2 Electric circuit elements

2.1 ELECTRICITY

The atoms which make up all things consist of a number of particles including the electron, the proton and the neutron. The others are more of interest to physicists than to engineers. The electron has a mass of 9.11×10^{-31} kg and carries a negative electric charge; the proton has a mass of 1.6×10^{-27} kg and carries a positive electric charge equal in magnitude to the negative charge of the electron; the neutron has the same mass as the proton but carries no electric charge. Apart from the hydrogen atom, which has one electron and one proton but no neutrons, all atoms contain all three of these subatomic particles. Atoms are normally electrically neutral because they have the same number of electrons as they have protons. If some electrons are removed from the atoms of a body, that body becomes positively charged because it will have lost some negative electricity. Conversely, a body which gains electrons becomes negatively charged (if you comb your hair the comb will gain some electrons and your hair will lose some). Positively charged bodies attract negatively charged bodies and repel other positively charged bodies (which is why the comb can make your hair stand on end!).

The total surplus or deficiency of electrons in a body is called its charge. The symbol for electric charge is Q and its SI unit is the coulomb (C) in honour of Charles Coulomb (1736–1806), a French physicist. The smallest amount of known charge is the charge on an electron which is 1.6×10^{-19} C. It follows that 6.25×10^{18} electrons $(1/1.6 \times 10^{-19})$ are required to make up 1 C of charge. When electric charges are in motion they constitute an electric current which we call electricity.

Electricity is a very convenient form of energy. It is relatively easy to produce in bulk in power stations whether they be coal fired, oil fired or nuclear (using steam turbines to drive the generators) or hydro (using water turbines to drive the generators). A modern coal-fired or nuclear power station typically produces 2000 MW using four 500 MW generators driven by steam turbines at 3000 r/min. The steam required to drive the turbines is raised by burning coal or from the heat produced in a nuclear reactor. Once generated it is transmitted, by means of overhead lines or underground cables, to load centres where it is used. Since generation takes place at about 25 kV, transmission at up to 400 kV, and utilization at around 240 V to 415 V, transformation of voltage levels is required and is conveniently carried out using the transformer. Finally, in use it is extremely flexible and most industrial and domestic premises rely heavily on it for lighting and power.

2.2 ELECTRIC CIRCUITS

Electric circuits or networks are the assemblage of devices and or equipment needed to connect the source of energy to the user or the device which exploits it. Communications systems, computer systems and power systems all consist of more or less complicated electric circuits which themselves are made up of a number of circuit elements. The devices and equipment mentioned above may be represented by 'equivalent circuits' consisting of these circuit elements, and an equivalent circuit must behave to all intents and purposes in the same way as the device or equipment which it represents. In other words, if the device were put into one 'black box' and the equivalent circuit were put into another 'black box', an outside observer of the behaviour of each would be unable to say which black box contained the real device and which contained the equivalent circuit. In practice it is virtually impossible to achieve exact equivalence.

2.3 CIRCUIT ELEMENTS

Circuit elements are said to be either active (if they supply energy) or passive and the elements which make up a circuit are:

- a voltage or current source of energy (active elements);
- resistors, inductors and capacitors (passive elements).

Energy sources

There are two basic variables in electric circuits, namely electric current and electric potential difference (which we will often call voltage for short). A source of energy is required to cause a current to flow and thereby to produce electric voltages in various parts of the circuit. Energy is work and is measured in joules (J) in honour of James Prescott Joule (1818–89), a British scientist. When a force (F newtons) moves a body through a distance (d metres) the work done is ($F \times d$) joules.

Example 2.1

Calculate the work done when a force of 10 N moves a body through a distance of 5 m.

Solution

The work done is force times distance moved = $F \times d = 10 \times 5 = 50$ J.

Voltage source

An ideal voltage source is independent of the current through it. Its electromotive force (emf) or voltage is a function of time only. If a thick copper wire were connected across its ends the current through it would be infinite. The symbol for an ideal voltage source is shown in Fig. 2.1.

