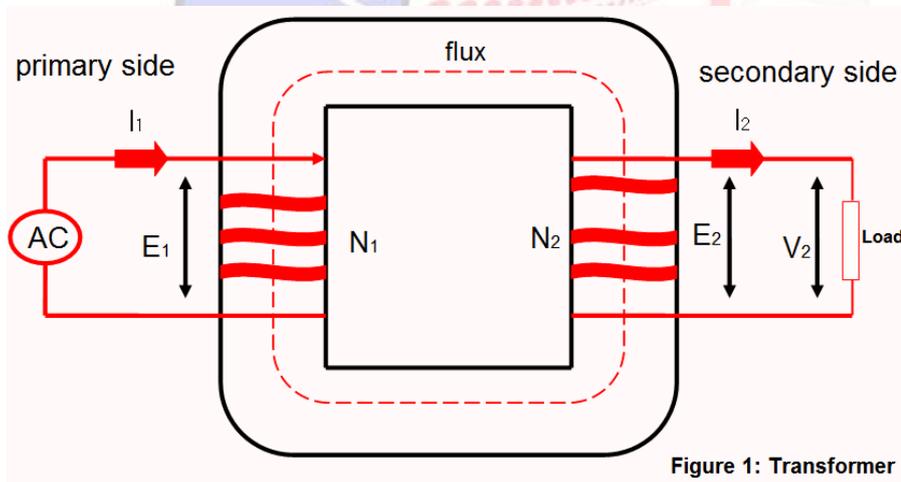


4- Transformers

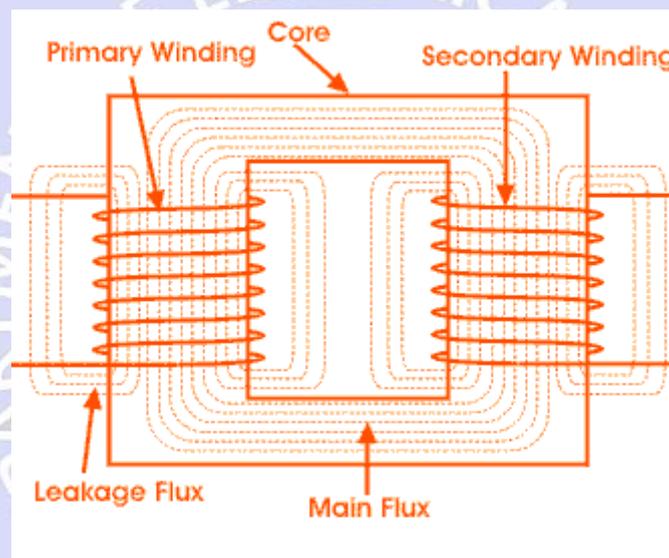
4-1 Introduction:

One of the main advantage of A.C transmission and distribution is the case with which an alternating voltage can be increase or reduce. The general practice in this country is to general at voltage of about 1-22 kV, then step up by means of transformers to higher voltages, for transmission lines, at suitable points other transformers are installed to step the voltage down to values suitable for motor, lamps heater, etc. Medium size transformers have a full-load efficiency of about 97-98 per cent, so that the losses at each point of transformation is very small. Also, since there are no moving parts, the amount of supervision required is practically negligible.



4-2 Leakage Flux Responsible for The Induced Reactance of a Transformer:

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding but, in practice it's impossible to realize this condition. It's found, however that all the flux linked with primary does it link the secondary but, part of it known Φ_{L1} completes its magnetic cct. By passing air rather than around the core. The preceding section it was explain that the leakage flux is proportional to the primary and secondary current and that has effect is to induce e.m.f of self-induction in the winding. Consequently, the effect of leakage flux can be considered as equivalent to inductive reactors X_1 and X_2 connected in series with a transformer having no leakage flux.



4-2-1 Methods of reducing leakage flux:

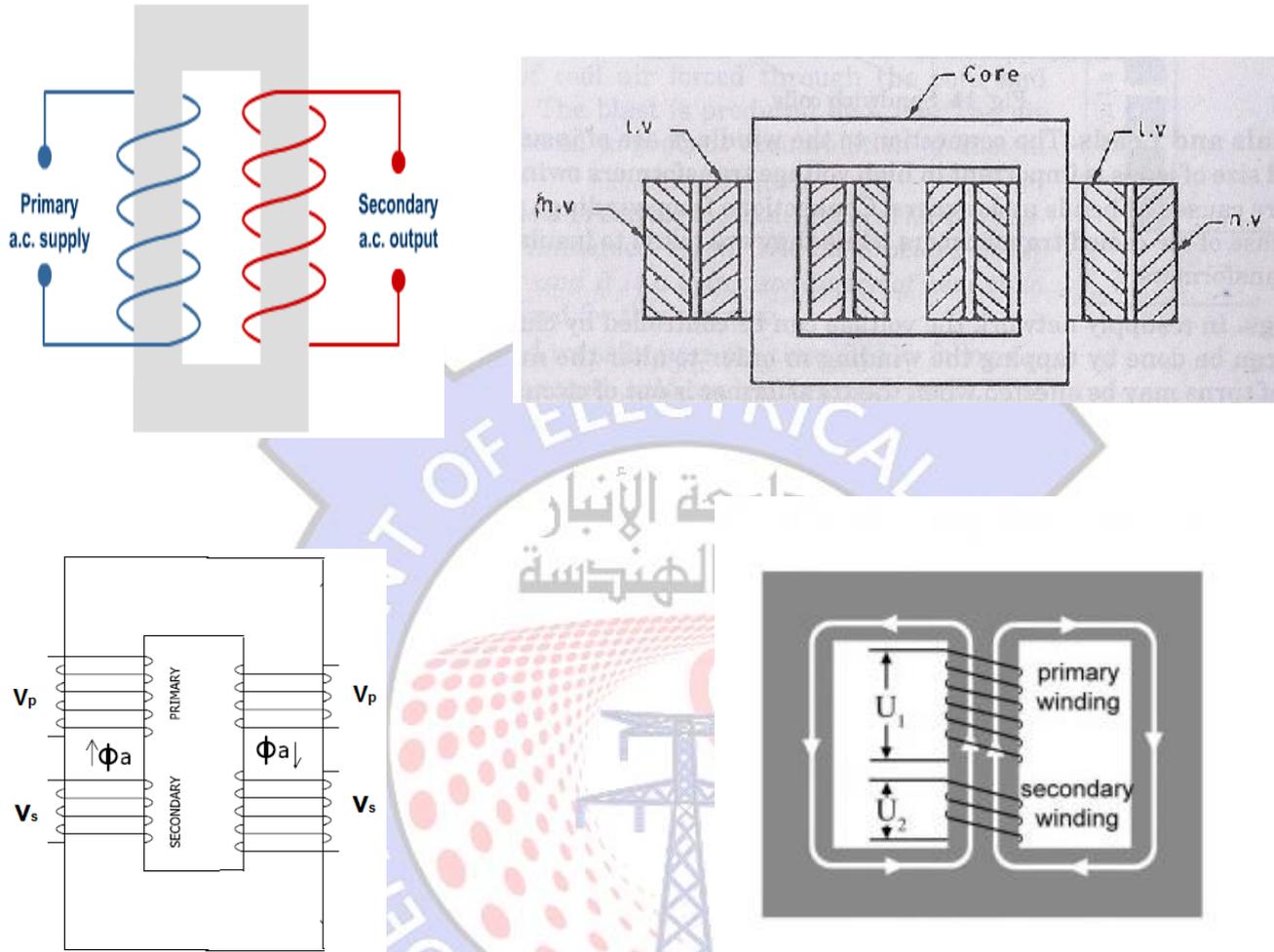
The leakage flux can be practically eliminated by winding the primary and secondary, on over the other, uniformly around a laminate iron ring of uniform cross- section but, such an arrangement is not commercially practicable except is very small sizes. Owing to the cost of threading a large number of turns through the ring.

The principal methods used in practice are:

- 1-Making the transformer (window) long and narrow.
- 2-Arrangement the primary and secondary winding concentrically.

3-Sandwiching the primary and the secondary winding

4-using shell-type construction.



4-3 Principle of Action of a Transformer:

Fig (1) shown the general arrangement of transformer iron core (C) consist of laminated sheets, about 0.35mm thick, insulated from one another by thin layers paper or varnish or by spraying the laminations with a mixture of flour. The purpose of laminating the core is to reduce the loss due to eddy current induced by alternating magnetic flux. The primary coil is connected to supply and the secondary coil is connect to the load. An alternating voltage applied to primary circulated an alternating current through primary and this current produces an alternating flux in the iron core. The mean path of this flux being represented by



the dotted D_1 . If the whole of the flux produced by primary passes through secondary, the e.m.f induced in each turn is the same for P and S . Hence, if N_1 and N_2 be the number of turns on (P) and (S) respectively.

$$\frac{\text{Total e.m.f induced in } S}{\text{Total e.m.f induced in } P} = \frac{N_2 \times \text{e.m.f per turn}}{N_1 \times \text{e.m.f per turn}} = \frac{N_2}{N_1}$$

When the secondary is on open circuit, its terminal voltage is the same as the induced e.m.f. The primary current is then very small, so that the applied voltage V_1 is practically equal and opposite to the induced in P hence:

$$\frac{V_2}{V_1} \approx \frac{N_2}{N_1}$$

Since the full-load efficiency of a transformer is, in early 100 per cent.

