A common concern has been that cracked members are less durable than uncracked ones. However, many such structures have been in service for more than 25 years and have performed satisfactorily.

9.9 CHOICE OF SECTION

In the previous examples the simplest shape of cross-section, namely rectangular, was used, chosen primarily to illustrate the basic principles of design. Where there is freedom to choose a more economical section the designer must decide which shape of section to use for a particular situation.

The solid rectangular section is one of the least economical sections, since the mid-depth regions are not usually highly stressed and the material is not used to its full extent. One way of overcoming this deficiency is to provide voids in the central region of the section; these provide a similar structural efficiency with less weight. A typical hollow-core slab is shown in Fig. 9.8(a). As with steel sections, an I-section is a very efficient shape, Fig. 9.8(b), providing maximum area of concrete at the furthest distance from the neutral axis. An alternative section, one with similar efficiency for bending but with far greater torsional stiffness, is the box-section, shown in Fig. 9.8(c).

The T-section shown in Fig. 9.8(b) is suitable for long-span beams, generally in bridges. For buildings, the T-section shown in Fig. 9.9(a) is

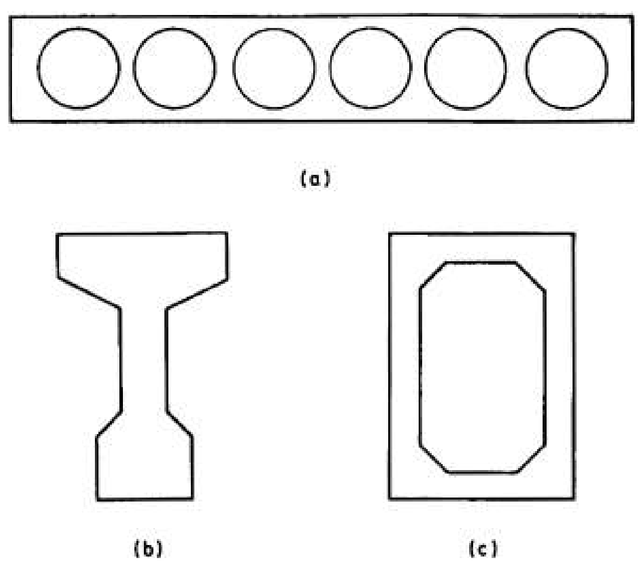

Figure 9.8 Sections: (a) hollow-core slab; (b) I-section; (c) box-section.

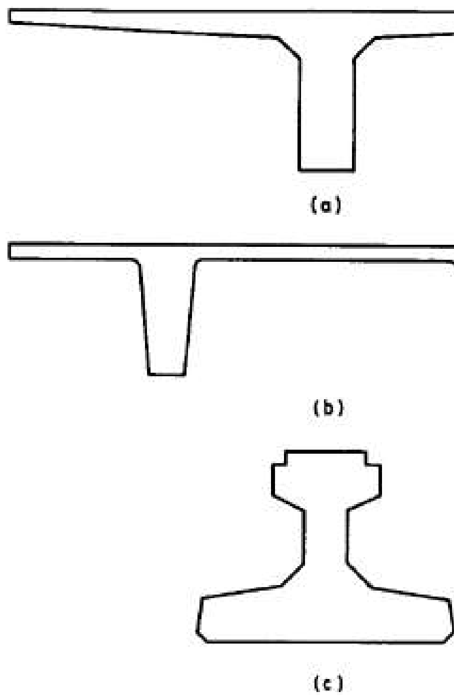

Figure 9.9 T-sections: (a) single; (b) double; (c) inverted.

often used. This has a large compression flange for the total design load but it is necessary to ensure that the compressive stresses in the rib at transfer are not excessive. If the rib is slender the possibility of buckling at transfer must also be considered. Single T-sections are not very stable during construction and a common solution is to use double T-sections, shown in Fig. 9.9(b). Both types of T-section are commonly used with a composite structural topping; the design of such members is dealt with in Chapter 10.

Another shape often used in composite construction is the inverted T-section, or 'top-hat' section shown in Fig. 9.9(c). The large flange at the soffit of the beam can accommodate a large bending moment due to the beam self weight and the weight of the *in situ* slab. Under total design load the compression flange is provided by the *in situ* slab, which acts compositely with the inverted T-section.

9.10 FLOW CHARTS FOR DESIGN

The methods of design for uncracked and cracked concrete members outlined in the preceding sections may be combined with the design elements considered in the previous chapters to give an overall view of the design process. This is most conveniently summarized in the form of flow charts, and Figs 9.10 and 9.11 show these for uncracked and cracked members, respectively.

Many steps in the design process are common to both types of member and the main difference is the determination of the amount of prestressing steel. For uncracked members emphasis is usually placed on the stresses at the serviceability limit state, with checks for ultimate strength made afterwards. For cracked members the ultimate strength capacity is generally ensured first, and conditions at the serviceability limit state checked later.

The steps in Figs 9.10 and 9.11 are intended only as a guide, and with experience many of them may be combined or bypassed completely.

9.11 DETAILING

There are some practical details concerning the layout of tendons which may affect the design, and it is important to be aware of these when deciding on the number and shape of tendons to be provided.

Unless a prestressed concrete member remains in compression under the rare load combination a certain minimum percentage of steel is required to ensure that, when the concrete cracks, the additional force transferred to the steel does not cause immediate yield or rupture. The minimum amount of reinforcement stated in EC2 is given by

$$A_s \geq k_c k f_{ct} A_{ct} / f_{yk},$$

where A_s is the area of bonded reinforcement, both untensioned and bonded prestressed, in the tensile zone (unbonded tendons should be ignored); k_c is a coefficient which takes account of the form of the stress distribution within the section prior to cracking and varies between 0 and 0.4 (a method of determining which value of k_c to use is given in Beeby and Narayanan (1995)); k is a coefficient which allows for non-linear self-equilibrating stresses within the member (a value of 0.8 should normally be used, except in the case of externally applied deformations, such as foundation settlement, where the stress distribution remains linear and a value of 1.0 should be used); f_{ct} is the tensile strength of the concrete at the time when the cracks are first expected to form (the value of f_{ct} should not be less than 3 N/mm^2); A_{ct} is the area of concrete in tension just prior to the formation of the first crack; and f_{yk} is the tensile strength of the reinforcement (or prestressing steel).

A minimum number of tendons is required in members which are statically determinate and where no load redistribution can take place. The minimum numbers of different types of tendon are shown in Table 9.2, but it is also sufficient to provide one tendon comprising a seven-wire strand, with wires of diameter not less than 4 mm.

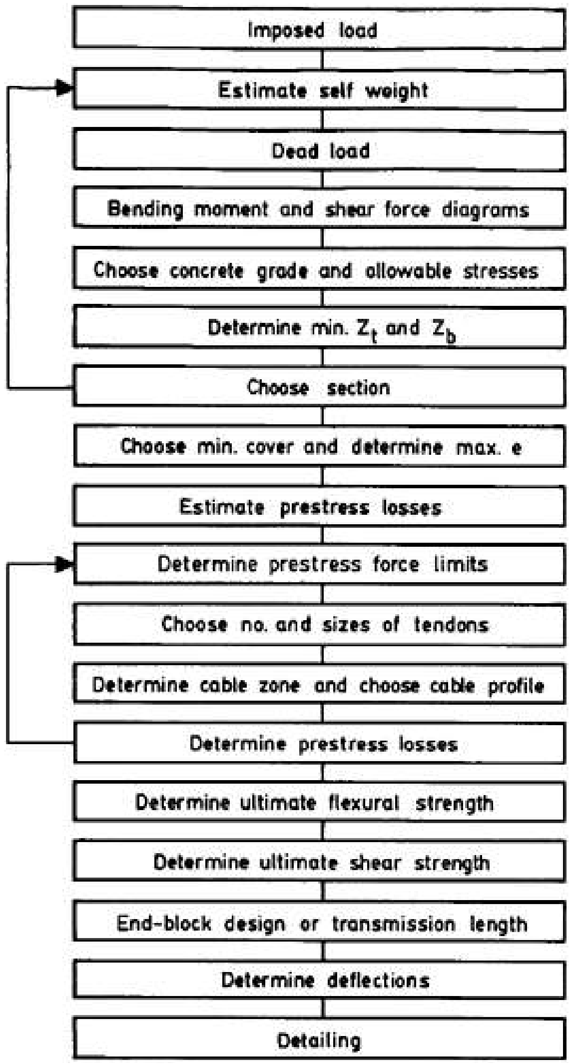

Figure 9.10 Flow chart for uncracked members.