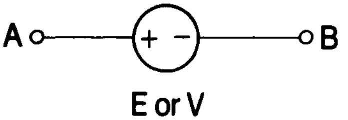

Figure 2.1

The electric potential difference between two points is defined as being the work required to move a unit positive charge (i.e. 1 C) between them. The unit is called the volt (V) in honour of Alessandro Volta (1745–1827), the Italian inventor of the electric battery. A potential difference of 1 V exists between two points when one joule of work (1 J) is required to move 1 C from the point of lower potential to that of the higher potential.

Example 2.2

Calculate (1) the work done when 300 C of charge is moved between two points having a potential difference of 100 V between them; (2) the potential difference between two points A and B if 500 J of work is required to move 2 mC from A to B.

Solution

1 Work done = charged moved × potential difference through which it is moved

$= QV$

$= 300 \times 100 = 30$ kJ

2 Potential difference = work done/charge moved = $500/2 \times 10^{-3} = 250$ kV with point B at the higher potential.

Current source

An ideal current source is independent of the voltage across it and if its two ends are not connected to an external circuit the potential difference across it would be infinite. The symbol for a current generator is shown in Fig. 2.2.

A steady flow of electric charges which does not vary with time is called a

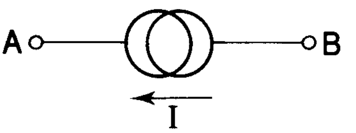

Figure 2.2

direct current. The symbol for current is I and its unit is called the ampere (A) in honour of Andre Ampere (1775–1836), a French mathematician and scientist. When 1 C of charge passes a given plane of reference in one second it represents a current of 1 A, thus

$$I = \mathrm{d}Q/\mathrm{d}t \quad (I \text{ is the rate of change of charge}) \tag{2.1}$$

It follows that when a current of I amperes flows for T seconds the charge moved is given by

$$Q = \int_0^T I \,\mathrm{d}t \tag{2.2}$$

Example 2.3

Calculate (1) the time needed for a current of 10 A to transfer 500 C of charge across a given plane of reference; (2) the current flowing if 200 C of charge passes between two points in a time of 10 s.

Solution

1 From Equation (2.1) we have that $I = \mathrm{d}Q/\mathrm{d}t$, therefore
$t = Q/I = 500/10 = 50$ s

2 Again $I = \mathrm{d}Q/\mathrm{d}t = 200/10 = 20$ A

Resistance

Materials within which charges can move easily are called conductors. Examples of good conductors are copper and aluminium in which electrons can move easily but cannot easily move away from the surface and out of the metal. These materials are said to have a low resistance. Materials within which charges cannot move or can move only with great difficulty are called insulators. These materials are said to have a high resistance, and examples of good insulators are glass and mica.

Ohm's law

Experiment shows that for many conducting materials the current (I) passing through the material from one end to the other is proportional to the potential difference appearing across its ends. Mathematically this is stated as $I \propto V$ or $V \propto I$. We can replace the proportionality sign ($\propto$) by an equality sign if we introduce a constant of proportionality. Thus we write

$$V = RI \tag{2.3}$$

where R is the constant of proportionality and is called the resistance of the conducting material. This is known as Ohm's law.

Rearranging Equation (2.3), we obtain the defining equation $R = V/I$ and we note that the unit of resistance is the unit of voltage divided by the unit of current, i.e. the volt per ampere. This is called the ohm, symbol Ω, in honour of Georg Ohm (1787–1854), a German scientist. Materials which obey Ohm's law are known as linear or ohmic materials.

Virtually all devices and equipment have inherent resistance. A circuit element designed specifically to have resistance is called a resistor. There are two circuit symbols commonly used for resistance and either is perfectly acceptable. These are shown in Fig. 2.3 together with the characteristic graph.