$I_a V_1 \times$ primary power factor $\approx I_2 V_2 \times$ secondary power factor, but the primary and secondary power factor at full-load are nearly equal, $\frac{I_2}{I_1} = \frac{V_1}{V_2}$ And the we have:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{V_1}{V_2}$$

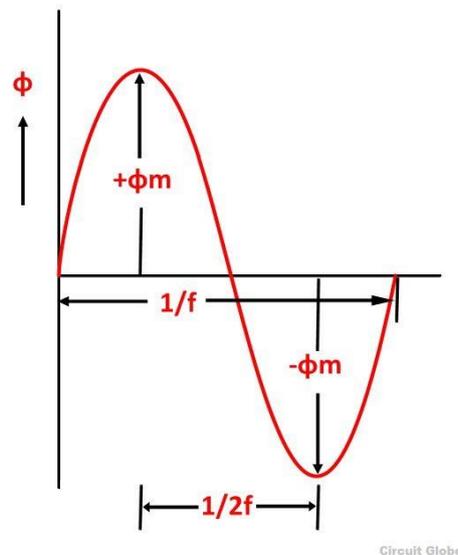
4-4 E.M.F Equation of a Transformer:

Suppose the maximum value of the flux to be Φ_m Webbers and the frequency to be f hertz (or cycles /second). From fig(2) is seen that the flux has to change from $+\Phi_m$ to $-\Phi_m$ in half cycle, namely in $\frac{1}{2f}$ second. Average rate of change of

$$\text{flux} = 2\Phi_m \div \frac{1}{2f}$$

$$= 4f\Phi_m \text{ weber/second.....}$$

And average e.m.f induced turn $= 4 f \Phi_m$ volts



Circuit Globe

For a sinusoidal wave the r.m.s or effective value is 1.11 times the average value.

r.m.s value of e.m.f induced /turn = $1.11 \times 4 f \Phi_m$

Hence r.m.s value of e.m.f induced in primary = E_1 .

$$E_1 = 4.44 N_1 f \Phi_m \text{ volts}$$

And r.m.s value of e.m.f induced in secondary = E_2

$$E_2 = 4.44 N_2 f \Phi_m \text{ volts}$$

$$\Phi_m = B_m \times A$$

B_m = maximum flux density

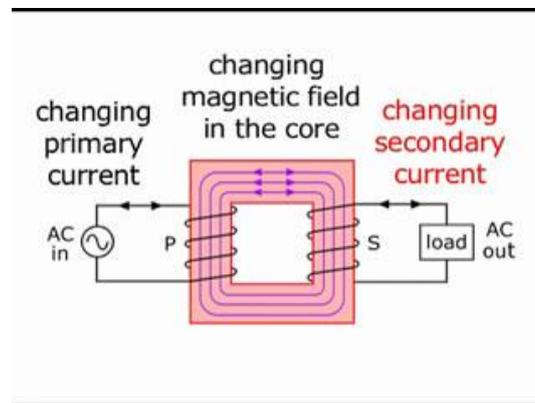
A = area of the core .

K = voltage transformation ratio

$$K = \frac{N_2}{N_1}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

$$V_1 I_1 = V_2 I_2 \rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$



4-5 Windings the Ideal Transformer P And S:

R_1 and R_2 are resistance equal to the resistance of the primary and secondary windings of the actual transformer. Similarly, inductive reactance X_1 and X_2 represent the reactance of the windings due to leakage flux in the actual transformer. The inductive reactor X is that it takes a reactive current equal the magnetizing current I_{mag} of the actual transformer. The core losses due to hysteresis and eddy currents are allowed for by a resistor R of such value that it takes a current to equal to the core loss of the actual transformer.

4-6 Phasor Diagram for Transformer On no load:

It is most convenient to commence the phasor diagram with the phasor representing the quantity that is common to the two windings, namely the flux Φ . This phasor can be convenient length and may be regarded merely as reference phasor, relative to which another phasor has to be drawn. The e.m.f induced by sinusoidal flux lags the flux by a quarter of a cycle. Consequently the e.m.f E_2 and E_1 induced in the secondary and primary winding are represented by phasor drawn 90° behind Φ , as in fig(3). The values of E_2 and E_1 are proportion to the number of turns on the secondary and primary windings, since practically the whole of the flux set up by the primary is linked with the secondary when the latter is on open cct. For convenience in drawing phasor diagram for transformer, it will be assume that N_1 and N_2 are equal, so that $E_2 = E_1$, as shown in fig(3). Since the difference between the value of the applied voltage V_1 and that of the induced e.m.f

E_1 is only about 0.05 per cent when the transformer is on no load, the phasor representing V_1 can be drawn equal and opposite to that representing E_1 .

The no-load current I_o taken by the primary consist of two components: a reactive or magnetizing component (I_{mag}) producing the flux and therefore in phasor with the the latter, and an active or power component (I_c) supplying the hysteresis and eddy current losses in the iron core and the negligible (I_2R) loss in the primary winding. Components I_c in phase with the applied voltage ($I_c V_1 = \text{core losses}$) this component is usually very small compared with I_{mag} , so that the no-load power factor is very small.

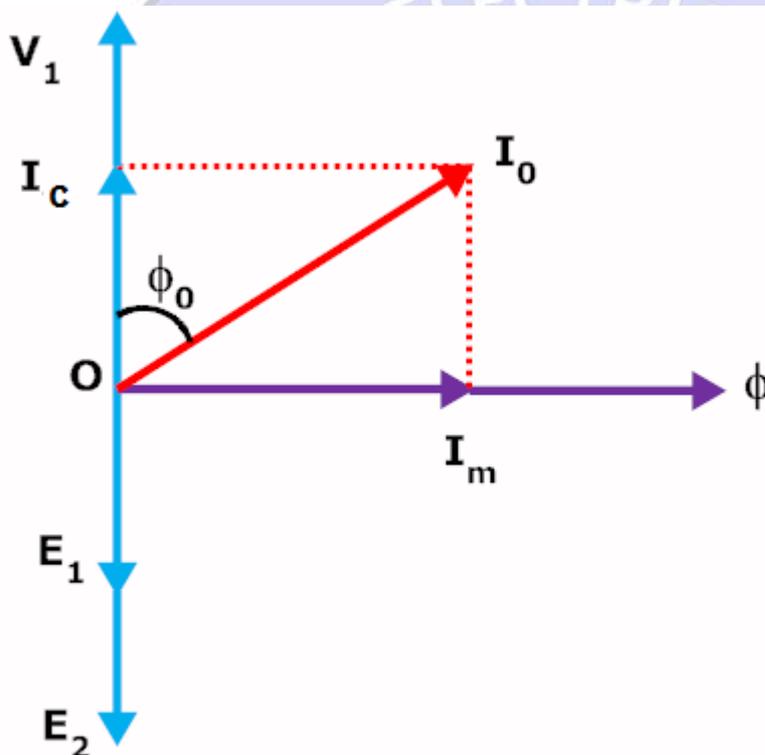


Figure 3 : Phasor diagram of practical transformer on no load

From (fig 3) it will see that:

$$\text{No-load current} = I_o = \sqrt{i_c^2 + i_{mag}^2}$$

$$\text{At power factor on no load} = \cos \phi_o = \frac{I_c}{I_o}$$

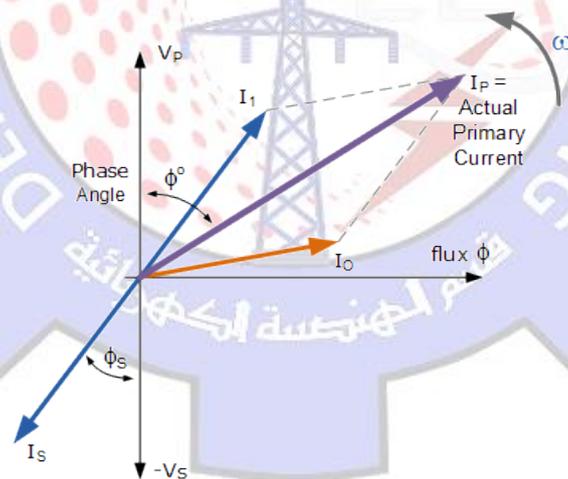


$$I_c = I_o \cos \phi_o$$

$$I_{mag} = I_o \sin \phi_o$$

4-7 Phasor Diagram for A Loaded Transformer:

Assuming the voltage drop in the winding to be negligible with this assumption, it follows that the secondary voltage V_2 is the same as the e.m.f E_1 induced in the secondary, and the primary applied voltage V_1 is equal and opposite in phase to the e.m.f. E_1 induced in primary winding, also, if we again assume equal number of turns on the primary and secondary winding then $E_1 = E_2$. Let us consider the general case of a load having lagging power factor $\cos \phi_1$, hence the phasor representing the secondary current I_2 lags V_2 by angle ϕ_2 , as shown in fig (4), phasor I_1 represents the component of the primary current to neutralize the demagnetizing effect of the secondary current and is drawn equal and opposite to I_2 . I_o is the no-load current of the transformer, the phasor sum of I_1 and I_o gives the total current I_1 taken from the supply, and the power factor on the primary side is $\cos \phi_1$, where ϕ_1 is phase difference between V_1 and I_1 .