Table 9.2 Minimum numbers of different types of prestressing steel

Individual bars or wires	*Bars and wires forming a strand or tendon*	*Tendons (but see text)*
3	7	3

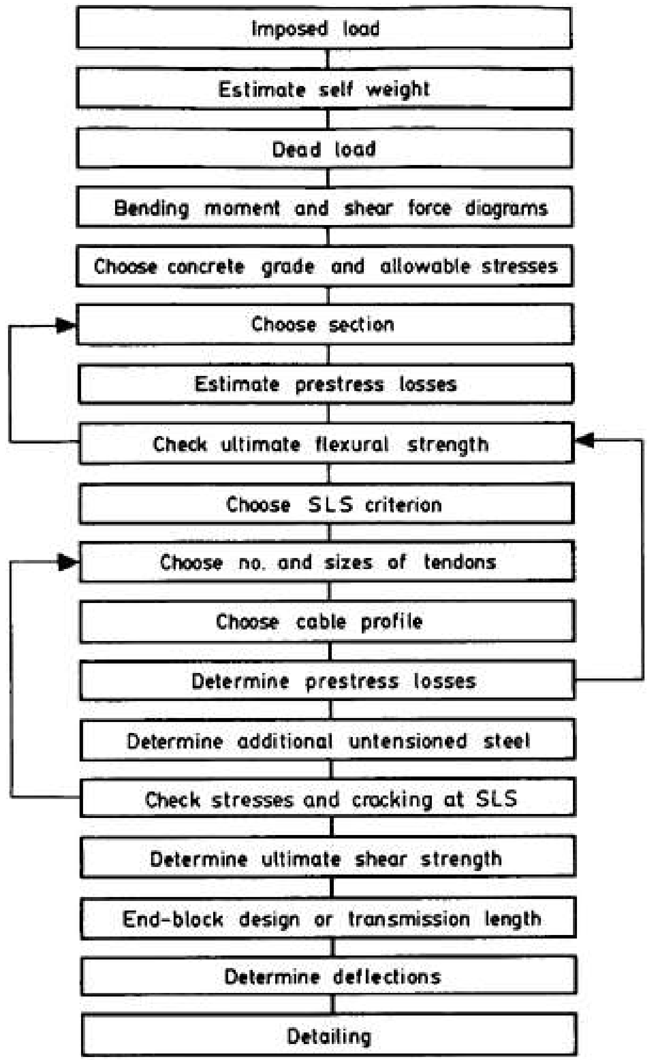

Figure 9.11 Flow chart for cracked members.

A factor which affects the choice of numbers and sizes of individual tendons is the clear space that must be provided between tendons in order to ensure proper placement and compaction of the surrounding concrete. These minimum distances are shown in Table 9.3, where d_g is the nominal aggregate size and φ is the diameter of the tendon or duct.

If a curved post-tensioned duct is placed near the surface of a concrete member, bursting of the concrete may occur in a direction perpendicular to the plane of curvature of the duct. In order to prevent this, a

Table 9.3 Minimum clear distances between prestressing tendons and ducts

Pretensioned		*Post-tensioned*	
Vertically	*Horizontally*	*Vertically*	*Horizontally*
$\geq d_g$;$\geq \varphi$; $\geq$10 mm	$\geq d_g$+5 mm; $\geq \varphi$;$\geq$20 mm	$\geq \varphi$;$\geq$50 mm	$\geq \varphi$;$\geq$40 mm

minimum cover to the duct should be specified. No such cover is given in EC2, but the values recommended in the former BS8110 are given in Table 9.4. In order to prevent the crushing of the concrete between curved post-tensioning ducts which are in the same plane of curvature, the clear spacing between the ducts should be not less than the values given in Table 9.5.

The minimum cover to ducts and tendons is usually determined from durability and fire resistance considerations, and is described in Chapter 3. Manufacturers of prestressing systems usually also specify the minimum cover to be used with their products.

Most prestressed concrete members will contain untensioned reinforcement fabricated into a cage. This serves several purposes:

(a) to facilitate the placing of the post-tensioning ducts;
(b) to enhance the ultimate flexural and shear strength of the member;
(c) to resist any tensile stresses which may be set up by restraint of shrinkage of the member by the formwork before transfer; and
(d) to enable the member to withstand any sudden load applied to it (the reinforcement should preferably be mild steel).

The detailing of the untensioned reinforcement is covered in the relevant sections of EC2. An example of a reinforcement cage in a post-tensioned beam is shown in Fig. 9.12.

PROBLEMS

9.1 A post-tensioned I-section beam of overall depth 1220 mm spans 15 m and has the following section properties:

w_o=8.7 kN/m — A_c=3.694×10^5 mm^2
Z_t=80.75×10^6 mm^3 — Z_b=119.32×10^6 mm^3
e_{max}=400 mm — y_b=490 mm.

Short- and long-term loss factors are 10% and 25%, respectively. The dead load is 15 kN/m and the imposed load of 60 kN/m is applied in two

Table 9.4 Minimum cover (mm) to curved ducts

	Duct internal diameter (mm)															
	19	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*	*110*	*120*	*130*	*140*	*150*	*160*	*170*
	Tendon force	*(0.8 $f_{pk}A_p$)(kN)*														
Radius of curvature of duct (m)	*296*	*387*	*960*	*1337*	*1920*	*2640*	*3360*	*4320*	*5183*	*6019*	*7200*	*8640*	*9424*	*10338*	*11248*	*13200*
2	50	55	155	220	320	445								Radii not normally used		
4		50	70	100	145	205	265	350	420							
6			50	65	90	125	165	220	265	310	375	460				
8				55	75	95	115	150	185	220	270	330	360	395		
10				50	65	85	100	120	140	165	205	250	275	300	330	
12					60	75	90	110	125	145	165	200	215	240	260	315
14					55	70	85	100	115	130	150	170	185	200	215	260
16					55	65	80	95	110	125	140	160	175	190	205	225
18					50	65	75	90	105	115	135	150	165	180	190	215
20						60	70	85	100	110	125	145	155	170	180	205
22						55	70	80	95	105	120	140	150	160	175	195
24						55	65	80	90	100	115	130	145	155	165	185
26						50	65	75	85	100	110	125	135	150	160	180
28							60	75	85	95	105	120	130	145	155	170
30							60	70	80	90	105	120	130	140	150	165
32							55	70	80	90	100	115	125	135	145	160
34							55	65	75	85	100	110	120	130	140	155
36							55	65	75	85	95	100	115	125	140	150
38							50	60	70	80	90	105	115	125	135	150
40	50	50	50	50	50	50	50	60	70	80	90	100	110	120	130	145

