Electric Circuit 1
Al-Anbar University

LECTURE 08
NODAL ANALYSIS WITH VOLTAGE SOURCE

Topics

Nodal Analysis in the presence of Voltage Sources

Objectives

- Apply Nodal Analysis when voltage sources are present
- Use the concept of Supernode in the Nodal Analysis
- Determine when (and when not) to apply KCL at a super node
- Develop a strategy for dealing with voltage sources in the Nodal Analysis

Voltage Sources connected to the Reference node

The case of voltage sources connected to the reference node is taken up first and it is illustrated with the help of an example.

Example: Calculate the nodal voltages V_1, V_2, V_3.

Solution:

Nodes 1 & 2

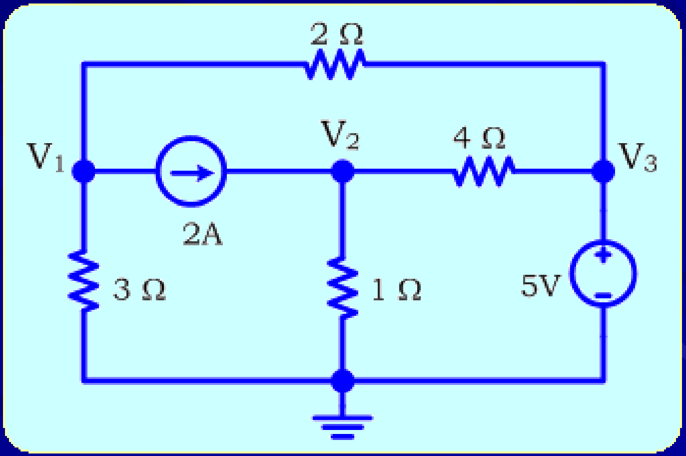
⟹ No voltage sources connected

⟹ No special treatment required

Node 3

⟹ Voltage source connected

⟹ Needs special treatment



Solution: Applying KCL

KCL at node 1:

$$\Rightarrow \frac{V_1 - 0}{3} + 2 + \frac{V_1 - V_3}{2} = 0$$

$$\Rightarrow 5V_1 - 3V_3 = -6 \qquad (1)$$

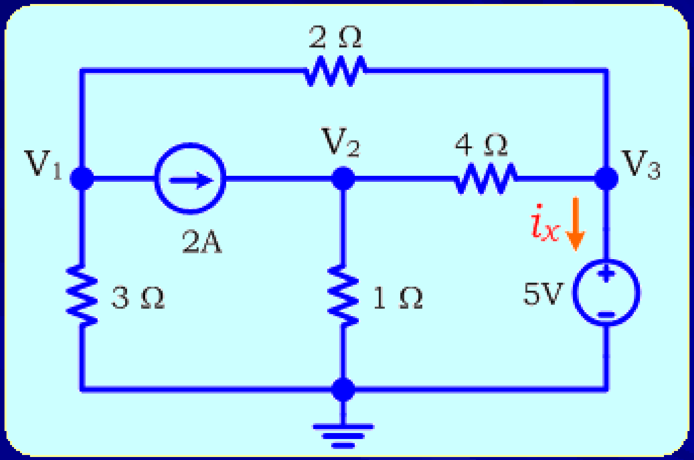
KCL at node 2:

$$\Rightarrow -2 + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{4} = 0$$

$$\Rightarrow 5V_2 - V_3 = 8 \qquad (2)$$

KCL at node 3:

$$\Rightarrow \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{2} + i_x = 0 \quad \text{(problem!)}$$



i_x cannot directly be replaced with nodal voltages, because Ohm's law does not apply to voltages sources

Solution: Solve equations

We have 3 unknowns ⟹ We need 3 equations ⟹ one equation is missing

For node 3, the basic Nodal Analysis procedure must be revised.

The 5V source is connected to the reference node.