4-8 Phasor Diagram for A Transformer on Load:

For convenience let us assume an equal number of turns on the primary and secondary windings, so that $E_1 = E_2$. Both E_1 and E_2 lag the flux by 90° as shown in fig (10) and \dot{V}_1 represents the voltage applied to the primary to



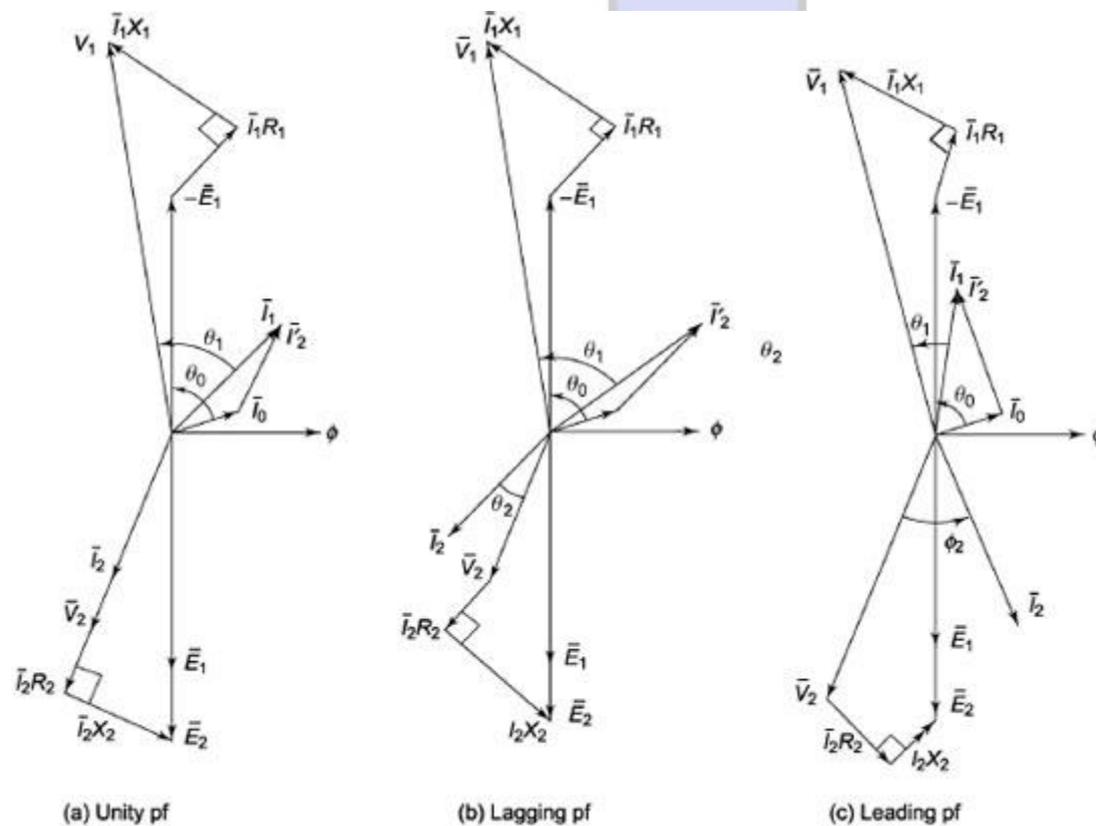
neutralized the induced e.m.f E_1 , and is therefore equal and opposite to the latter. The general case of a load having a lagging P.f by 45° then

$I_1 R_1$ = Voltage drop due to primary resistance.

$I_2 R_2$ = Voltage drop due to secondary resistance.

$I_1 X_1$ = Voltage drop due to primary leakage reactance.

$I_2 X_2$ = Voltage drop due to secondary leakage reactance.



4-9 Equivalent Resistance:

A fig (5) is shown a transformer whose primary and secondary windings have resistance of R_1 and R_2 respectively. At would how be shown that the resistance of the two windings can be transferred to any on of the two windings. The advantage of concentrating both of the resistance in one winding is that it makes calculations very simple and easy because on has then two works in one winding



only. As will be proved that the resistance of R_2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. The value $\frac{R_2}{K^2}$ will be denoted by \hat{R}_2 – the equivalent secondary resistance referred primary. The copper losses in secondary is $I_2^2 R_2$. This loss is supplied by primary which takes a current of I_1 if \hat{R}_2 is the equivalent resistance in primary which would be caused the same loss as R_2 in secondary, then:

$$I_1^2 \hat{R}_2 = I_2^2 R_2 \text{ or } \hat{R}_2 = \left(\frac{I_2}{I_1}\right)^2 R_2$$

$$\frac{I_2}{I_1} = \frac{1}{K} \rightarrow \hat{R}_2 = \frac{R_2}{K^2}$$

$$\text{And } I_1^2 R_1 = I_2^2 R_1 \text{ Or } \hat{R}_1 = \left(\frac{I_1}{I_2}\right)^2 R_1$$

$$\frac{I_1}{I_2} = K \rightarrow \hat{R}_1 = K^2 R_1$$

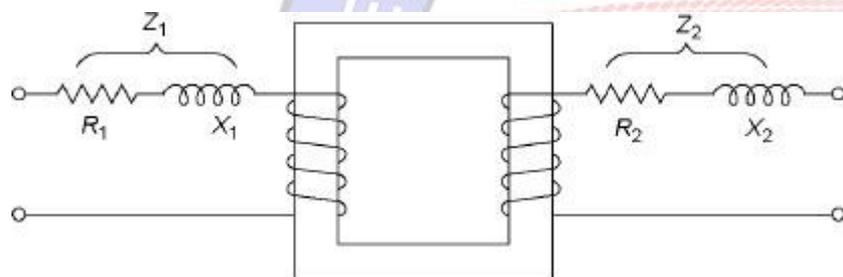


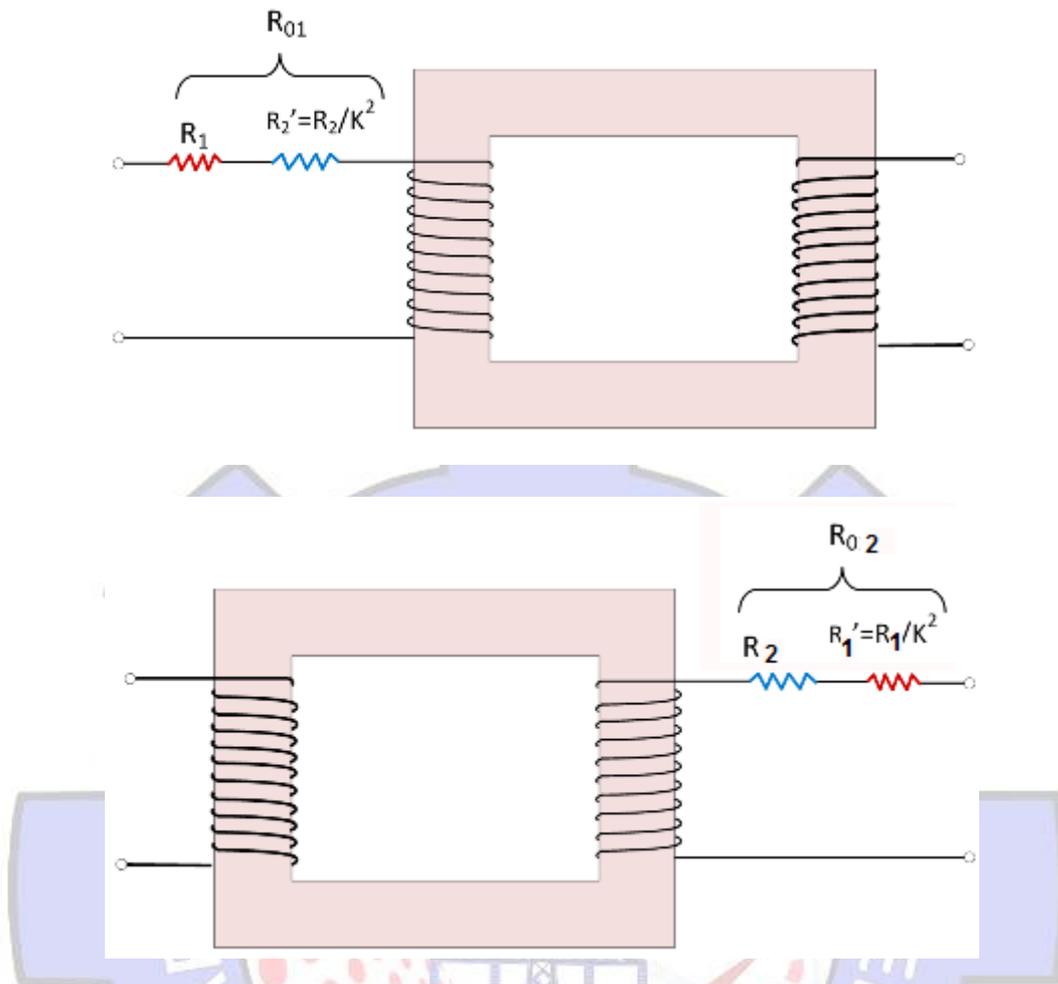
Fig (6, a), secondary resistance has been transferred to primary side leaving secondary circuit resistance less. Resistance $R_1 + \hat{R}_2 = R_1 + \frac{R_2}{K^2}$ is known as equivalent or effective resistance of the transformer as referred to primary and may be designated as R_{01} :