Table 9.5 Minimum distance (mm) between ducts in plane of curvature

	Duct internal diameter (mm)															
	19	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*	*110*	*120*	*130*	*140*	*150*	*160*	*170*
	Tendon force $(0.8 f_{pk} A_p)$(kN)															
Radius of curvature of duct (m)	*296*	*387*	*960*	*1337*	*1920*	*2640*	*3360*	*4320*	*5183*	*6019*	*7200*	*8640*	*9424*	*10336*	*11248*	*13200*
2	110	140	350	485	700	960								Radii not normally used		
4	55	70	175	245	350	480	610	785	940							
6	38	60	120	165	235	320	410	525	630	730	870	1045				
8			90	125	175	240	305	395	470	545	655	785	855	940		
10			80	100	140	195	245	315	375	440	525	630	685	750	815	
12						160	205	265	315	365	435	525	570	625	680	800
14						140	175	225	270	315	375	450	490	535	585	785
16							160	195	235	275	330	395	430	470	510	600
18								180	210	245	290	350	380	420	455	535
20									200	220	265	315	345	375	410	480
22											240	285	310	340	370	435
24												265	285	315	340	400
26												260	280	300	320	370
28																345
30																340
32																
34																
36																
38																
40	38	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340

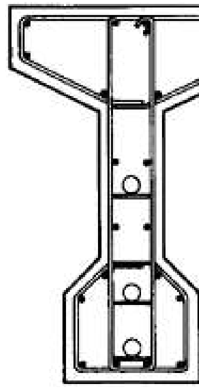

Figure 9.12 Reinforcement cage in a post-tensioned beam.

equal increments. Initially, only a limited number of the tendons required for the second loading stage are tensioned, the remainder being tensioned after the first stage imposed load has been applied. If each tendon has a value of P_o of 600 kN, determine the total number of tendons required, based on the stresses at midspan, and how many of them can be tensioned during the first stage. Assume grade C40/50 concrete with grade C30/37 achieved at transfer.

9.2 For the beam in Problem 9.1, determine the economical maximum total imposed load which can be applied in two stages, and the corresponding number of tendons to be used.

REFERENCE

Beeby, A.W. and Narayanan, R.S. (1995) *Designer's Handbook to Eurocode 2, Part 1.1: Design of Concrete Structures,* Thomas Telford, London.

10

Composite construction

10.1 INTRODUCTION

Many applications of prestressed concrete involve the combination of precast prestressed concrete beams and *in situ* reinforced concrete slabs. Some examples of such composite construction are shown in Fig. 10.1. An *in situ* infill between precast beams is shown in Fig. 10.1(a) while an *in situ* topping is shown in Fig. 10.1(b). The former type of construction is often used in bridges, while the latter is common in building construction. The beams are designed to act alone under their own weight plus the weight of the wet concrete of the slab. Once the concrete in the slab has hardened and provided that there is adequate horizontal shear connection between them, the slab and beam behave as a composite section under design load. The beams act as permanent formwork for the slab, which provides the compression flange of the composite section. The section size of the beam can thus be kept to a minimum, since a compression flange is only required at the soffit at transfer. This leads to the use of inverted T-, or 'top-hat', sections.

10.2 SERVICEABILITY LIMIT STATE

The stress distributions in the various regions of the composite member are shown in Fig. 10.2(a)–(d). The stress distribution in Fig. 10.2(a) is due to the self weight of the beam, with the maximum compressive stress at the lower extreme fibre. Once the slab is in place, the stress distribution in the beam is modified to that shown in Fig. 10.2(b), where the bending moment at the section, M_d is that due to the combined self weight of the beam and slab.

Once the concrete in the slab has hardened and the imposed load acts on the composite section, the additional stress distribution is shown in Fig. 10.2(c). This is determined by ordinary bending theory, but using the composite section properties. The final stress distribution is shown

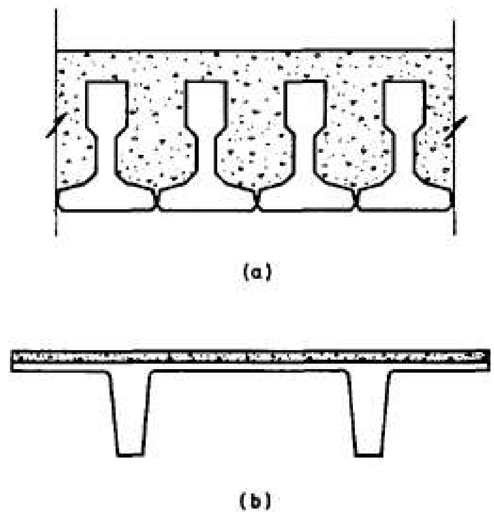

Figure 10.1 Examples of composite construction: (a) *in situ* infill; (b) *in situ* topping.

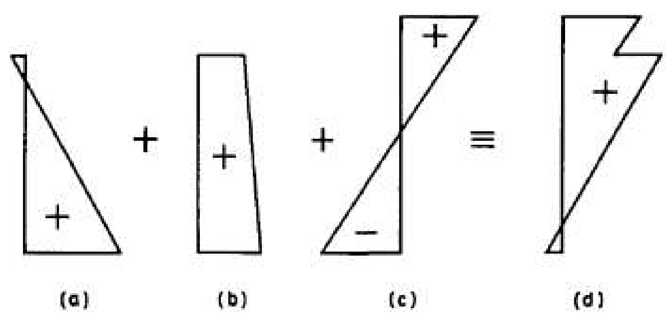

Figure 10.2 Stress distribution within a composite section.

in Fig. 10.2(d) and is a superposition of those shown in Fig. 10.2(b) and Fig. 10.2(c). The important feature to note is that there is a discontinuity in the final stress distribution under design load at the junction between the beam and slab. The beam has an initial stress distribution before it behaves as part of the composite section, whereas the slab only has stresses induced in it due to the composite action.

Example 10.1 ■■

The floor slab shown in Fig. 10.3 comprises precast pretensioned beams and an *in situ* concrete slab. If the span of the beams is 5 m and the imposed load is 5 kN/m^2 (including finishes), determine the stress

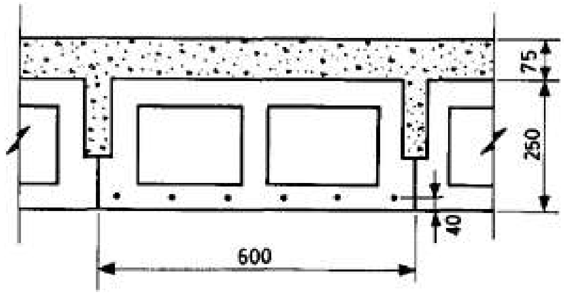

Figure 10.3

distributions at the various load stages. Assume all long-term losses have occurred before the beams are erected and that the net force in each wire is 19.4 kN.

Section properties of the beams:

A_c=1.13×10^5 mm^2
I_c=7.5×10^8 mm^4
Z_t=Z_b=6×10^6 mm^3.

Eccentricity of the wires=125−40=85 mm.

(i) Self weight of the beams=0.113×24
=2.7 kN/m.
M_o=(2.7×5^2)/8
=8.4 kNm.

Total prestress force after all losses have occurred is given by

βP_o=6×19.4
=116.4 kN.

