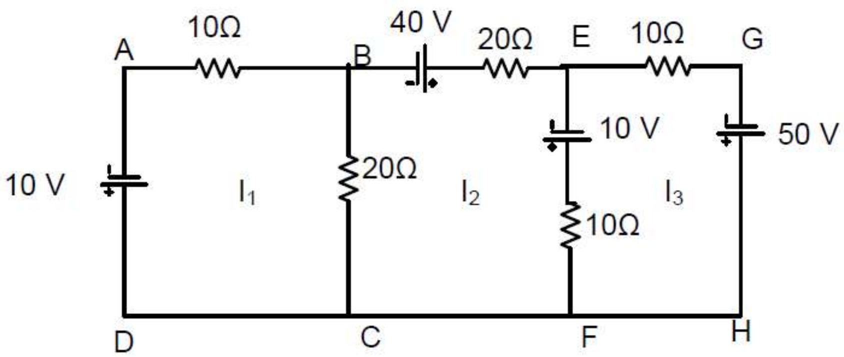
Loop 3

$$-3 + V_1 - V_3 = 0$$
$$V_1 = V_3 + 3$$
$$= -11 + 3$$
$$= -8V$$

Loop 4

$$-V_1 - V_2 - 2 = 0$$
$$V_2 = -V_1 - 2$$
$$= 6V$$

Example: Find I_1, I_2 and I_3 in the network shown in Fig below using loop current method



Solution:- For mesh ABCDA,

$$-I_1\times 10-(I_1-I_2)\times 20-10=0$$
$$\Rightarrow\ 3I_1-2I_2=-1 \qquad (1)$$

For mesh BEFCB,

$$40-I_2\times 20+10-(I_2-I_3)\times 10-(I_2-I_1)\times 20=0$$
$$\Rightarrow\ 2I_1-5I_2+I_3=-5 \qquad (2)$$

For mesh EGHFE,

$$-10I_3+50-(I_3-I_2)\times 10-10=0$$
$$\Rightarrow\ I_2-2I_3=-4 \qquad (3)$$

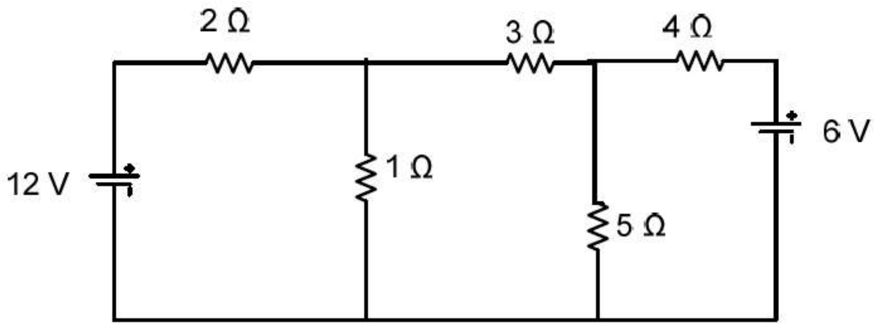
Equation (2) x 2 + Equation (3)

$$4I_1-9I_2=-14 \qquad (4)$$

Solving eq^n (1) & eq^n (4)

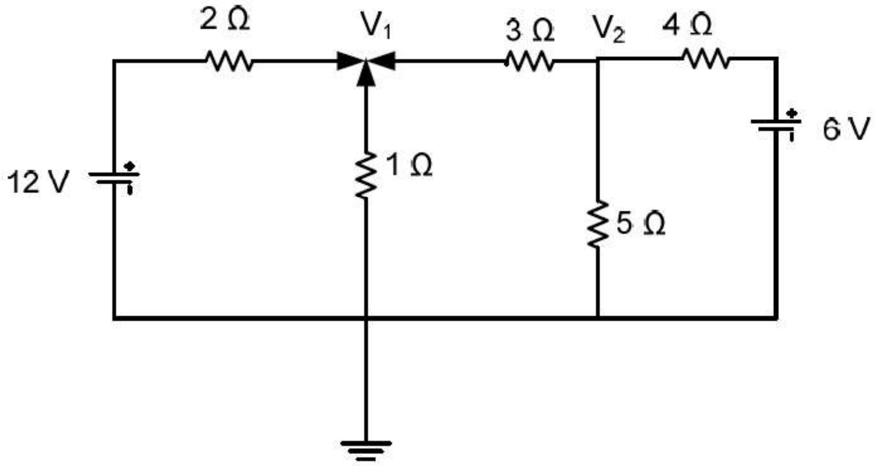
$I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A

Example: - Use nodal analysis to find currents in the different branches of the circuit shown below.