$$R_{01} = R_1 + \hat{R}_2 = R_1 + \frac{R_2}{K^2}$$

Similarly, the equivalent resistance of the transformer as referred to secondary is:

$$R_{02} = R_2 + \hat{R}_1 = R_2 + R_1 K^2$$

As shown in fig (6, b)



4-10 Equivalent Circuit of A Transformer:

The behavior of a transformer may be conveniently considered by assuming it to be equivalent to an ideal transformer, a transformer having no losses and no magnetic leakage and an from core of infinite permeability requiring no magnetizing current, and then allowing for the imperfections of the actual transformer by means of additional circuits or impedance inserted between the supply and the primary winding and between the secondary and the load, thus in fig(9).

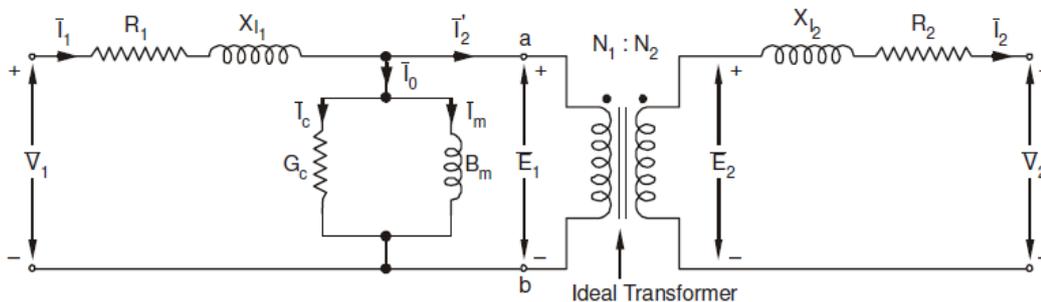
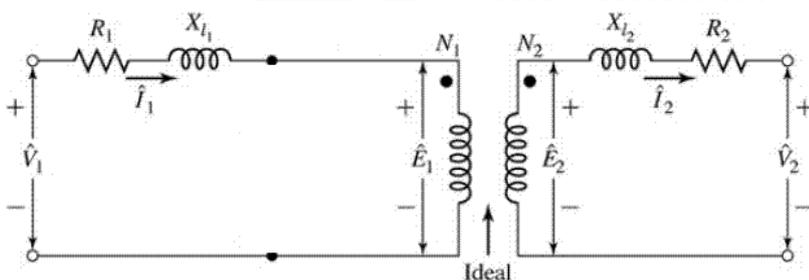


Fig. (9) Equivalent circuit of a transformer

4-11 Approximate Equivalent cct. of Transformer:

Since the no-load of transformer is only about 3.5 per cent of the full-load primary current, we can omit the parallel circuits R and X in fig (9) without introducing an appreciable error when we are considering the behavior of the transformer on full-load. Thus, we have the equivalent cct. Of fig (11)



Simplification of The Approximate Equivalent Circuit Of A Transformer:

We can replace the resistance R_2 of the secondary of fig(11) by additional resistance \hat{R}_2 in the primary circuit such the power absorbed in \hat{R}_2 when carrying the primary current is equal to that in R_2 due to the secondary current.

$$I_1^2 \hat{R}_2 = I_2^2 R_2 \rightarrow \hat{R}_2 = \left(\frac{I_2}{I_1}\right)^2 \approx R_2 \left(\frac{V_1}{V_2}\right)^2$$

Hence if R_e be a single resistance in the primary circuit equivalent. The primary and secondary resistances of the actual transformer.

$$R_e = R_1 + \hat{R}_2 = R_1 + R_2 \left(\frac{V_1}{V_2}\right)^2$$



Similarly, since the inductance of a coil is proportional to the square of the number of turns, the secondary leakage reactance X_2 can be replaced by an equivalent reactance \dot{X}_2 in the primary circuit, such that:

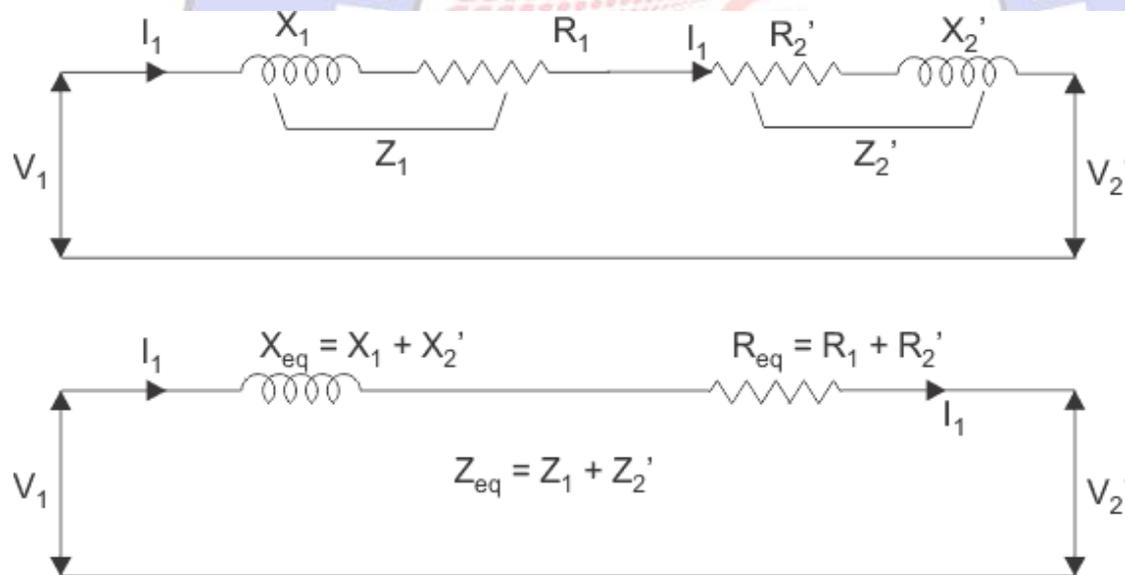
$$\dot{X}_2 = X_1 \left(\frac{N_1}{N_2} \right)^2 \approx X_1 \left(\frac{V_1}{V_2} \right)^2$$

X_e be the single reactance in the primary ckt. equivalent X_1 and X_2 of the actual transformer.

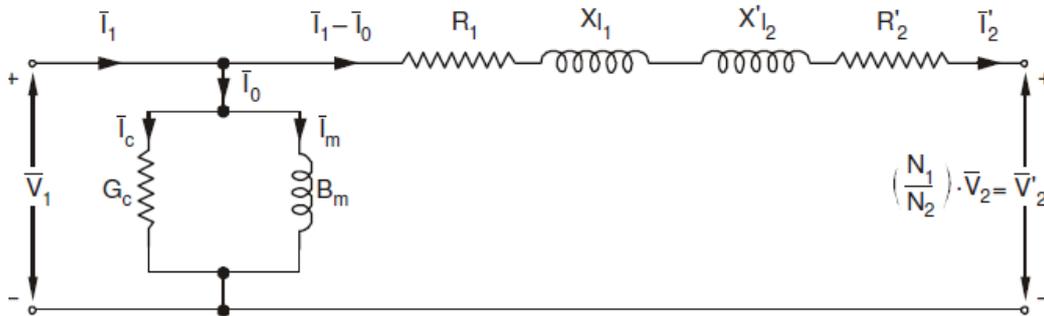
$$X_e = X_1 + \dot{X}_2 = X_1 + X_2 \left(\frac{V_1}{V_2} \right)^2$$

Z_e be the equivalent impedance of the primary and secondary winding referred to primary ckt.