Apply KVL: Node 3 and reference node

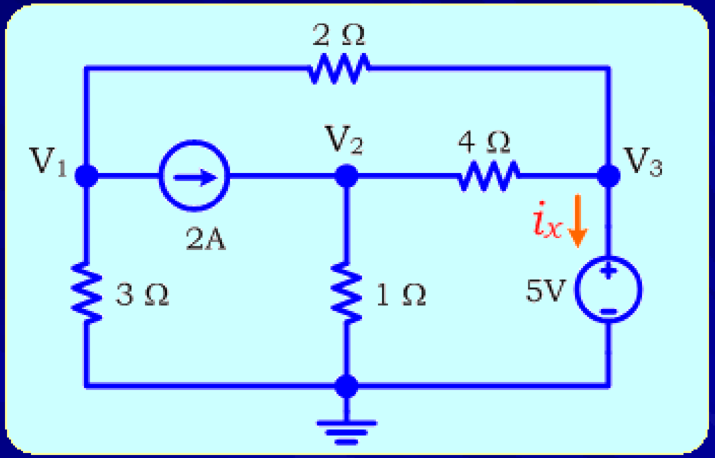
⟹ $V_3 - 0 = 5$

⟹ $V_3 = 5$ (3)

From the previous slide,

$5V_1 - 3V_3 = -6$ (1)

$5V_2 - V_3 = 8$ (2)



Solving the above set of equations, we get:

$V_1 = 1.8V$ & $V_2 = 2.6V$ & $V_3 = 5V$

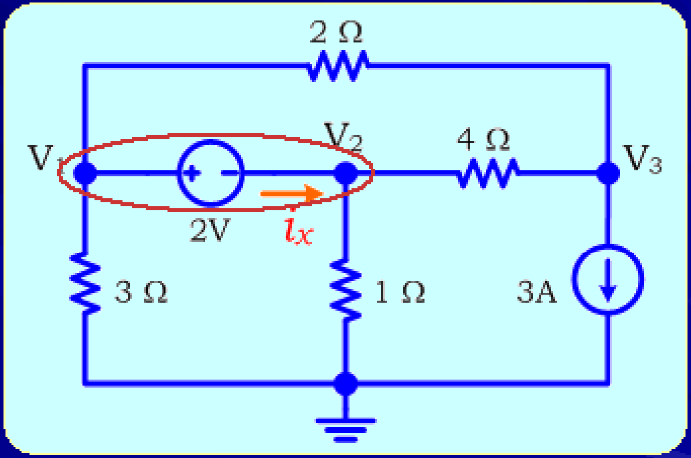
Voltage source connected to reference ⟹ Use KVL only (do not use KCL)

Voltage Sources NOT connected to the Reference node

The case of voltage sources not connected to the reference node is illustrated here with the help of an example.

Example: Calculate the nodal voltages V_1, V_2, V_3.

Solution:



KCL at node 1:

$$\Longrightarrow \quad \frac{V_1 - 0}{3} + i_x + \frac{V_1 - V_3}{2} = 0 \qquad \text{(problem!)}$$

KCL at node 2:

$$\Longrightarrow \quad -i_x + \frac{V_2}{1} + \frac{V_2 - V_3}{4} = 0 \qquad \text{(problem!)}$$

The 2V source node is connected to nodes 1 & 2. $\Longrightarrow$ KCL at nodes 1 and 2 contain i_x

We need two equations, one for each node. How should we proceed now?

Solution: Draw super node

1. Draw a super node around nodes 1 & 2.

2. KCL at super node: (avoids i_x)

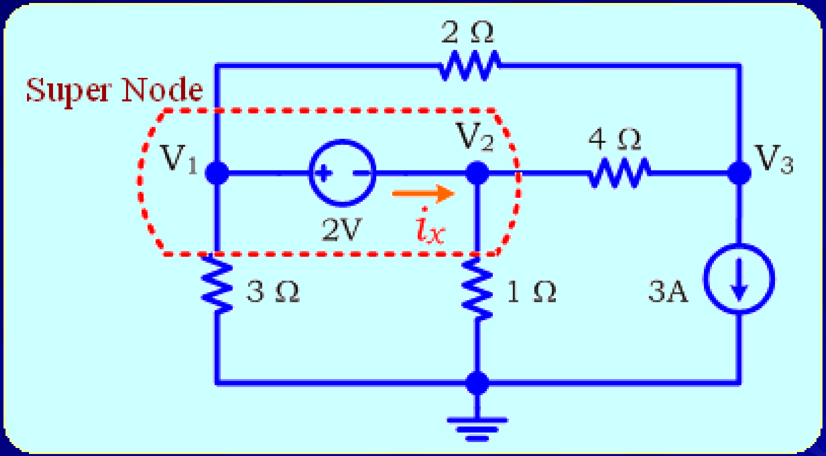
$$\frac{V_1-0}{3}+\frac{V_2-0}{1}+\frac{V_2-V_3}{4}+\frac{V_1-V_3}{2}=0$$

$$\Rightarrow \quad 10V_1+9V_2-3V_3=0 \qquad (1)$$

3. KVL $\Rightarrow$ $V_1-V_2=2 \qquad (2)$

KCL: node 3 $\dfrac{V_3-V_2}{4}+3+\dfrac{V_3-V_1}{2}=0$

$$\Rightarrow \quad -2V_1-V_2+3V_3=-24 \qquad (3)$$



[We obtained the required number of equations]