The stress distribution in the beams is thus given by

$$\sigma_t = \frac{116.4 \times 10^3}{1.13 \times 10^5} - \frac{116.4 \times 85 \times 10^3}{6 \times 10^6} + \frac{8.4 \times 10^6}{6 \times 10^6}$$
$$= 1.03 - 1.65 + 1.40$$
$$= 0.78 \text{ N/mm}^2$$

$$\sigma_b = 1.03 + 1.65 - 1.40$$
$$= 1.28 \text{ N/mm}^2.$$

(ii) The weight of the slab is supported by the beams acting alone, so that

M_d=8.4+0.075×0.6×24×5^2/8
=11.8 kNm.

The stress distribution within the beams is now given by

$$\sigma_t = 1.03 - 1.65 + \frac{11.8 \times 10^6}{6 \times 10^6}$$
$$= -0.62 + 1.97$$
$$= 1.35 \text{ N/mm}^2$$

$$\sigma_b = 1.03 + 1.65 - 1.97$$
$$= 0.71 \text{ N/mm}^2.$$

(iii) The imposed load of 5 kN/m^2 is supported by the composite section and the section properties of this are now required. To find the neutral axis of the composite section, taking moments about the soffit of the beams gives

$(1.13\times10^5+75\times600)y=(1.13\times10^5\times125+75\times600\times288)$
$\therefore y=171$ mm.
$I_{comp}=7.5\times10^8+1.13\times10^5\ (171-125)^2$
$+(75^3\times600)/12+(75\times600)/(288-171)^2$
$=1.63\times10^9$ mm^4.

The imposed load bending moment, $(M_{des}-M_d)=0.6\times5\times5^2/8$
$=9.4$ kNm.

The stress distribution within the composite section under this extra bending moment is given by

$$\sigma_{t,slab} = \frac{9.4 \times 10^6}{1.63 \times 10^9} \times (325 - 171) = 0.89 \text{ N/mm}^2$$

$$\sigma_{t,beam} = \frac{9.4 \times 10^6}{1.63 \times 10^9} \times (250 - 171) = 0.46 \text{ N/mm}^2$$

$$\sigma_{b,beam} = \frac{-9.4 \times 10^6}{1.63 \times 10^9} \times 171 = -0.99 \text{ N/mm}^2.$$

The total stress distributions under the three load cases are shown in Fig. 10.4.
■■

The maximum compressive stress occurs at the upper fibres of the beams, but is significantly lower than the level of stress had the beam carried the total imposed load alone. This explains the advantage of inverted T-sections in composite construction, where only a small compression flange is required for bending moments M_o and M_d, the

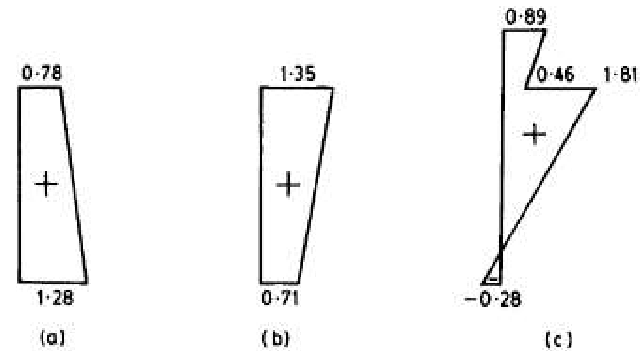

Figure 10.4 Stress distribution for composite section in Example 10.1 (N/mm^2): (a) beam; (b) beam and slab; (c) beam and slab and imposed load.

compression flange for bending moment M_{des} being provided by the slab.

The maximum compressive stress in the slab is much lower than in the beam and, for this reason, in many composite structures a lower grade of concrete is used for the *in situ* portion. The modulus of elasticity for this concrete is lower than that for the beam and this effect can be taken into account in finding the composite section properties by using an approximate modular ratio of 0.8.

The *in situ* slab in Example 10.1 lies above the composite section neutral axis and, therefore, the slab is in compression over its full depth under the total design load. However, for composite sections as shown in Fig. 10.1(a) the *in situ* portion of the section extends well below the neutral axis, so that the lower region is in tension. If the tensile strength of this concrete is exceeded then the composite section properties must be determined on the basis of the *in situ* section having cracked below the neutral axis.

10.3 ULTIMATE STRENGTH

The basic principles for the analysis of prestressed concrete sections at the ultimate limit state of flexural strength described in Chapter 5 are also applicable to composite sections. For the section shown in Fig. 10.5(a), it may be assumed initially that, at the ultimate limit state, the neutral axis lies within the slab and the section may then be treated effectively as a rectangular beam. The position of the neutral axis should later be checked to see whether it does, indeed, fall within the slab.

For the section shown in Fig. 10.5(b), the position of the neutral axis may be determined on the assumption that the section is rectangular,

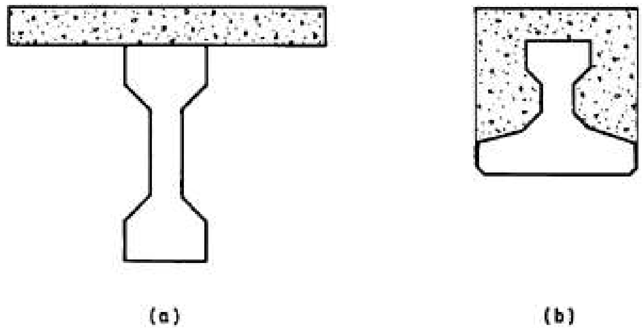

Figure 10.5

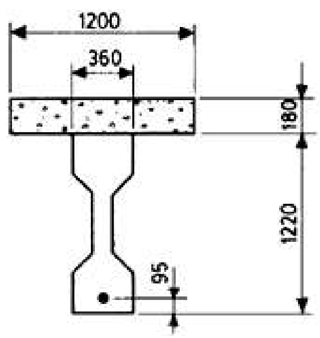

Figure 10.6 Composite section in Example 10.2.

but the different strengths of the concrete in the slab and beam regions of the compression zone should be taken into account.

Example 10.2 ■■

Determine the ultimate moment of resistance of the composite section shown in Fig. 10.6, if the concrete grades in the slab and beam are C25/ 30 and C50/60, respectively, f_{pk}=1820 N/mm^2, *and* A_p=2640 mm^2. Long-term losses are 25%.

The strain and stress distributions are shown in Fig. 10.7.

Initially it is assumed that the tendons have yielded and that the concrete stress block is a distance *y* below the top fibre of the beam.

For equilibrium:

$$0.78\times1820\times2640=0.57\ (25\times180\times1200+50\times360\ y)$$

$\therefore$ y=65 mm.

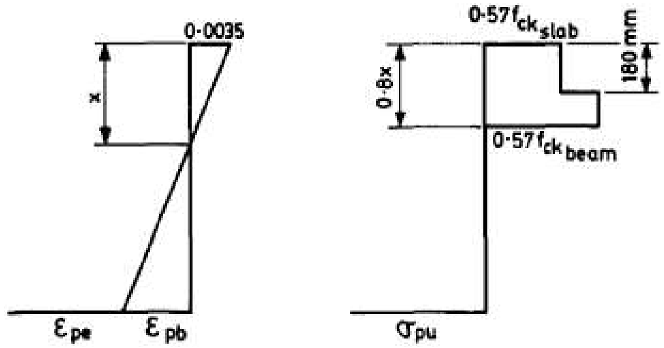

Figure 10.7 Ultimate strain and stress distributions for beam in Example 10.2.

The depth of the neutral axis is thus given by

$0.8\,x=(180+65)$
$\therefore\ x=306$ mm.