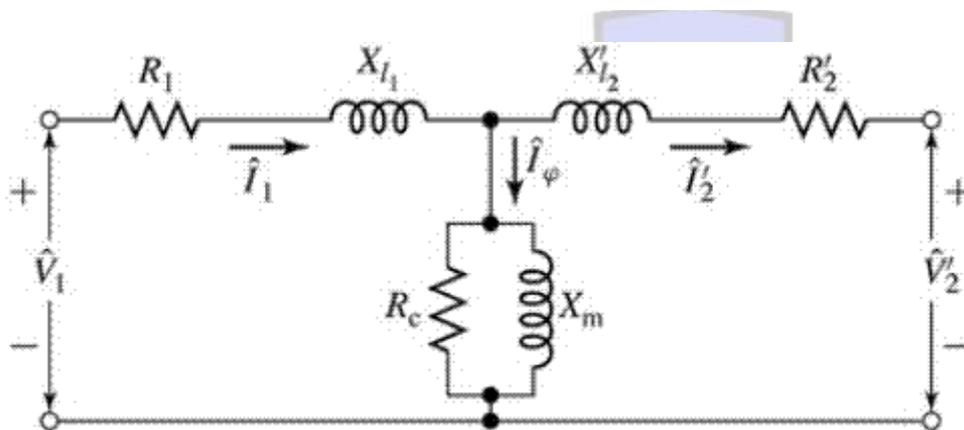
$$Z_e = \sqrt{R_e^2 + X_e^2}, \quad R_e = Z_e \cos \varphi, \quad X_e = Z_e \sin \varphi$$



Approximate Equivalent Circuit of Transformer referred to Primary



(a) Equivalent Circuit Referred to Primary Side



4-12 Efficiency of A Transformer:

The losses which occur in a transformer on load can be divided into two groups:

- 1-Copper losses in primary and secondary windings namely $I_1^2 R_1 + I_2^2 R_2$.
- 2-Iron losses in the core due to hysteresis and eddy currents. The factor determining these losses have already been discussed in A.C machines.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 per cent between no load and full load its usual to assume the iron losses constant at all loads.

$P_c = \text{total iron loss in core}$

$$\sum_{\text{loss}} = P_c + P_{cu1} + P_{cu2} = P_c + I_1^2 R_1 + I_2^2 R_2$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{losses}} = \frac{I_2 V_2 \times Pf}{I_2 V_2 \times Pf + P_c + I_1^2 R_1 + I_2^2 R_2}$$



$$\text{OR } \frac{P_{in} - \text{losses}}{P_{in}} = 1 - \frac{\text{losses}}{P_{in}}$$

$$P_c = V_1 I_o \cos \phi_o$$

$$P_{cu} = P_{cu1} + P_{cu2} = I_1^2 R_1 + I_2^2 R_2$$

4-13 Condition for Maximum Efficiency of Transformer:

R_{2e} the equivalent resistance of the primary and secondary windings referred to the secondary circuit.

$$R_{2e} = R_1 \left(\frac{N_2}{N_1} \right)^2 + R_2$$

= a constant for a given transformer

For any load current I_2 ,

$$\text{Total copper loss} = I_2^2 R_{2e}$$

$$\text{And } \eta\% = \frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_c + P_{cu2}} = \frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_c + I_2^2 R_{2e}}$$

For a normal transformer, V_2 is approximately constant, hence for a load of given power factor. The efficiency is maximum when the denominator of η is a minimum.

$$\frac{d}{dI_2} (V_2 I_2 \times P.f + P_c + I_2^2 R_{2e}) = 0$$

$$\frac{d}{dI_2} \left(V_2 \times P.f + \frac{P_c}{I_2} + I_2 R_{2e} \right) = 0$$

$$\frac{-P_c}{I_2^2} + R_{2e} = 0 \rightarrow I_2^2 R_{2e} = P_c$$



Example1:

A 200 KVA, 6600V /400V, 50 HZ single phase transformer has 80 turns on the secondary. Calculate:

- a- The approximate values of the primary and secondary current.
- b- The approximate number of primary turns.
- c- The maximum value of the flux.

Solution:

a- full-load primary current $I_1 = \frac{S}{V_1} = \frac{200 \times 10^3}{6600} = 30.3 \text{ A}$

- full-load secondary current $I_2 = \frac{S}{V_2} = \frac{20 \times 10^3}{400} = 500 \text{ A}$

b- $N_1 = N_2 \times \frac{V_1}{V_2} = \frac{80 \times 6600}{400} = 1320$

c- $E_2 = 4.44 \times N_2 \times f \times \Phi_m$
 $400 = 4.44 \times 80 \times 50 \times \Phi_m$
 $\Phi_m = 0.0225 \text{ wb.}$

Example 2: A 2200/200 V transformer draws a no-load primary current of 0.6 A and absorb 400 watts, find the magnetic and iron loss current.

Solution: iron loss current $= I_c = \frac{P}{V} = \frac{400}{2200} = 0.182 \text{ A}$

$$I_o = \sqrt{I_o^2 + I_{mag}^2} \rightarrow I_o^2 = I_c^2 + I_{mag}^2$$

$$0.6^2 = 0.182^2 + I_{mag}^2$$

$$I_{mag} = 0.572 \text{ A}$$



Example 3: A single phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current 3A at a power factor 0.2 lagging. Calculate the primary current and power factor when the secondary current is 280 A, power factor of 0.8 lagging. (voltage drop-in winding be negligible).

Solution:

If \dot{I}_1 represents the component of the primary current to neutralize the demagnetizing effect of the secondary current. The power –turns due to \dot{I}_1 must be equal and opposite to those due to I_2 .

$$\frac{I_2}{\dot{I}_1} = \frac{N_1}{N_2} = \frac{280}{\dot{I}_1} = \frac{1000}{200}$$

$$\dot{I}_1 \times 1000 = 280 \times 200 \rightarrow \dot{I}_1 = 56 \text{ A}$$

$$\cos \phi_2 = 0.8 \rightarrow \sin \phi_2 = 0.6$$

$$\cos \phi_o = 0.2 \rightarrow \sin \phi_o = 0.98$$

$$\begin{aligned} I_1 \cos \phi_1 &= \dot{I}_1 \cos \phi_2 + I_o \cos \phi_o \\ &= 56 \times 0.8 + 3 \times 0.2 = 45.4 \text{ A} \end{aligned}$$

$$\begin{aligned} I_1 \sin \phi_1 &= \dot{I}_1 \sin \phi_2 + I_o \sin \phi_o \\ &= 56 \times 0.6 + 3 \times 0.98 = 36.54 \text{ A} \end{aligned}$$

$$I_1 = \sqrt{45.4^2 + 36.54^2} = 58.3 \text{ A}$$

$$\tan^{-1} \frac{I_1 \sin \phi_2}{I_1 \cos \phi_1} = \tan^{-1} \frac{36.54}{45.4} = 38^\circ$$

Primary power factor = $\cos \phi_1 = \cos 38^\circ = 0.78 \text{ lagging}$

Example 4:

A 50 KVA, 4400/220 V transformer has $R_1=3.45\Omega$, $R_2=0.009$ have values of reactance are $X_1=5.2\Omega$ and $X_2=0.015\Omega$. Calculate for the transformer: 1-equivalent resistance as referred to primary. 2-equivalent resistance as referred to secondary. 3-equivalent reactance as referred to both primary and secondary. 4-equivalent impedance as referred to both primary and secondary. 5-total P_{cu} loss. first used



individual resistance two windings and secondary using equivalent resistance as referred to each side.

Solution: $I_1 = \frac{S}{V_1} = \frac{50 \times 10^3}{4400} = 11.36 \text{ A}$

$$I_2 = \frac{S}{V_2} = \frac{50 \times 10^3}{220} = 227 \text{ A}$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{220}{4400} = 0.05$$

$$R_{e1} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{0.05^2} = 7.05 \Omega$$

$$R_{e2} = K^2 R_{e1} = 0.05^2 \times 7.05 = 0.0176 \Omega$$

Or $R_{e2} = R_2 + \frac{R_1}{K^2} = 0.009 + \frac{3.45}{0.05^2} = 0.0176 \text{ A}$

$$X_{e1} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{0.05^2} = 11.2 \Omega$$

$$X_{e2} = X_2 + \frac{X_1}{K^2} = 0.015 + \frac{5.2}{0.05^2}$$

$$X_{e1} = k^2 X_{e1} = 0.05^2 \times 11.2 = 0.028 \Omega$$

$$4- Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{7.05^2 + 11.2^2} = 13.23 \Omega$$

$$Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{0.0176^2 + 0.028^2} = 0.03311 \Omega$$

$$5- P_{cu} = I_1^2 R_1 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = 910 \text{ W}$$

$$P_{cu} = I_1^2 R_{e1} = 11.36^2 \times 7.05 = 910 \text{ W}$$

$$= I_2^2 R_{e2} = 227^2 \times 0.017 = 910 \text{ W}$$



Example 5: The primary and secondary of a 500 KVA transformer have resistance of 0.42Ω and 0.0011Ω respectively. The primary and secondary voltages are 6600 V and 400 V, and the iron losses is 2.9 kW. Calculate the efficiency at:

a-full-load, b-half-load. Assuming the p.f of the load 0.8.