Solving (1), (2), (3), we get: $V_1=-0.5V$ & $V_2=-2.5V$ & $V_3=-9.17V$

Voltage source not connected to reference $\Rightarrow$ KCL at super node and KVL

Intersecting Super nodes

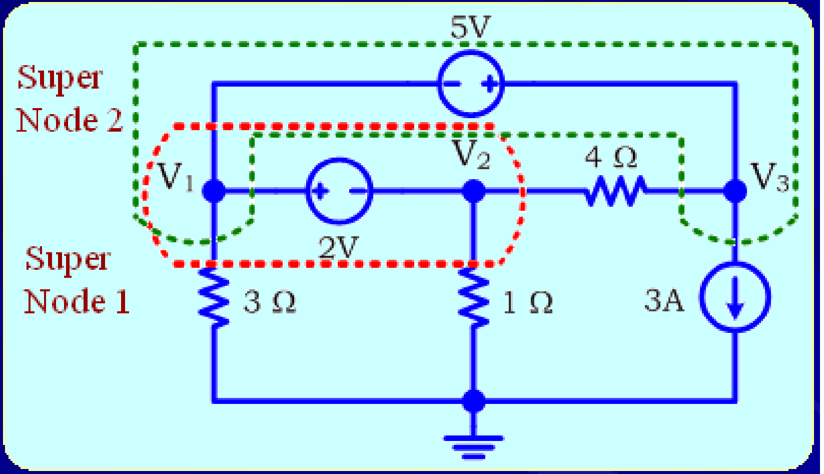
The case of intersecting super nodes is illustrated here with the help of an example.

Example: Calculate the nodal voltages V_1, V_2, V_3.

Solution:

Nodes 1 & 2 are connected by the 2V source ⟹ Draw super node 1

Nodes 1 & 3 are connected by the 5V source ⟹ Draw super node 2



If we apply KCL at super node 1

⟹ KCL contains current through the 5V source

If we apply KCL at super node 2

⟹ KCL contains current through the 2V source

Do not apply KCL at super nodes 1 or 2

Solution: combine super nodes into a single super node

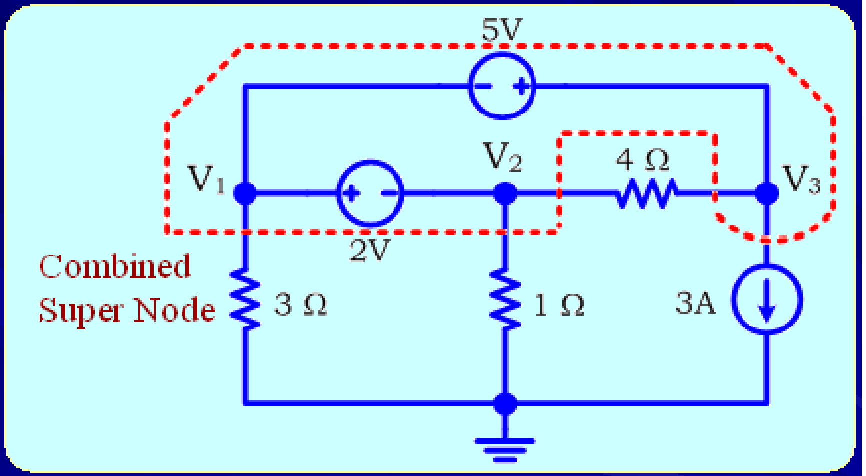
KCL at the new super node.

$$\frac{V_1-0}{3}+\frac{V_2-0}{1}+\frac{V_2-V_3}{4}+\frac{V_3-V_2}{4}+3=0$$

(actual current through 4 Ohms leaves and enters the super node)

$$\Rightarrow \frac{V_1-0}{3}+\frac{V_2-0}{1}+3=0$$

$$\Rightarrow V_1+3V_2=-9 \qquad (1)$$



To obtain the remaining 2 equations, apply KVL

KVL $\Rightarrow V_1-V_2=2 \qquad (2)$

KVL $\Rightarrow V_1-V_3=5 \qquad (3)$

Solving (1), (2), (3), we get:

$$\Rightarrow V_1=-0.75V \quad V_2=-2.75V \quad V_3=-5.75V$$

Nodal Analysis with Voltage sources: Strategy A

Every voltage source provides one nodal equation using KVL.