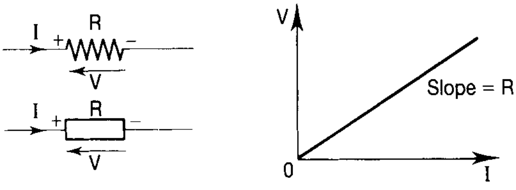

Figure 2.3

The point of entry of the current in a resistor is always positive with respect to the point of exit so far as potential difference is concerned.

Example 2.4

Find the unknown quantities in the diagrams of Fig. 2.4.

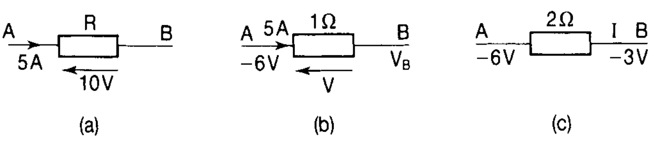

Figure 2.4

Solution

(a) Using Ohm's law we have that $R = V/I = 10/5 = 2\ \Omega$

(b) Again from $V = IR$ we see that V (the voltage across the resistor) $= 5 \times 1 = 5$ V. Since the current enters end A, it is at a higher potential than end B, so $V_B = V_A - 5 = -6 - 5 = -11$ V

(c) Since end B is at a higher potential than end A, the current must enter end B. The potential difference across the resistor is 3 V so that $I = V/R = 3/2 = 1.5$ A, flowing from right to left through the resistor.

Resistivity

The resistance of a conductor is directly proportional to its length (l) and inversely proportional to its cross-sectional area (A). Mathematically then $R \propto l/A$. This may also be written as

$$R = \rho l/A \tag{2.4}$$

where ρ is the constant of proportionality and is called the resistivity of the material of the conductor. Its unit is obtained by rearranging the above equation to make ρ the subject so that $\rho = RA/l$ and we see that the unit of ρ is the unit of R (Ω) multiplied by the unit of A(m^2) divided by the unit of l (m), i.e. $(\Omega\,\mathrm{m}^2)/\mathrm{m} = \Omega\,\mathrm{m}$. The unit of ρ is therefore the ohm-metre. Sometimes it is convenient to use the reciprocal of resistance which is called conductance (G) for which the unit is the siemens (S). Ernst Werner von Siemens (1816–92) was a German inventor. The reciprocal of resistivity is conductivity (σ) for which the unit is the siemens per metre ($\mathrm{S\,m^{-1}}$). Thus we have that $G = 1/R = A/\rho l$ and since $\sigma = 1/\rho$ we have

$$G = \sigma A/l \tag{2.5}$$

Example 2.5

A copper rod, 20 cm long and 0.75 cm in diameter, has a resistance of 80 $\mu\Omega$. Calculate the resistance of 100 m of wire, 0.2 mm in diameter drawn out from this rod.

Solution

From Equation (2.4), the resistance of the rod is given by $R_R = \rho l_R/A_R$ so that $\rho = R_R A_R/l_R$ where A_R is the cross-sectional area of the rod and l_R is its length. Putting in the values

$$\rho = \{80 \times 10^{-6} \times [\pi(0.0075)^2/4]\}/0.2 = 1.77 \times 10^{-8}\ \Omega\,\mathrm{m}$$

For the wire $R_W = \rho l_W/A_W$ where R_W is the resistance of the wire, A_W is the cross-sectional area of the wire and l_W is its length. Putting in the values,

$$R_W = [1.77 \times 10^{-8} \times 100]/\pi(0.0001)^2 = 56\ \Omega$$

Table 2.1 illustrates the enormous range of values of resistivity (and conductivity) exhibited by various materials. We shall see in the next section that resistance (and resistivity and conductivity) varies with temperature; the values given here are at 20 °C. Remember: the higher the conductivity the better the conductor:

Table 2.1

Material	*Conductivity* ($S\ m^{-1}$)	*Resistivity* (Ω m)
Silver	6.1×10^7	1.64×10^{-8}
Copper	5.7×10^7	1.75×10^{-8}
Carbon	3×10^4	3.33×10^{-5}
Distilled water	1×10^{-4}	1×10^4
Glass	1×10^{-12}	1×10^{12}
Mica	1×10^{-15}	1×10^{15}
Quartz	1×10^{-17}	1×10^{17}

Resistors in series

If a number of resistors are connected as shown in the diagram of Fig. 2.5 they

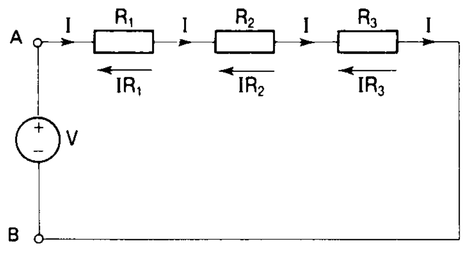

Figure 2.5

are said to be in series. Resistors are in series, therefore, if the same current flows through each of them. In the diagram of Fig. 2.6, for example, only the resistors R_5 and R_6 are in series with each other. Resistor R_1 is in series with the combination of all the others.

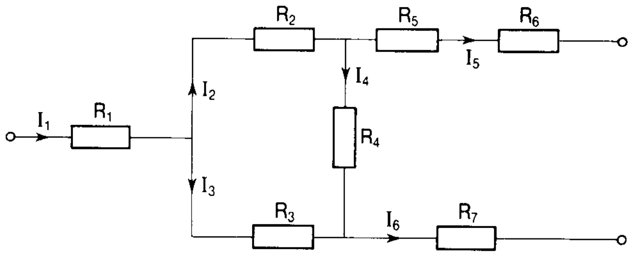

Figure 2.6

By Ohm's law the potential difference across the resistors R_1, R_2 and R_3 in Fig. 2.5 is given by IR_1, IR_2 and IR_3, respectively. The total potential difference between the terminals A and B is therefore $IR_1 + IR_2 + IR_3 = I[R_1 + R_2 + R_3]$. Although this seems obvious, we have, in fact, anticipated Kirchhoff's voltage law which will be stated formally in Chapter 3. A single resistor which would take the same current (I) from the same source (V) would have to have a resistance of $[R_1 + R_2 + R_3]$. The equivalent resistance (R_{eq}) of the three resistors in series is therefore the sum of the three individual resistances. In general, for n resistors is series, $R_{eq} = R_1 + R_2 + \cdots + R_n$. In short this can be written

$$R_{eq} = \sum_{a=1}^{n} R_a \tag{2.6}$$

Example 2.6

Determine (1) the current flowing in the circuit of Fig. 2.7, (2) the voltage across each resistor.

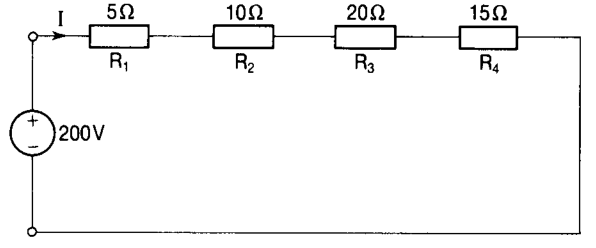

Figure 2.7

Solution

1 Using Ohm's law $I = V/R_{eq}$ and from Equation (2.6), $R_{eq} = R_1 + R_2 + R_3 + R_4$, so $R_{eq} = 5 + 10 + 20 + 15 = 50\ \Omega$. Therefore $I = 200/50 = 4$ A

2 Again, from Ohm's law
$$\begin{aligned} V_{R1} &= IR_1 = 4 \times 5 = 20\text{ V} \\ V_{R2} &= IR_2 = 4 \times 10 = 40\text{ V} \\ V_{R3} &= IR_3 = 4 \times 20 = 80\text{ V} \\ V_{R4} &= IR_4 = 4 \times 15 = 60\text{ V} \end{aligned}$$

Note that these add up to 200 V, which is the voltage of the supply.