solution: $I_2 = \frac{S}{V_2} = \frac{500 \times 10^3}{400} = 1250 \text{ A}$

$$I_1 = \frac{S}{V_1} = \frac{500 \times 10^3}{6600} = 75.8 \text{ A}$$

$$P_{cu2} = I_2^2 R_2 = 1250^2 \times 0.0011 = 1720 \text{ W}$$

$$P_{cu1} = I_1^2 R_1 = 75.8^2 \times 0.42 = 2415 \text{ W}$$

$$P_{cu} = P_{cu1} + P_{cu2} = 4135 + 2.9 \times 10^3 = 7035 \text{ W}$$

$$P_2 = P_{out} = S_x \cos \varphi = 500 \times 10^3 \times 0.8 = 400 \text{ KW}$$

$$P_1 = P_{in} = P_2 + \sum P_{loss} = 400 + 7.035 = 407.035 \text{ KW}$$

$$\frac{400}{407.035} \times 100 = 98.27\% \eta =$$

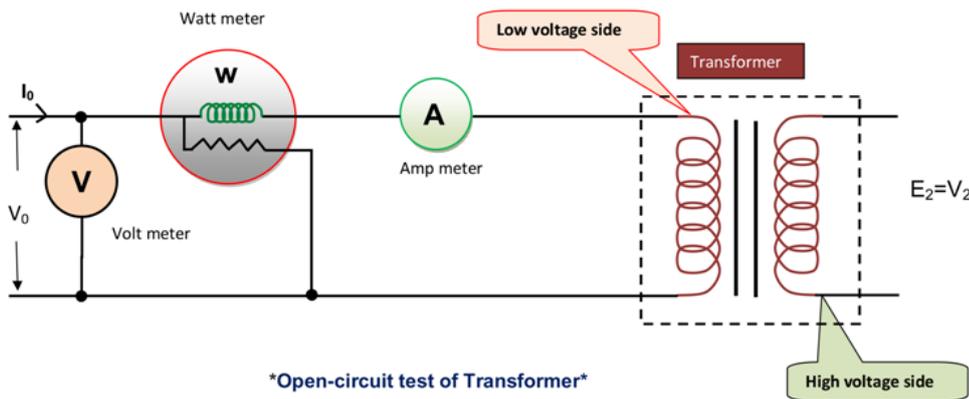
b-half load $P_{cu} = 4135 \times 0.5^2 = 1034 \text{ W}$

$$\sum P_{loss} = 1034 + 2.9 = 3.934 \text{ KW}$$

$$\frac{200}{203.9} \times 100 = 98.08\% \eta =$$

4-14 Open Circuit or No-Load Test:

The purpose of this test is to determine no-load loss or core loss and no-load current I_o which is helpful in finding X_o and R_o .



$$P(w) = I_o V_1 \cos \phi_o$$

$$I_{mag} = I_o \sin \phi_o$$

$$I_c = I_o \cos \phi_o$$

$$X_o = \frac{V_1}{I_{mag}} \quad , \quad R_o = \frac{V_1}{I_c}$$

Wher: $X_o =$ no load inductance, $R_o =$ no load resistance

$I_o =$ no load current, $V_1 =$ supply voltage

$P =$ no load power

If W or P_o is wattmeter reading, then:

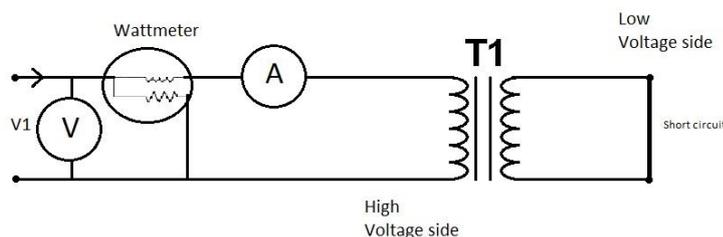
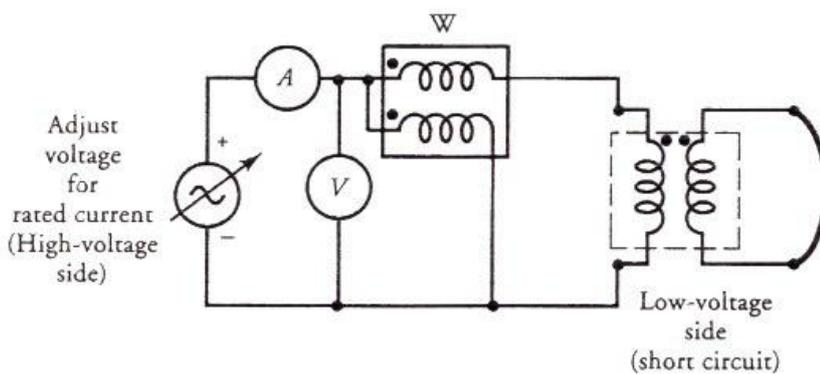
$$P_o = V_1 I_o \cos \phi_o \quad \text{and} \quad \cos \phi_o = \frac{P_o}{V_1 I_o}$$

4-15 Short Circuit or Impedance Test:

This is an economical method for determining the follow:

$Z_{e1}, Z_{e2}, X_{e1}, X_{e2}, P_{cu}$. Equivalent impedance (Z_{e1} or Z_{e2}), leakage reactance (X_{e1} or X_{e2}) and total resistance (R_{e1} or R_{e2}) of the transformer as referred to the winding.

P_{cu} losses at full load and at any load and then used to calculate the efficiency of transformer. Knowing Z_{e1} and Z_{e2} , the total voltage drops in the transformer as referred to primary or secondary can be calculate and hence regulation of the transformer determined.



If V_{sc} is the voltage required to circulate rated load current, then:

$$Z_{e1} = \frac{V_{sc}}{I_1}, \quad X_{eq} = \frac{V_1}{I_1}$$

ALSO



$$P_{sc} = I_1^2 R_{e1}$$

$$R_{e1} = \frac{P_{sc}}{I_1^2} \quad \text{and} \quad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

Note:

P_o = input power in watts on the open cct test

$$= \text{Iron loss} = P_c$$

P_{sc} = input power in watts on the short cct test with full load current.

P_{sc} = total copper loss on full load,

$$\Sigma \text{ loss in full load} = P_o + P_{sc}$$

$$\frac{\text{full load VA} \times P.f}{\text{full load VA} \times P.f + P_o + P_{sc}} \times 100 \eta =$$

4-16 Voltage Drop in A Transformer:

Total transformer drops as referred to secondary

$$= I_2 R_{e1} \cos \varphi \mp I_2 X_{e2} \sin \varphi$$

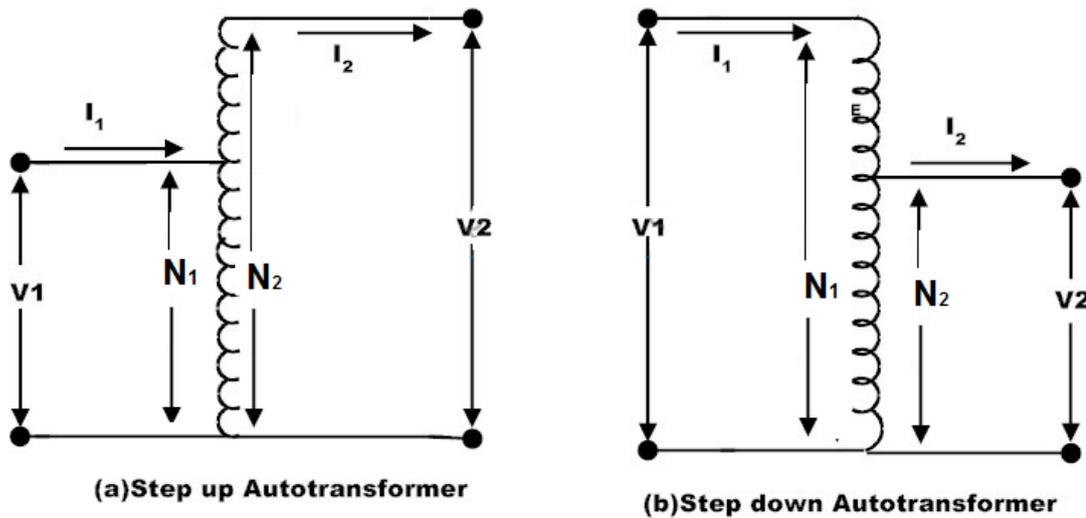
Total transformer drops as referred to primary

$$= I_1 R_{e1} \cos \varphi \mp I_1 X_{e1} \sin \varphi$$

Note: the upper signs are to be used +Ve for lagging power factor and – Ve for leading power factor.