Strategy A

1. Apply KVL to every voltage source.
2. Draw a super node around every voltage source.
3. If two super nodes intersect --- join them.
4. Do not apply KCL to any super node that contains the reference node.
5. Apply KCL to the remaining nodes and super nodes.

Nodal Analysis with Voltage sources: Strategy B

Strategy B

1. Apply KVL to each voltage source.
2. If two super nodes intersect --- join them.
3. Apply KCL at a node, super node, or a combined super node (as neccessary)

Strategies A and B are basically the same. They produce exactly the same set of equations.

Let us apply Strategy B to the next example.

Example: Write down the nodal equations (do not simplify and do not solve)

Solution:

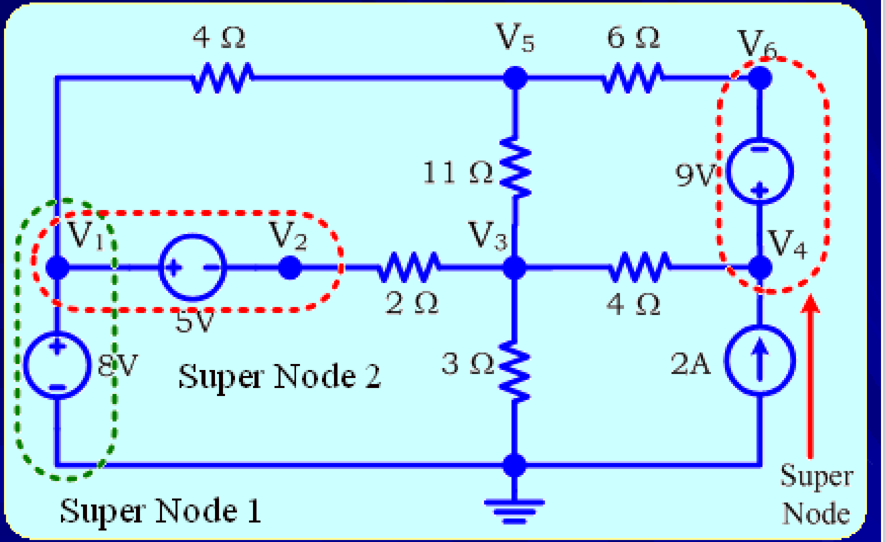
Apply KVL for every voltage source.

$\Rightarrow$

$$V_1 = 8 \qquad (1)$$

$$V_1 - V_2 = 5 \qquad (2)$$

$$V_4 - V_6 = 9 \qquad (3)$$



3 more equations are required.

$\Rightarrow$ Apply KCL

KCL at node 3 $\Rightarrow$

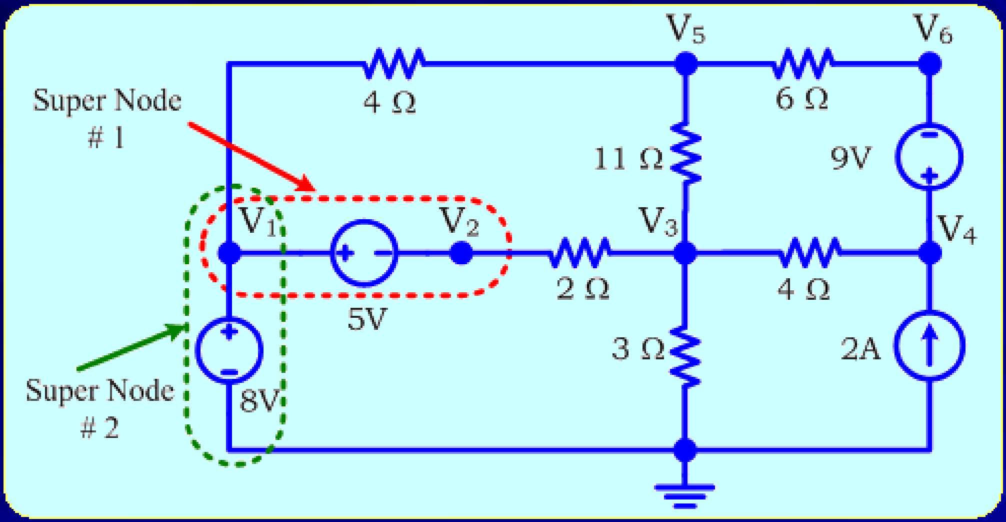
$$\frac{V_3 - V_2}{2} + \frac{V_3}{3} + \frac{V_3 - V_4}{4} + \frac{V_3 - V_5}{11} = 0 \qquad (4)$$