It must now be checked whether the steel has, indeed, yielded:

$$\varepsilon_{pe} = \frac{0.7\times 0.75\times 1820}{200\times 10^3} = 0.00478.$$

Thus:

$$\varepsilon_p = 0.00478 + \frac{(1305-306)}{306}\times 0.0035$$
$$= 0.0162 \quad (\geqslant \varepsilon_{pk} = 0.0017).$$

The ultimate moment of resistance is then given by

$M_u=[0.57\,(25\times1200\times180\times155+50\times360\times65^2/2)$
$+0.78\times1820\times2640\,(1305-245)]\times10^{-6}$
$=4471.4$ kNm.

If necessary, the effect of additional untensioned reinforcement can be taken into account, as described in Chapter 5. The ultimate strength of the precast beam supporting its own weight plus that of the slab should also be checked.

■■

10.4 HORIZONTAL SHEAR

The composite behaviour of the precast beam and *in situ* slab is only effective if the horizontal shear stresses at the interface between the two regions can be resisted. For shallow members, such as that shown in Fig. 10.3, there is usually no mechanical key between the two types of concrete, and reliance is made on the friction developed between the contact surfaces. For deeper sections, mechanical shear connectors in the form of links projecting from the beam are used, which provide a much better shear connection.

The determination of the horizontal shear resistance is based on the ultimate limit state, and if this condition is satisfied it may be assumed that satisfactory horizontal shear resistance is provided at the serviceability limit state.

A simply supported composite section carrying a uniformly distributed load is shown in Fig. 10.8(a) and the free-body diagram for half the length of the *in situ* slab is shown in Fig. 10.8(b). At the simply supported end there must be zero force in the slab, while the maximum force occurs at the midspan. The distribution of shear forces on the underside of the slab is also shown in Fig. 10.8(b), being zero at midspan and reaching a maximum at the support. This behaviour is similar to that in an elastic beam, where the vertical and horizontal shear stresses increase towards the support for a uniformly distributed load.

The following expression is given in Part 1–3 of EC2 for the horizontal shear stress, τ_{sdj}:

$$\tau_{sdj}=\beta V_{sd}/(z\, b_j),$$

where β is the ratio of the longitudinal force in the slab to the total longitudinal force, given by M_{sd}/z, both calculated for a given section; V_{sd} is the transverse ultimate shear force; z is the lever arm; and b_j is the width of the interface.

The design shear resistance for horizontal joints with vertical shear reinforcement is given by

$$\tau_{Rdj}=k_T\tau_{Rd}+\mu\sigma_N+0.87\, f_{yk}\varrho\mu\leq 0.33\ \mathrm{v}f_{ck},$$

where k_T is a coefficient from Table 10.1, with k_T=0 if the joint is subjected to tension; τ_{Rd} is the basic design shear strength from Table

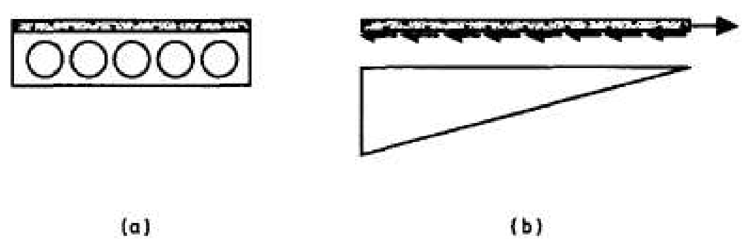

Figure 10.8 Horizontal shear: (a) composite section; (b) free-body diagram for *in situ* slab.

Table 10.1 Values for coefficients k_T and μ

Type of surface	k_T	μ
Rough	1.8	0.7
Smooth	1.4	0.6
Very smooth	0	0.5

7.1, based on the strength of the beam or slab, whichever is the lower; μ is the coefficient of shear friction from Table 10.1; σ_N is the stress per unit area arising from external normal forces across the joint, with a maximum value of 0.4 f_{ck}; v is an efficiency factor, defined in Section 7.2; and ϱ is the ratio of the shear reinforcement to the area of the joint, A_{sv}/A_j.

Thus the amount of vertical shear reinforcement required is given by

$$0.87 f_{yk}\varrho\mu=\tau_{sdj}-(k_T\tau_{Rd}+\mu\sigma_N).$$

Example 10.3 ■■

The composite section shown in Example 10.2 supports a uniformly distributed ultimate load of 60 kN/m over a span of 24 m. Determine the horizontal shear reinforcement required.

Maximum ultimate shear force and bending moments are given by

$V_{sd}=60\times24/2=720$ kN
$M_{sd}=60\times24^2/8=4320$ kNm.

From Example 10.2

$d=1400–95=1305$ mm
$z=1305–0.4\times306=1183$ mm
$M_{sd}/z=4320/1.183=3651.7$ kN.

The total force in the slab

$=0.57\times25\times1200\times180\times10^{-3}$
$=3078$ kN
$\therefore \beta=3078/3651.7=0.84$

and

$$\tau_{sdj}=\frac{0.84\times720.0\times10^3}{360\times1183}=1.42\text{ N/mm}^2.$$

From Table 10.1, for a rough surface, $k_T=1.8$ and $\mu=0.7$. From Table

7.1, τ_{Rd}=0.3 N/mm^2. The value of σ_N may conservatively be taken as zero.
Thus, for no shear reinforcement:

$$\tau_{sdj}=1.8\times0.3=0.54 \text{ N/mm}^2$$

and shear reinforcement is required. The amount is given by

$$0.87\times460\times0.7\varrho=(1.42-0.54)$$
$$\therefore \varrho=0.00314$$
$$\therefore A_{sv}=0.00314\times360\times10^3=1130 \text{ mm}^2/\text{m}.$$

Thus the maximum shear reinforcement to be provided is T12 links at 200 mm centres, with A_{sv}=1130 mm^2/m. This can be reduced away from the supports, in steps according to the shear force diagram.

Alternative arrangements for the anchorage of the links in the slab are shown in Fig. 10.9(a) and (b).

■■

For composite sections of the type shown in Fig. 10.1(a) the horizontal shear stress between the slab and the top of the beam may be determined using the method described above. However, it is not necessary to check the shear stresses down the sides of the beam and along its lower flange, since these will generally be satisfactory if adequate provision has been made for horizontal shear resistance at the top of the beam. Further information on horizontal shear resistance may be found in FIP (1982).

10.5 VERTICAL SHEAR

As with the flexural strength of composite sections, the vertical shear resistance must be checked at two stages: firstly for the beam carrying

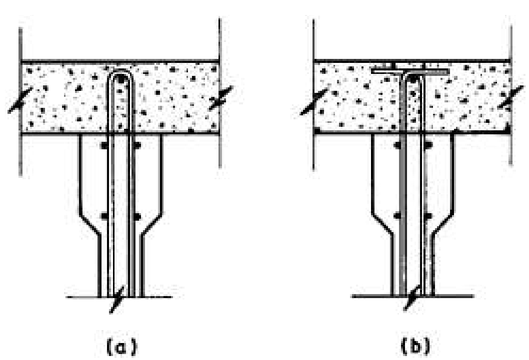

Figure 10.9 Horizontal shear connection alternatives.

the weight of the slab, and secondly for the composite section under the total design load. For the first stage, the shear resistance may be determined using the method described in Chapter 7. The same method may be used for the composite section, allowance being made for the increased effective depth.

10.6 DEFLECTIONS

The deflections of composite prestressed concrete members may be found using the methods described in Chapter 6, depending on whether the member is cracked or uncracked. However, as with the determination of stresses, account must be taken of the different types of section for each loading stage.

Example 10.4 ■■

For the composite section in Example 10.2 determine the maximum deflections at the various load stages. Assume that the tendon has a parabolic profile with eccentricity of 375 mm at midspan and zero at the supports. The self weight of the beam is 8.7 kN/m and the quasi-permanent imposed load is 18 kN/m.