4-17 Auto – transformer:

It is transformer with one winding only. Because of on winding it use less copper and hence is cheaper. Figure below shows both step down and step up auto-transformer.



As compared to an ordinary 2-winding transformer of same output, an auto-transformer has higher efficiency but ,smaller size.

Uses of auto-transformer:

- 1-To give small boost to a distribution cable to current the voltages drop.
- 2-To give up to 50 to 60 at full voltage to an induction motor during starting.
- 3- As furnace transformers for getting a convenient supply to suit the furnace winding from 230 v supply.
- 4- As interconnecting transformer in 132kv/330kv system.
- 5- In control equipment for 1-phase and 3-phase electrical locomotives.

5 Parallel operation of single-phase transformer:

For supply a load excess of the rating of an existing transformer, a second transformer may the connected in parallel. There are certain definite conditions



which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their KVA ratings. The conditions are:

- 1-Primary windings of the transformer should be suitable for the supply system voltage and frequency.
- 2-The transformers should be properly connected with regard to polarity.
- 3- The voltage ratings of both primaries and secondaries should be identical.
- 4-The percentage impedance should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factor.
- 5- With transformers having different KVA ratings. The equivalent impedances should be inversely proportional to the individual KVA rating if circulating currents are to be avoided.

There are two cases for represent of parallel connections of single-phase transformers:

5-1 Ideal case:

Equal voltage ratios:

At no-load of both secondaries is the same $E_A = E_B = E$, and that the two voltage are coincident . There is no phase different between E_A and E_B which would be turn if the magnetizing different from each other.

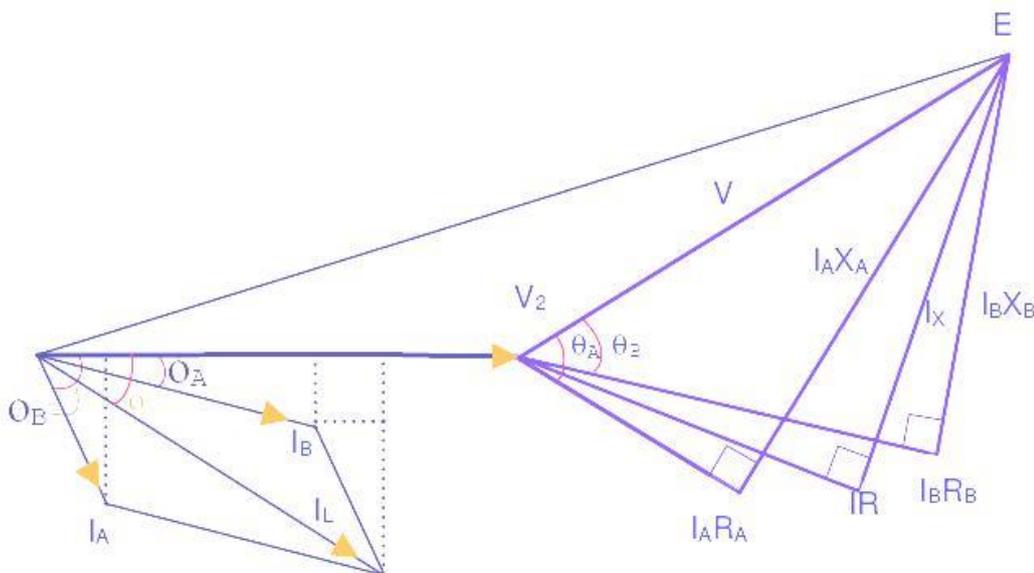
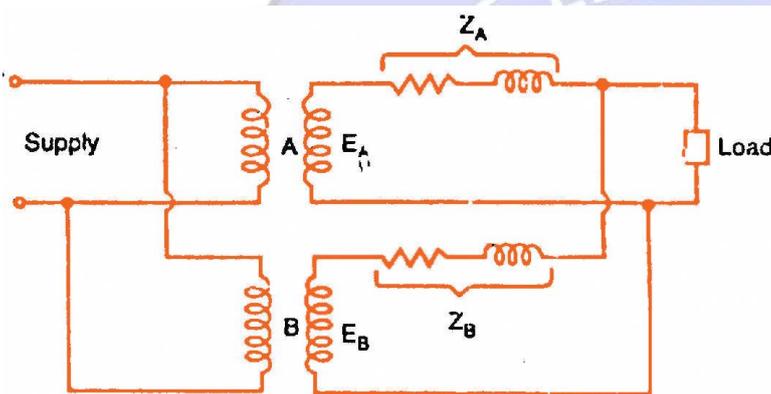
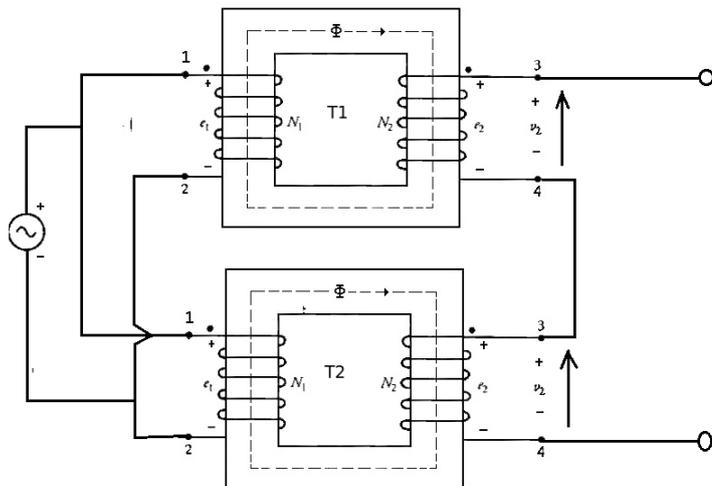


Figure 39: Phasor Diagram of Operation for two Transformers working in Parallel

Phasor diagram of 2 transformers in parallel:



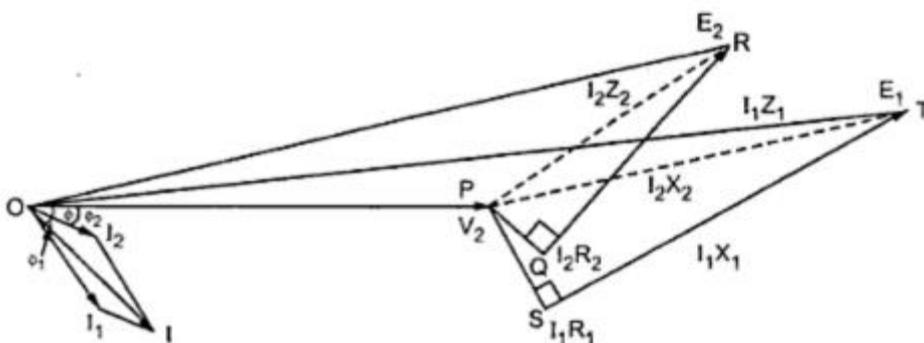
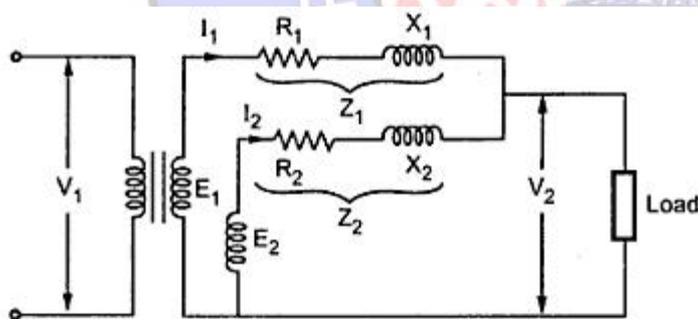
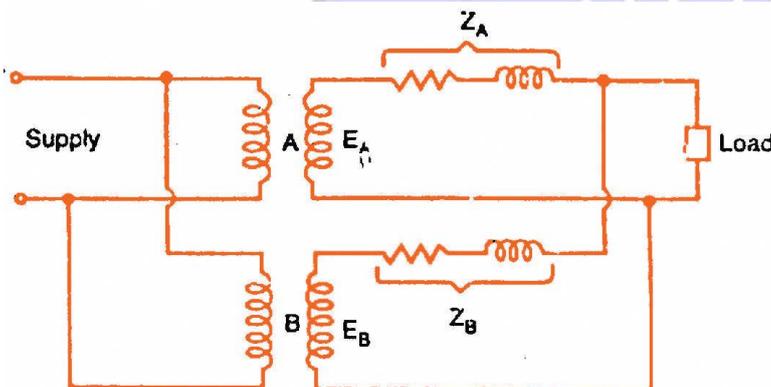
$$I_A = \frac{I Z_B}{Z_A + Z_B} \quad \text{and} \quad I_B = \frac{I Z_A}{Z_A + Z_B}$$

$$V_2 I_A = \frac{V_2 I Z_B}{Z_A + Z_B} \quad \text{and} \quad V_2 I_B = \frac{V_2 I Z_A}{Z_A + Z_B}$$

$$S_A = S \frac{Z_B}{Z_A + Z_B} \quad \text{and} \quad S_B = S \frac{Z_A}{Z_A + Z_B}$$