KCL at node 5 $\Rightarrow$ $\frac{V_5 - V_1}{4} + \frac{V_5 - V_3}{11} + \frac{V_5 - V_6}{6} = 0$ (5)

KCL at super node $\Rightarrow$ $\frac{V_4 - V_3}{4} - 2 + \frac{V_6 - V_5}{6} = 0$ (6)

Nodal Analysis with Voltage sources: Analysis

In the previous example, we *did not* apply KCL at the Super node 1, or Super node 2 because we *do not need* KCL there (since we have enough equations from nodes 1 & 2). Another way to think about this point.



Super node 1 contains current through the 8V source.
⟹ Do not apply KCL at super node 1

Super node 2 contains current through the 5V and 8V sources.
⟹ Do not apply KCL at super node 2

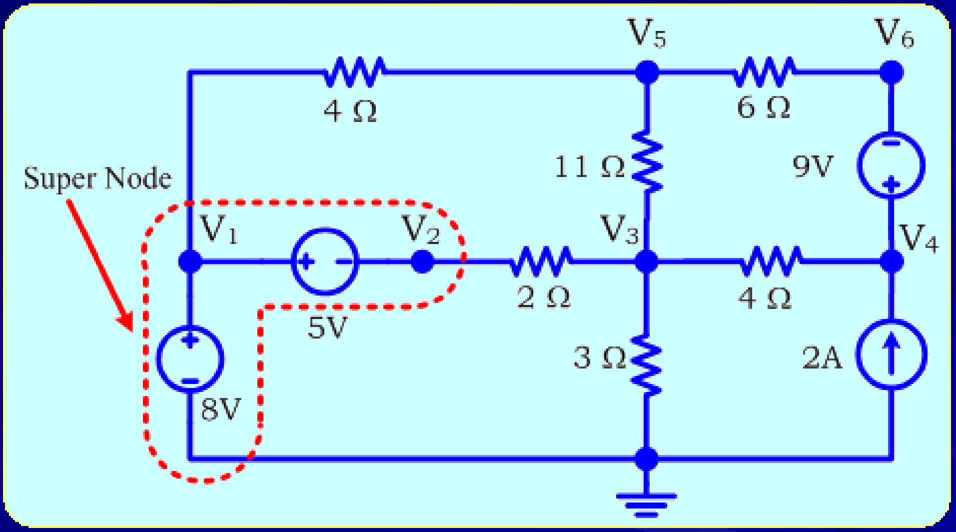
Nodal Analysis with Voltage sources: Analysis

Combine Super node 1 and Super node 2 into a new Super Node.

The new Super node still contains current through the 8V source.

⟹ Do not apply KCL at the new super node

The basic reason is that the new super node contains the reference node.



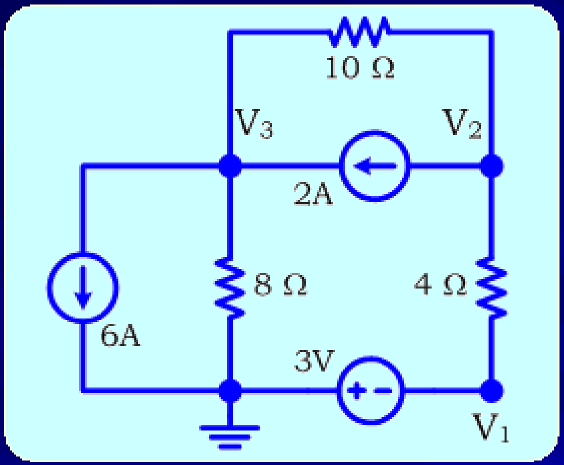
Important:

Super node contains the reference node

⟹ Do not apply KCL

Practice Problem

Write simplified equations for the nodal voltages V_1, V_2 & V_3



Answer:

$$V_1 = -3$$
$$7V_2 - 2V_3 = -55$$
$$-4V_2 + 9V_3 = -160$$