(i) The bending moment distributions under the beam self weight, prestress force and under a central point load are shown in Fig. 10.10(a), (b) and (c), respectively. Using the method of virtual work described in Chapter 6:

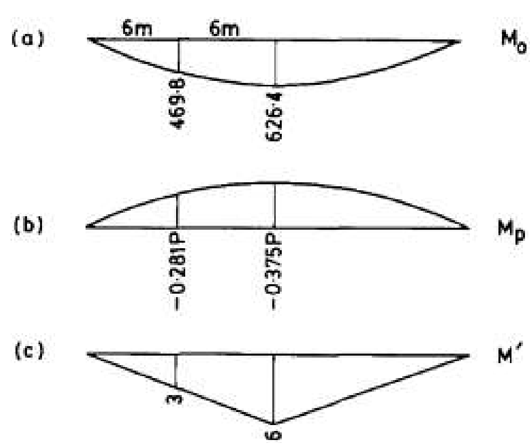

Figure 10.10 Bending moment diagrams for composite section in Example 10.4 (kNm): (a) beam self weight; (b) prestress force; (c) central point load.

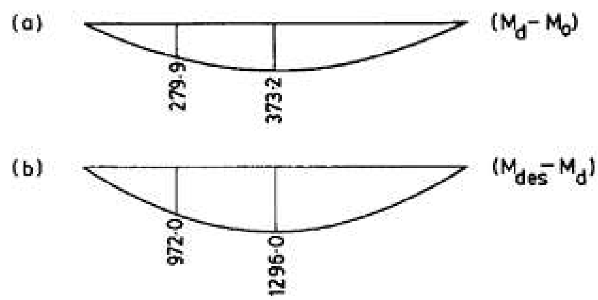

Figure 10.11 Bending moment diagrams for composite section in Example 10.4 (kNm): (a) weight of slab; (b) imposed load.

$$\delta_o = \int_0^L [M'(M_o + M_p)/EI]dx$$
$$= (2 \times 12/6EI)\ [4(469.8 - 0.281P)(3) + (626.4 - 0.375P)(6)]$$

With $P=\alpha P_o$=3000 kN, I_{beam}=0.059 m^4 and E_{cm} at transfer=33.5 ×10^3 N/mm^2,
δ_o=−0.0151 m.

(ii) The bending moment distribution for the weight of the slab is shown in Fig. 10.11(a):

$$\delta_d - \delta_o = \int_0^L [M'(M_d - M_o)/EI]dx$$
$$= (2 \times 12/6EI)\ [4(279.9)(3) + (373.2)(6)]$$
$$= 0.0113 \text{ m.}$$
$$\therefore \delta_d = 0.0113 - 0.0152 = -0.0039 \text{ m.}$$

(iii) The bending moment distribution for the imposed load is shown in Fig. 10.11(b):

$$\delta_{qp} - \delta_d = \int_0^L [M'(M_{qp} - M_d)/EI]dx$$
$$= (2 \times 12/6EI)\ [4(972)(3) + (1296)(6)].$$

For the composite section, I_{comp}=0.151 m^4 and the value of E_{cm} at the age of loading is 37×10^3 N/mm^2.

∴ $\delta_{qp}-\delta_d$=0.0139 m.

Also, the prestress force has been reduced to βP_o=2500 kN and the above value of E_{cm} should now be used for the deflections under both self weight and weight of the slab. Thus,

δ_{qp}=0.0139–0.0086+0.0113×33.5/37
=0.0155 m, or 15.5 mm.

A load-deflection curve for the composite section is shown in Fig. 10.12. This clearly shows the stiffening effect produced by the composite action of the slab and beam. Long-term deflections can be determined using an effective value of E_{cm}, as described in Chapter 6.

■■

10.7 DIFFERENTIAL MOVEMENTS

The fact that the slab of a composite member is usually cast at a much later stage than the beam means that most of the time-dependent effects of shrinkage of the slab take place with the section acting compositely. Most of the shrinkage of the beam will already have occurred by the time the slab is in place, and the movement due to the shrinkage of the slab will induce stresses throughout the whole of the composite section. The water content of the slab concrete is often higher than that of the beam, since a lower strength is required, and this aggravates the problem of differential shrinkage. These extra stresses, which occur even under zero applied load, are not insignificant and should be considered in design.

Both the slab and beam undergo creep deformations under load and, although some of the creep deformations in the beam may have taken place before casting of the slab, the level of compressive stress is higher in the beam, and so the creep deformations are larger. The differential creep deformations between the slab and beam set up stresses in the

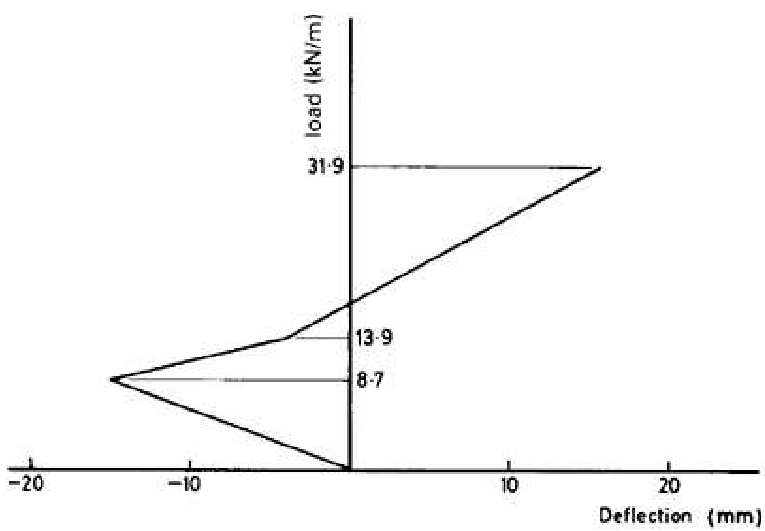

Figure 10.12 Load-deflection curve for composite section in Example 10.4.

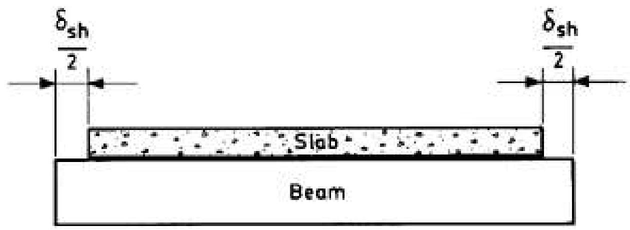

Figure 10.13 Differential movements.

composite section which tend to reduce those set up by differential shrinkage.

A problem which is encountered, particularly in connection with bridge decks, is that of varying temperature across a composite section, although this may still be a problem in composite members used as roof structures. The hotter upper surface tends to expand more than the cooler lower surface and stresses are induced throughout the composite section.

A method for determining the stresses due to differential shrinkage will now be outlined, and this can be adapted to find the stresses due to differential creep and temperature movements.

Consider a composite member as shown in Fig. 10.13, where the slab is shown to have a free shrinkage movement of δ_{sh} relative to the beam. In reality this movement is restrained by the shear forces which are set up between the slab and beam, putting the slab into tension and the beam into compression. The magnitude of the tensile force in the slab is given by

$$T=\varepsilon_{sh}A_{c,slab}E_{c,slab},$$

where $A_{c,slab}$ and $E_{c,slab}$ are the cross-sectional area and modulus of elasticity of the slab, respectively, and ε_{sh} is the free shrinkage strain of the slab concrete. The compressive force in the beam must be numerically equal to this tensile force.