Unequal Voltage Ratios:

In this case the voltage ratios of the two transformers are different. It means that their no-load secondary voltages are unequal.



$$E_A = I_A Z_A + V_2$$



$$E_B = I_B Z_B + V_2$$

$$V_2 = I Z_L = (I_A + I_B)Z_L$$

$$E_A = I_A Z_A + (I_A + I_B)Z_L$$

$$E_B = I_B Z_B + (I_A + I_B)Z_L$$

$$E_A - E_B = I_A Z_A - I_B Z_B$$

$$I_A = \frac{E_A - E_B + I_B Z_B}{Z_A} \quad \text{Substituting } I_A \text{ in}$$

$$I_B = \frac{E_B Z_A - (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

$$I_A = \frac{E_A Z_B - (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

$$I = I_A + I_B = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)} \times \frac{1/Z_A Z_B}{1/Z_A Z_B} \times Z_L$$

5-3 Transformers in Three-Phase Circuits

Three single-phase transformers can be connected to form a *three-phase transformer bank* in any of the four ways shown in Fig. 1. In all four parts of this figure, the windings at the left are the primaries, those at the right are the secondaries, and any primary winding in one transformer corresponds to the secondary winding drawn parallel to it. Also shown are the voltages and currents resulting from balanced impressed primary line-to-line voltages V and line currents I when the ratio of primary-to-secondary turns $N_1/N_2 = a$ and ideal transformers are assumed. 4 Note that the rated voltages and currents at the primary and secondary of the three-phase transformer bank depends upon the connection used but that the rated kVA of the three-phase bank is three times that of the individual single-phase transformers, regardless of the connection.

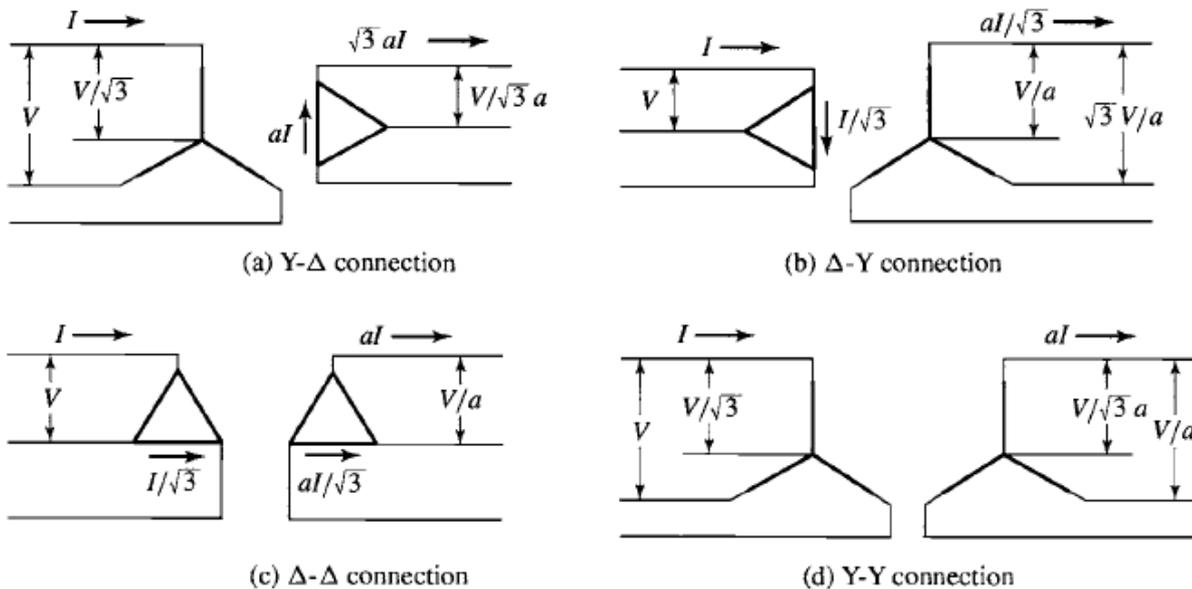


Figure 1 Common three-phase transformer connections; the transformer windings are indicated by the heavy lines

The Y- Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage. One reason is that a neutral is thereby provided for grounding on the high-voltage side, a procedure which can be shown to be desirable in many cases. Conversely, the Δ-Y connection is commonly used for stepping up to a high voltage. The Δ - Δ connection has the advantage that one transformer can be removed for repair or maintenance while the remaining two continue to function as

a three-phase bank with the rating reduced to 58 percent of that of the original bank; this is known as the *open-delta*, or *V*, connection. The Y-Y connection is seldom used because of difficulties with exciting-current phenomena.

Instead of three single-phase transformers, a three-phase bank may consist of one *three-phase transformer* having all six windings on a common multi-legged core and contained in a single tank. Advantages of three-phase transformers over connections of three single-phase transformers are that they cost less, weigh less, require less floor space, and have somewhat higher efficiency. A photograph of the internal parts of a large three-phase transformer is shown in Fig. 2.

Circuit computations involving three-phase transformer banks under balanced conditions can be made by dealing with only one of the transformers or phases and recognizing that conditions are the same in the other two phases except for the phase displacements associated with a three-phase system. It is usually convenient to carry out the computations on a single-phase (per-phase-Y, line-to-

neutral) basis, since transformer impedances can then be added directly in series with transmission line impedances. The impedances of transmission lines can be referred from one side of the transformer bank to the other by use of the square of the ideal line-to-line voltage

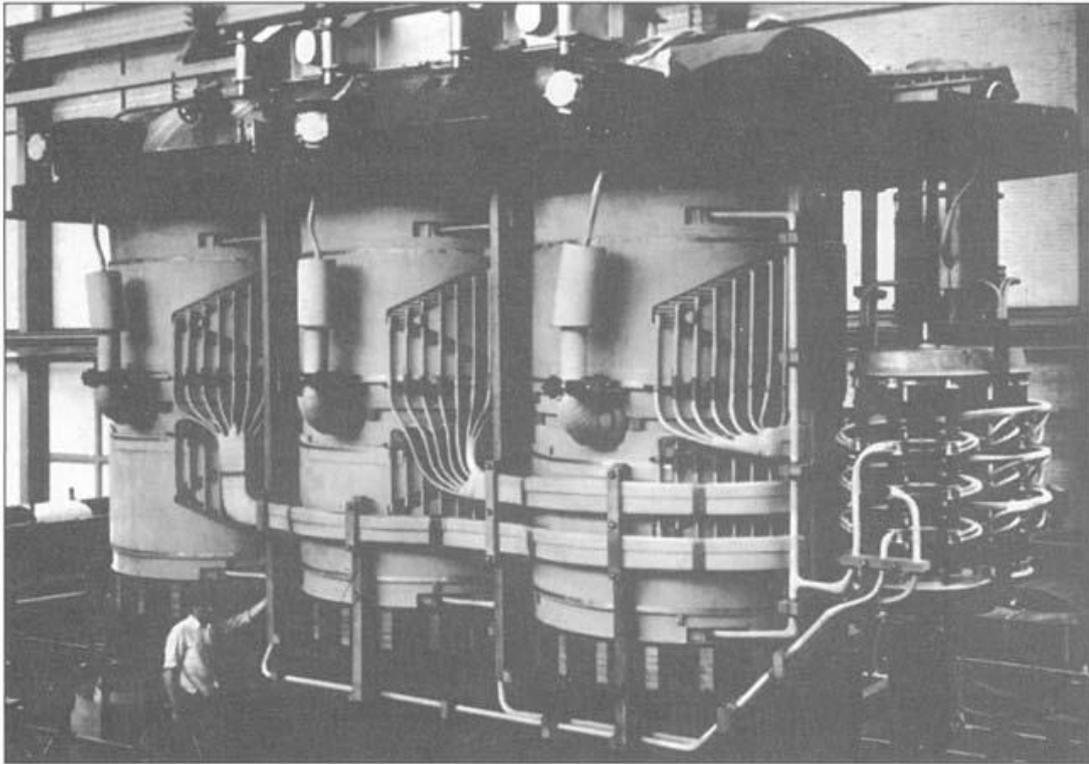


Figure 2 A 200-MVA, three-phase, 50-Hz, three-winding, 210/80/10.2-kV transformer removed from its tank. The 210-kV winding has an on-load tap changer for adjustment of the voltage.

ratio of the bank. In dealing with Y- Δ or A-Y banks, all quantities can be referred to the Y-connected side. In dealing with A-A banks in series with transmission lines, it is

convenient to replace the Δ -connected impedances of the transformers by equivalent Y-connected impedances. It can be shown that a balanced A-connected circuit of Z_{Δ} Ω /phase is equivalent to a balanced Y-connected circuit of Z_Y Ω /phase if

$$Z_Y = \frac{1}{3} Z_{\Delta}$$