Voltage division

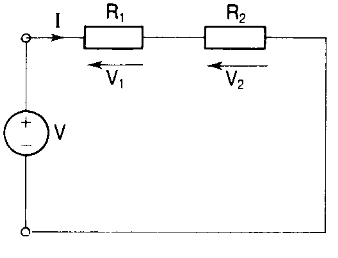

Figure 2.8

For the two resistors shown connected in series in Fig. 2.8, $V = I[R_1 + R_2]$. Also $V_1 = IR_1$ so that

$V_1/V = IR_1/[I(R_1 + R_2)]$

and

$$V_1 = R_1V/(R_1 + R_2) \quad (2.7)$$

Similarly,

$$V_2 = R_2V/(R_1 + R_2) \quad (2.8)$$

This shows that the ratio of the voltage across a resistor in a series circuit to the total voltage is the ratio of the resistance of that resistor to the total resistance.

Example 2.7

The diagram of Fig. 2.9 shows a variable resistor R_1 in series with a fixed resistor $R_2 = 30\ \Omega$. Determine (1) the voltage V_2 appearing across R_2 when R_1 is set at $20\ \Omega$; (2) the value to which R_1 must be set to make the voltage across $R_2(V_2) = 150$ V.

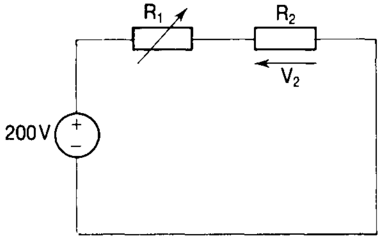

Figure 2.9

Solution

1 From Equation (2.8) we have
$V_2 = R_2V/(R_1 + R_2) = 30 \times 200/(20 + 30) = 120$ V

2 Rearranging Equation (2.8) to make R_1 the subject, we have $R_1 = (R_2V/V_2) - R_2$. Putting in the numbers,

$R_1 = \{(30 \times 200)/150\} - 30 = 40 - 30 = 10\ \Omega$

Resistors in parallel

If a number of resistors are connected as shown in Fig. 2.10 they are said to be in parallel. Resistors are in parallel if the same voltage exists across each one.

The total current I is made up of I_1 flowing through R_1, I_2 flowing through R_2 and I_3 flowing through R_3 and by Ohm's law these currents are given by V/R_1, V/R_2 and V/R_3, respectively. It follows that $I = I_1 + I_2 + I_3$ (again this seems obvious but this time we have anticipated Kirchhoff's current law which is formally introduced in Chapter 3). So

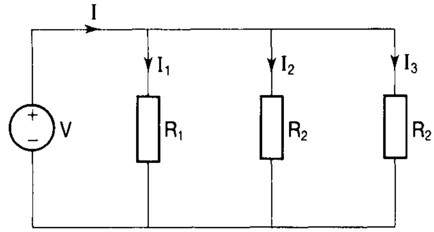

Figure 2.10

$$I = (V/R_1) + (V/R_2) + (V/R_3) = V[(1/R_1) + (1/R_2 + (1/R_3)] \quad (2.9)$$

If a single resistor R_{eq} connected across the voltage source (V) were to take the same current (I) then

$$I = V/R_{eq} \quad (2.10)$$

Comparing Equations (2.9) and (2.10) we see that

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$$

In general for n resistors connected in parallel

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots + 1/R_n \quad (2.11)$$

Since conductance (G) is the reciprocal of resistance ($G = 1/R$) we see that

$$G_{eq} = G_1 + G_2 + \cdots + G_n \quad (2.12)$$

The equivalent conductance of a number of conductances in parallel is thus the sum of the individual conductances.

Example 2.8

Determine the current I flowing in the circuit of Fig. 2.11.