In addition to the direct stresses described above, bending stresses are also introduced by restraint of the free differential shrinkage. In order to determine these stresses, the free bodies of the slab and beam are considered, as shown in Fig. 10.14. Initially, the slab can be regarded as having a force T applied through its centroid, so that its length is equal to that of the beam. There must be no net external force on the composite member due to differential shrinkage alone, so a pair of equal and opposite compressive forces must be applied to maintain equilibrium. However, these compressive forces act on the composite section and induce a bending moment at the ends of the member of

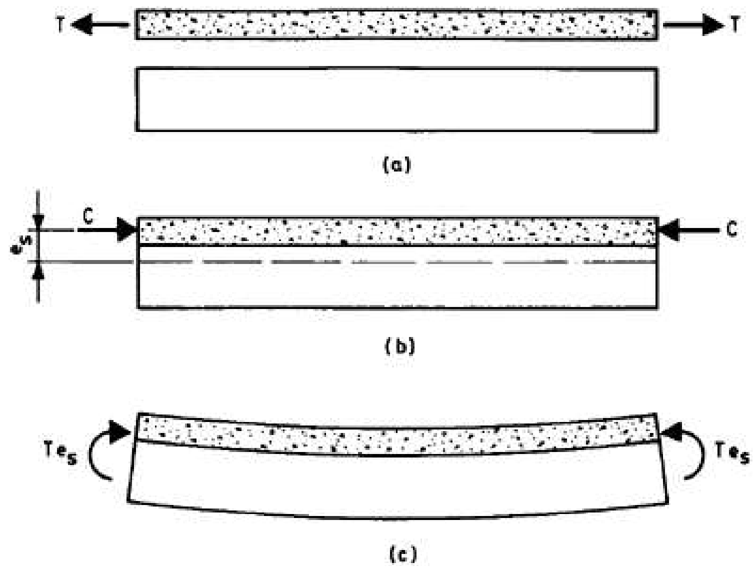

Figure 10.14 Internal stress resultants due to differential movements.

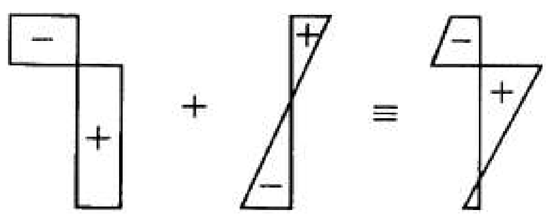

Figure 10.15 Stresses due to differential movements.

magnitude Te_s, where e_s is the distance between the centroids of the slab and composite sections. The total stress distribution across the section is shown in Fig. 10.15.

Example 10.5 ■■

For the composite section in Example 10.2 determine the stress distribution across the section if the slab undergoes a shrinkage strain of 100×10^{-6}. Assume $E_{c,slab}=30.5\times10^3$ N/mm^2, $I_{comp}=1.51\times10^{11}$ mm^4 and $A_{c,beam}=3.69\times10^5$ mm^2.

For the slab:

$A_{c,slab}=1200\times180=2.16\times10^5$ mm^2

$\therefore\ T=100\times10^{-6}\times30.5\times10^3\times2.16\times10^5\times10^{-3}$

$=658.8$ kN.

$$\text{Thus the average stress in the slab} = -\frac{658.8 \times 10^3}{2.16 \times 10^5}$$
$$= -3.05 \text{ N/mm}^2.$$

$$\text{Thus the average stress in the beam} = \frac{658.8 \times 10^3}{5.85 \times 10^5}$$
$$= 1.13 \text{ N/mm}^2.$$

The centroid of the composite section can be shown to lie 606 mm from the top of the slab. Thus the eccentricity of the slab centroid about the centroid of the composite section is (606−90)=516 mm, and the moment about this centroid is

=658.8×0.516
=339.9 kNm.

Thus the bending stresses at the top of the slab, at the junction between slab and beam, and at the soffit of the beam are, respectively:

$$\sigma_{t,slab} = \frac{339.9 \times 10^6 \times 606}{1.51 \times 10^{11}}$$
$$= 1.36 \text{ N/mm}^2$$
$$\sigma_{b,slab} = \sigma_{t,beam} = \frac{339.9 \times 10^6 \times 426}{1.51 \times 10^{11}}$$
$$= 0.96 \text{ N/mm}^2;$$
$$\sigma_{b,beam} = -\frac{339.9 \times 10^6 \times 794}{1.51 \times 10^{11}}$$
$$= -1.79 \text{ N/mm}^2.$$

The resulting stress distribution is shown in Fig. 10.16. These stresses must be added to those due to the prestress force and applied load.

■■

The temperature distributions in real concrete structures are non-linear across the section. A full description of the analysis method for this case is given in Hambly (1991).

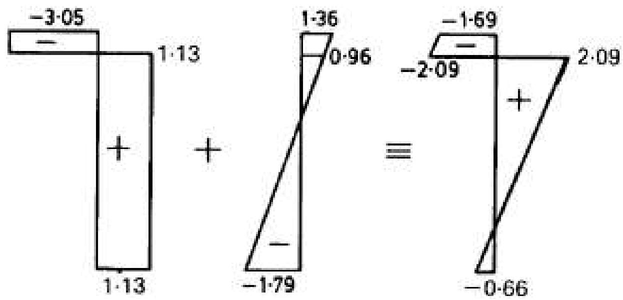

Figure 10.16 Stress distribution for composite section in Example 10.5 (N/mm^2).

10.8 PROPPING AND CONTINUITY

All the composite sections considered so far have comprised beams which are simply supported. This is particularly useful where continued access is required beneath the structure throughout construction, such as where a bridge passes over a busy road or railway. However, if it is possible to provide a temporary intermediate support to the beam during construction, a considerable saving may be made, since the loading condition for the beam carrying the weight of the slab concrete has a large influence on its design.

A beam with a temporary central support is shown in Fig. 10.17. Initially the beam supports its own weight, with distribution of bending moments as shown in Fig. 10.18(a). Once the temporary support is in place and the slab concrete poured, the extra bending moments are as in Fig. 10.18(b). The stresses in the beam in these two cases are found using the beam section alone. When the concrete has hardened sufficiently, the temporary support is removed and the beam stresses are found for the beam self weight and slab load acting on the simply supported composite section. Finally, the stresses induced by the imposed load bending moments, Fig. 10.18(c), acting on the composite section must be added. The final stresses in the beam will be less than if the beam had been unpropped, but the hogging bending moment induced in the beam when it is supporting the weight of the slab must be taken into consideration.

Clearly, the final stresses in the beam could be reduced further by introducing more temporary props at intermediate points in the span. The limiting case of this is to have the beam continuously propped. In this case, many of the advantages of using composite construction would be lost, but a good compromise may well be to have two or three intermediate supports, depending on the access required beneath the structure during construction.

Another extension of the basic form of composite construction is to

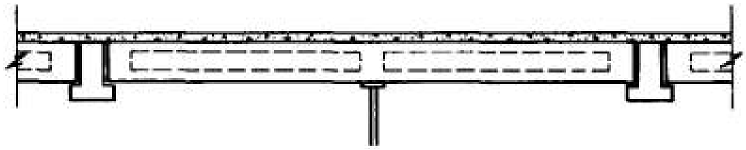

Figure 10.17 Propping.

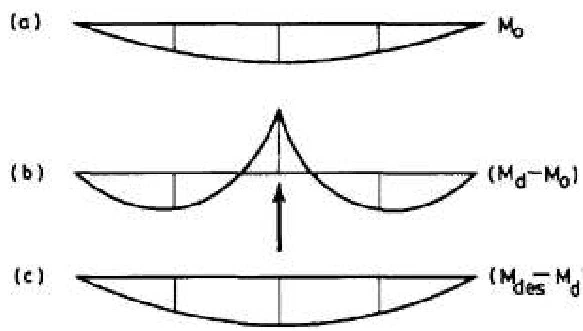

Figure 10.18 Effects of propping.