Solution:-

Let V_1 and V_2 are the voltages of two nodes as shown in Fig below



Applying KCL to node-1, we get

$$\frac{12 - V_1}{2} + \frac{0 - V_1}{1} + \frac{V_2 - V_1}{3} = 0$$

$$\Rightarrow 36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$

$$\Rightarrow -11V_1 + 2V_2 = 36 \ldots\ldots\ldots\ldots\ldots(1)$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 \ldots\ldots\ldots\ldots\ldots(2)$$

Solving Eq (1) and (2) we get $V_1 = 3.924$ Volt and $V_2 = 3.584$ volt

Current through 2 Ω resistance = $\frac{12-V_1}{2} = \frac{12-3.924}{2} = 4.038\,A$

Current through 1 Ω resistance = $\frac{0-V_1}{1} = -3.924\,A$

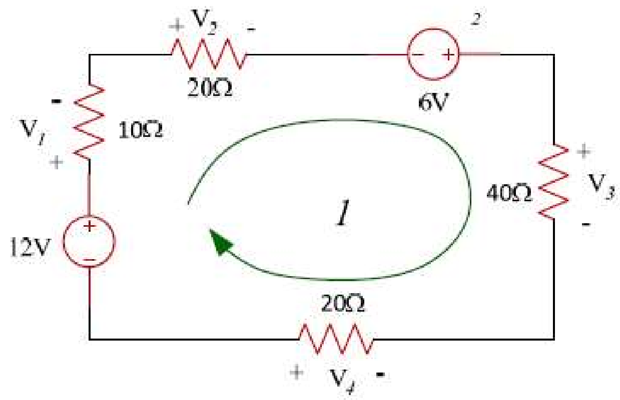
Current through 3 Ω resistance = $\frac{V_1-V_2}{3} = 0.1133\,A$

Current through 5 Ω resistance = $\frac{0-V_2}{5} = -0.7168\,A$

Current through 4 Ω resistance = $\frac{6-V_2}{4} = 0.604\,A$

As currents through 1Ω and 5Ω are negative, so actually their directions are opposite to the assumptions.

Example.: Find the current I for the circuit shown.



KVL equations for voltages

$$v_1 + v_2 + v_3 - v_4 = 18$$

Using Ohm's Law

$$v_1 = 10\Omega i \qquad v_2 = 20\Omega i \qquad v_3 = 40\Omega i \, , \qquad v_4 = -20\Omega i$$

Apply them into KVL eauation

$$10i + 20i + 40i + 20i = 18$$

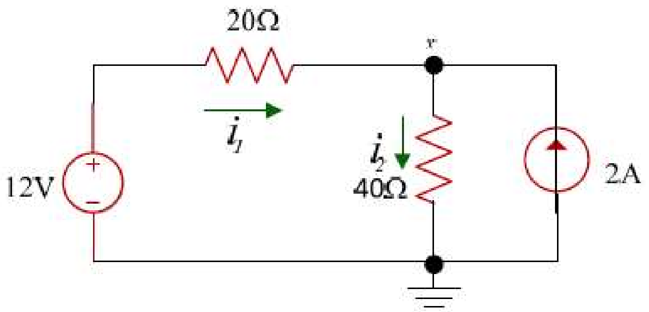
$$(90)i = 18$$

$$i = \frac{18}{90} = 0.2A$$

$$V_1 = 10\Omega i = 10(0.2) = 2V \qquad V_2 = 20\Omega i = 20(0.2) = 4V$$

$$V_3 = 40\Omega i = 40(0.2) = 8V \qquad V_4 = 20\Omega i = 20(0.2) = 4V$$

Example: Find the current through a 20 Ω resistor, and current through a 40 Ω resistor in the following circuit.



- Write KCL at node x

$$i_1 - 2_2 + 2A = 0$$

- Write v_x in the circuit using Ohm's Law

$$i_1 = \frac{12V - v_x}{20\Omega} \quad \text{and} \quad i_2 = \frac{v_x}{40\Omega}$$

- Apply them in to KCl equation

$$\frac{12V - v_x}{20\Omega} - \frac{v_x}{40\Omega} + 2A = 0$$

$$0.6 - 0.05v_x - 0.025v_x + 2A = 0$$

$$0.075v_x = 2.6A$$

$$v_x = 34.67V$$

$$i_1 = \frac{12V - v_x}{20\Omega} = \frac{12V - 34.67}{20\Omega} = -1.134A$$

$$i_2 = \frac{V_x}{40\Omega} = \frac{34.67}{40\Omega} = 0.867A$$

Example: For the circuit in Fig. shown, find voltages v_1 and v_2.

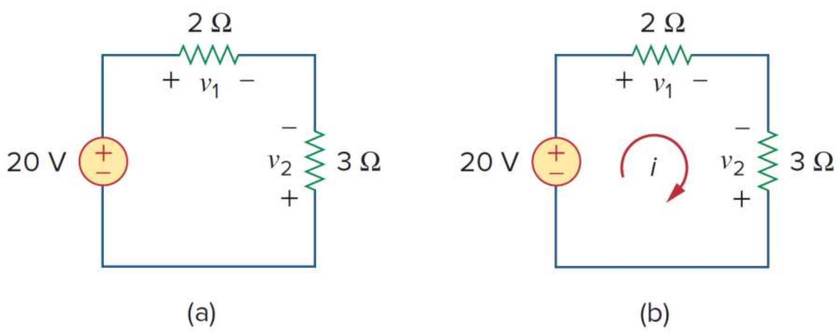


Figure 2.21
For Example 2.5.

Solution:

To find v_1 and v_2 we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Rightarrow \qquad i = 4\text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$