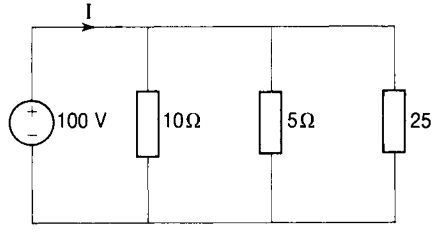

Figure 2.11

Solution

$I = V/R_{eq} = VG_{eq}$ where R_{eq} and G_{eq} are, respectively, the equivalent resistance and conductance of the parallel combination. From Equation (2.12), $G_{eq} = G_1 + G_2 + G_3 = 1/10 + 1/5 + 1/25 = 0.1 + 0.2 + 0.04 = 0.34$ S. Therefore

$I = VG_{eq} = 100 \times 0.34 = 34$ A

Often we meet just two resistors connected in parallel and it is useful to remember that since $1/R_{eq} = 1/R_1 + 1/R_2 = (R_1 + R_2)/R_1R_2$ then

$$R_{eq} = R_1R_2/(R_1 + R_2) \qquad (2.13)$$

i.e. the equivalent resistance of two resistors in parallel is their product divided by their sum.

Current division

In Fig. 2.12 the total current (I) is made up of I_1 flowing through resistor R_1 and

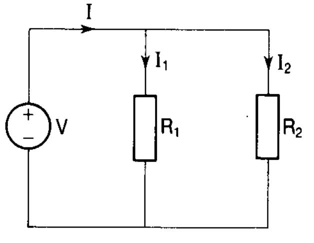

Figure 2.12

I_2 flowing through resistor R_2 and by Ohm's law $I_1 = V/R_1$, $I_2 = V/R_2$ therefore

$$I = I_1 + I_2 = V[(1/R_1 + (1/R_2)]$$
$$= V(R_1 + R_2)/R_1R_2$$
$$I_1/I = (V/R_1)/V(R_1 + R_2)/R_1R_2 = R_2/(R_1 + R_2)$$

So

$$I_1 = R_2I/(R_1 + R_2) \qquad (2.14)$$

Similarly

$$I_2 = R_1I/(R_1 + R_2) \qquad (2.15)$$

Example 2.9

Determine the current I_2 and the voltage V in the circuit of Fig. 2.13.

Solution

From Equation (2.15), $I_2 = R_1I/(R_1 + R_2) = 10 \times 20/(10 + 40) = 4$ A.
From Equation (2.13) $R_{eq} = R_1R_2/(R_1 + R_2) = 400/50 = 8\ \Omega$

$V = IR_{eq} = 20 \times 8 = 160$ V

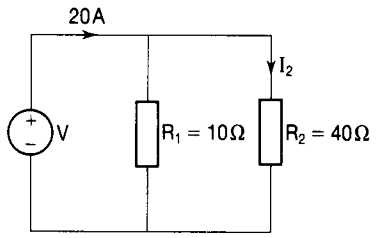

Figure 2.13

Example 2.10

The circuit of Fig. 2.14 is a series–parallel circuit. Calculate (1) the current drawn from the supply (I); (2) the potential difference across the resistor R_4 (V_4); (3) the current through the resistor R_6 (I_6).

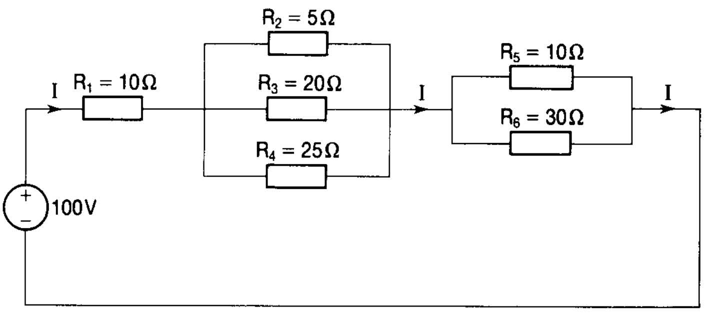

Figure 2.14

Solution

The equivalent resistance of the parallel combination of resistors R_5 and R_6 is given by