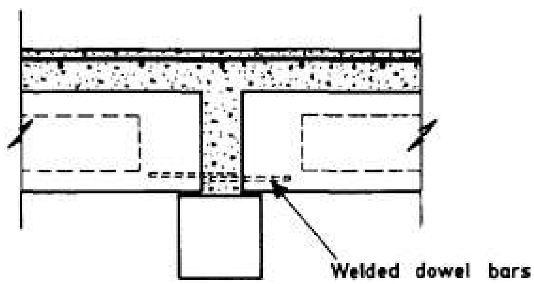

Figure 10.19 Continuity.

join adjacent simply supported spans so that, under imposed load, they behave as a continuous structure. Two such simply supported sections are shown in Fig. 10.19, which support their own weight and that of the slab concrete. The slab extends across the top of the supports and is reinforced so that it can resist the tensile stresses which are set up there once the whole structure behaves continuously under the imposed load. The design of this region of the continuous member should be carried out as for a reinforced concrete member.

The continuous behaviour of the composite structure shown in Fig. 10.19 under imposed load will induce tensile stresses in the top of the prestressed concrete beams adjacent to the supports. These stresses should be limited to those in Table 2.1. In determining the ultimate strength of continuous members such as those shown in Fig. 10.19, the support sections should be considered as reinforced concrete sections. In the regions just beyond the bearing, the precompression in the concrete may be ignored over the transmission length of the tendons.

Another consideration in the use of continuous composite constructions is the secondary bending moments set up at the supports due to creep and shrinkage in the adjacent spans. Long-term creep effects due to prestressing cause an upward camber in the spans which induces a sagging bending moment at the support. Differential shrinkage and long-term creep effects due to the vertical load on the spans cause a downward deflection in the adjacent spans, inducing a hogging bending moment at the supports. The overall effect is usually a net sagging bending moment, requiring reinforcement at the bottom of the support section, as shown in Fig. 10.19. Further information on the assessment of these secondary bending moments may be found in Clark (1978).

10.9 DESIGN OF COMPOSITE MEMBERS

The same considerations that were applied to the design of a prestressed concrete member in Chapter 9 may be applied when the member acts compositely with an *in situ* slab. However, there are now, in general, eight inequalities, since the stresses due to the dead load of the beam and slab, and the stress in the *in situ* concrete must also be considered. The stress in the last case should be limited to 0.6 f_{ck}. It is generally found that, of these eight stress conditions, the two most critical for determining the required prestress force and eccentricity are the upper fibre stress at transfer and the lower fibre stress under the total design load.

The minimum composite section size can be based on the stress conditions at the bottom fibre. When the beam is supporting its own weight, an inequality similar to 9.2(b) may be written:

$$\frac{\alpha P_o}{A_{c,beam}} + \frac{\alpha P_o}{Z_{b,beam}} - \frac{M_o}{Z_{b,beam}} \leq f'_{max},$$

where $A_{c,beam}$ and $Z_{b,beam}$ are the section properties of the beam. When the total design load is acting, the effect of the bending moment ($M_{ra}-M_d$) is found by using the composite section properties:

$$\frac{\beta P_o}{A_{c,beam}} + \frac{\beta P_o}{Z_{b,beam}} - \frac{M_d}{Z_{b,beam}} - \frac{(M_{ra} - M_d)}{Z_{b,comp}} \geq f_{min},$$

where $Z_{b,comp}$ is the lower fibre section modulus for the composite section. Combining the above inequalities gives

$$Z_{b,comp} \geqslant \frac{\alpha(M_{ra} - M_d)}{(\beta f'_{max} - \alpha f_{min}) + (1/Z_{b,beam})(\beta M_o - \alpha M_d)}.$$

The range for the prestress force required may be found for a given eccentricity from

$$P_o \geqslant \frac{(Z_{t,beam} f'_{min} - M_o)}{\alpha[(Z_{t,beam}/A_{c,beam}) - e]}$$

$$P_o \geqslant Z_{b,beam} \frac{\{f_{min} + (M_{ra}/Z_{b,comp}) + M_d[(1/Z_{b,beam}) - (1/Z_{b,comp})]\}}{\beta[(Z_{b,beam}/A_{c,beam}) + e]}.$$

Note that if the denominator in the first expression is negative, the inequality is reversed. Once the prestress force has been chosen, the limits to the eccentricity may be found from

$$e \leqslant \frac{Z_{t,beam}}{A_{c,beam}} + \frac{1}{\alpha P_o}(M_o - Z_{t,beam} f'_{min})$$

$$e \geqslant \frac{1}{\beta P_o}\left[Z_{b,beam} f_{min} + M_d\left(1 - \frac{Z_{b,beam}}{Z_{b,comp}}\right) + \frac{Z_{b,beam}}{Z_{b,comp}} M_{ra}\right] - \frac{Z_{b,beam}}{A_{c,beam}}.$$

Further information on the design of composite structures may be found in Bate and Bennett (1976).

PROBLEMS

10.1 Beams similar to that in Problem 9.1 are placed at 1.2 m centres and, with a 180 mm deep reinforced concrete slab, form a composite bridge deck spanning 24 m. If the weight of finishes is 2 kN/m^2 and the total imposed load is 10 kN/m^2, determine the minimum prestress force required.

10.2 For the bridge deck in Problem 10.1 determine the ultimate moment of resistance if f_{pk}=1860 N/mm^2, A_p=1840 mm^2 and the slab is of grade C25/30 concrete.

10.3 A precast hollow-core floor slab, 1200 mm wide and 150 mm deep, with a 50 mm concrete topping, is to span 6 m. The section properties are: Z_b=Z_t=4.38×10^6 mm^3, A_c=1.53×10^5 mm^2. If the long-term prestress force is 115 kN, and the eccentricity is 40 mm, determine the stresses at the bottom of the precast slab at midspan for the following situations:

(i) precast slab;
(ii) precast slab and topping;
(iii) precast slab and topping, with slab propped at midspan;
(iv) propping removed and an imposed load of 3 kN/m^2 applied.

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Hambly, E.C. (1991) *Bridge Deck Behaviour,* Chapman & Hall, London.

11

Indeterminate structures

11.1 INTRODUCTION

All of the prestressed concrete members so far considered have been statically determinate. This reflects the major use of prestressed concrete in building structures, since the most common type of prestressed concrete construction is in the form of simply supported beams. However, there are important applications of prestressed concrete in statically indeterminate structures. Many of the features of the analysis and design of these structures are similar to those used for statically determinate structures, as outlined in previous chapters. There are two important differences, however: the introduction of secondary moments and the behaviour at the ultimate limit state. These will be discussed in the following sections.

The most important application of prestressed concrete indeterminate structures is in the field of multi-span bridges. This is a specialized area of design and construction and is well beyond the scope of this book, but many excellent reference books on the subject may be found in the Bibliography.

In the field of building structures, continuous prestressed concrete beams are sometimes employed, but a more widespread use is in prestressed concrete flat slabs. The design of these will be discussed in detail in Chapter 12.

11.2 SECONDARY MOMENTS

It was shown in Chapter 1 that for a statically determinate prestressed concrete member the line of pressure in the concrete is coincident with the resultant force due to the prestressing tendons, provided that there is no applied axial load on the member. For statically indeterminate prestressed concrete structures, this is not necessarily the case. The prestress moment in a statically determinate member at any section is