$R_{56} = R_5R_6/(R_5 + R_6) = 10 \times 30/(10 + 30) = 300/40 = 7.5\ \Omega$

For the parallel combination of the resistors R_2, R_3 and R_4 the equivalent resistance is given by

$1/R_{234} = 1/R_2 + 1/R_3 + 1/R_4 = 0.2 + 0.05 + 0.04 = 0.29\ \text{S}$

Therefore $R_{234} = 1/0.29 = 3.45\ \Omega$.

The equivalent resistance of the whole series–parallel circuit is given by $R_{eq} = R_1 + R_{234} + R_{56}$ so $R_{eq} = 10 + 3.45 + 7.5 = 20.95\ \Omega$

$I = V/R_{eq} = 100/20.95 = 4.77\ \text{A}$

$V_4 = IR_{234} = 4.77 \times 3.45 = 16.46\ \text{V}$

$I_6 = R_5I/(R_5 + R_6) = 10 \times 4.77/40 = 1.19\ \text{A}$

Internal resistance

It was stated earlier in the chapter that an ideal voltage source is independent of the current flowing through it. Practical voltage sources have internal resistance which means that the voltage at its terminals varies as the current through it changes. The equivalent circuit of a practical voltage source then takes the form shown in Fig. 2.15 where r represents the internal resistance of the source and A and B are its terminals. The terminal voltage is thus V_{AB}.

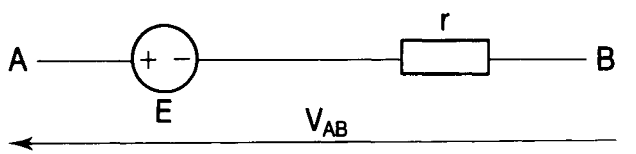

Figure 2.15

Example 2.11

A battery has an internal resistance of 0.5 Ω and a terminal voltage of 15 V when it supplies no current. Determine the terminal voltage when the current through it is 5 A.

Solution

The diagram is as shown in Fig. 2.15. Let the battery terminal voltage when it supplies no current be E (this is called the open circuit voltage). Then, when a current I flows, the terminal voltage $V_{AB} = E - Ir$ where r is the internal resistance. When $I = 5$ A, $V_{AB} = 15 - 5 \times 0.5 = 15 - 2.5 = 12.5$ V.

Effect of temperature

The resistance of metals increases with temperature while for insulators it decreases with temperature. There are some materials for which there is virtually no change in resistance over a wide range of temperatures.

For a given material it is found that

$$R = R_s[1 + \alpha_s(T - T_s)] \tag{2.16}$$

where R is the resistance at a temperature T, R_s is the resistance at temperature T_s, and α_s is the temperature coefficient of resistance corresponding to T_s and is defined as the change in resistance per degree change of temperature divided by the resistance at some temperature T_s. It is measured in (°C^{-1}) which is read as 'per degree Celsius'. For a standard temperature $T_s = 0$ °C, α_s for copper is 0.0043 per °C; for manganin (an alloy of copper, magnesium and nickel) it is 0.000 003 per °C.

If a certain material has a resistance of R_0 at a standard temperature of 0 °C and a resistance temperature coefficient of α_0, then at temperatures T_1 and T_2, respectively, its resistance will be given by $R_1 = R_0[1 + \alpha_0 T_1]$ and $R_2 = R_0[1 + \alpha_0 T_2]$ from which we see that

$$R_1/R_2 = [1 + \alpha_0 T_1]/[1 + \alpha_0 T_2] \quad (2.17)$$

Example 2.12

A copper coil has a resistance of 100 Ω at a temperature of 40 °C. Calculate its temperature at 100 °C. Take α_0 to be 0.0043 per degree C.

Solution

From Equation (2.17) we have that $R_1/R_2 = [1 + \alpha_0 T_1]/[1 + \alpha_0 T_2]$. In this case, $R_1 = 100\ \Omega$, $T_1 = 40$ °C and $T_2 = 100$ °C. Rearranging Equation (2.17) to make R_2 the subject, we have

$$R_2 = [1 + \alpha_0 T_2]R_1/[1 + \alpha_0 T_1] = [1 + 0.43] \times 100/[1 + 0.172] = 122\ \Omega$$

Colour code for resistors

Some resistors are coded by means of colour bands at one end of the body of the resistor. The first band indicates the first digit of the value of the resistance, the second band gives the second digit and the third band gives the number of zeros. If there is a fourth band this tells us the percentage tolerance on the nominal value. The colour codes are given in Table 2.2.

Table 2.2

First, second and third band		*Fourth band (% tolerance)*	
black	= 0	gold	= 5
brown	= 1	silver	= 10
red	= 2	none	= 20
orange	= 3		
yellow	= 4		
green	= 5		
blue	= 6		
violet	= 7		
grey	= 8		
white	= 9		

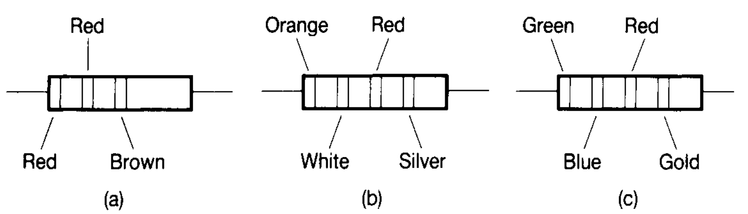

Figure 2.16

Example 2.13

Write down the nominal value of the resistance of each of the resistors shown in

Fig. 2.16. If they are connected in series, determine the maximum possible resistance of the combination.

Solution

(a) The first band is red so the first digit is 2; the second band is red so the second digit is 2; the third band is brown so there is one zero. There is no fourth band so that the tolerance is 20 per cent. The nominal value of this resistor is therefore 220 Ω and its tolerance is 20 per cent so that its resistance should lie between $220 - 44 = 176$ Ω and $220 + 44 = 264$ Ω.

(b) The first band is orange so the first digit is 3; the second band is white so the second digit is 9; the third band is red so there are two noughts; the fourth band (silver) means that the tolerance is 10 per cent. The nominal value of this resistor is therefore 3900 Ω (3.9 kΩ) and its value lies between 3510 Ω (−10 per cent) and 4290 Ω (+10 per cent).

(c) The bands on this resistor represent 5 (first digit), 6 (second digit) and red (two zeros) so its nominal value is 5600 Ω (5.6 kΩ). The fourth band (gold) means that its tolerance is ±5 per cent and so its value must be within the range 5320 Ω (5.32 kΩ) to 5880 Ω (5.88 kΩ).

If these resistors were to be connected in series the equivalent resistance of the combination would lie between 9006 Ω (9.006 kΩ) and 10 434 Ω (10.434 kΩ).

Non-linear resistors

A resistor which does not obey Ohm's law, that is one for which the graph of voltage across it to a base of current through it is not a straight line, is said to be non-linear. Most resistors are non-linear to a certain degree because as we have seen the resistance tends to vary with temperature which itself varies with current. So the term non-linear is reserved for those cases where the variation of resistance with current is appreciable. For example, a filament light bulb has a resistance which is very much lower when cold than when at normal operating temperature.

Capacitance

If we take two uncharged conductors of any shape whatever and move Q coulombs of charge from one to the other an electric potential difference will be set up between them (say V volts). It is found that this potential difference is proportional to the charge moved, so we can write $V \propto Q$ or $Q \propto V$. Introducing a constant we have

$$Q = CV \qquad (2.18)$$