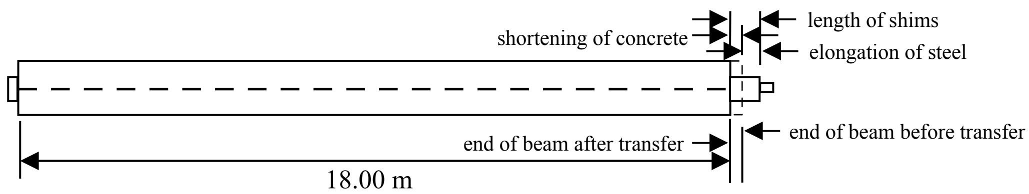
1. A Prescon cable, 18.00 m long is to be tensioned from one end to an initial prestressed of 1040 MPa immediately after transfer. Assume that there is no slack in the cable, that the shrinkage of concrete is 0.0002 at the time of transfer, and that the average compression in concrete is 5.50 MPa along the length of tendon. E_c = 26.2 GPa; E_s = 200 GPa. Compute the length of shims required, neglecting any elastic shortening of the shims and any friction along the tendon. Ans: 100.98mm

Fig. 1



Solution:

Elasttic elongation of steel:

$$\Delta_s = \frac{f_s L}{E_s} = \frac{1040\,(18x10^3)}{200\,x10^3} = 93.6mm$$

Shortening of concrete due to shrinkage:

$$\Delta_{c\ shrinkage} = 0.0002(18x10^3) = 3.6mm$$

Elastic shortening of concrete:

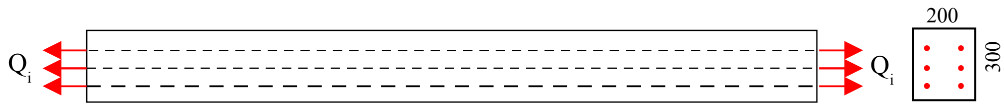
$$\delta_D = \frac{ML^2}{8EI} = \frac{(18.093)(10^2)x10^{12}}{8(27.5x10^3)(2.278x10^9)} = 3.61\,mm$$

Length of shims required:

$$\Delta_T = \Delta_{s\ elastic} + \Delta_{c\ shrinkage} + \Delta_{c\ elastic} = 93.6 + 3.6 + 3.78 = 100.98mm$$

2. A pretensioned member has a section shown 200mmx300mm. It is concentrically prestressed with 516mm^2 of high tensile steel wire which is anchored to the bulkheads of a unit stress of 1040 MPa. Assuming n = 6, compute the stresses in the concrete and steel immediately after transfer. Ans: f_c = 8.575 MPa; f_y = 988.55 MPa

Fig. 2



Solution:

Exact Method

$$f_c = \frac{Q_o}{A_c + (n-1)A_s} = \frac{516x1040}{(200x300) + (6-1)516} = 8.575MPa$$

$$nf_c = (6)8.575 = 51.45\,MPa$$

Stress in steel after transfer

$$f_s = f_{so} - nf_c = 1040 - 51.45 = 988.55\, MPa$$

Approximate Method

The loss of prestress in steel due to elastic shortening of concrete is approximated by:

$$f_s = n\frac{Q_o}{A_g} = (6)\frac{516x1040}{200x300} = 53.664\, MPa$$

Stress in steel after loss

$$f_s = f_{so} - nf_c = 1040 - 53.664 = 986.335\, MPa$$

Stress in concrete is:

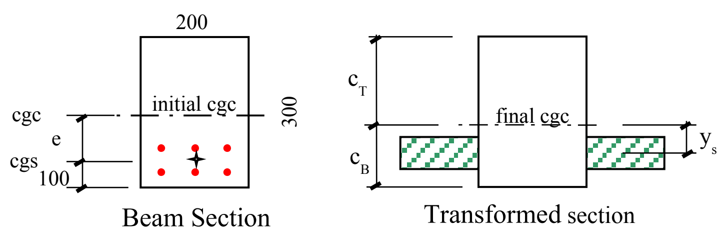
$$f_c = \frac{net\ stress\ of\ steel\ x\ A_s}{A_g} = \frac{Q_{net}}{A_g} = \frac{986.335x516}{(200x300)} = 8.482\, MPa$$

Approximations introduced:

1. using gross area instead of net area
2. using initial stress in steel instead of the reduced stress

3. A pretensioned member has a section 200mmx300mm. It is eccentrically prestressed with 516mm² of high tensile steel wire which is anchored to the bulkheads at a unit stress of 1040 MPa. The c.g.s. is 100mm above the bottom fiber. Assuming n = 6, compute the stresses in the concrete immediately after transfer. Ans: f_T = 0.00 MPa; f_B = +16.918 MPa

Fig. 3



Solution

Exact Method

$$(n-1)A_s = (6-1)(516) = 2580\ mm^2$$

$$A_g = 200\,x300 = 60000\ mm^2$$

$$e = (300/2) - 100 = 50\, mm$$

Summing up moment at initial cgc:

$$A_T y_o = A_1 y_1 + A_2 y_2$$

$$y_o = \frac{A_1 y_1 + A_2 y_2}{A_T} = \frac{(200x300)(0) + 2580(50)}{60000 + 2580} = 2.06\ mm$$

$$c_B = (300/2 - y_o) = 147.94\ mm$$

$$c_T = (300/2 + y_o) = 152.06\ mm$$

$$e = c_B - 100 = 47.94\ mm$$

Compute transformed section moment of inertia:

$$I_T = \frac{1}{12}bh^3 + A_g (y_o)^2 + (n-1)A_s y_s$$

$$= \frac{200x300^3}{12} + 60000(2.06)^2 + 2580(47.94)^2 = 4.562x10^8\ mm^4$$

Fiber stresses:

$$f = \frac{Q_i}{A_T} \pm \frac{Q_i ey}{I_T} = \frac{516x1040}{60000 + 2580} \pm \frac{(516x1040)(47.94)y}{4.562x10^8}$$

$$= 8.575 \pm 0.056393\ y$$

Top fiber stress:

$$f_T = 8.575 - 0.056393(152.06) = 0.00\ MPa$$

Bottom fiber stress:

$$f_B = 8.575 + 0.056393(147.94) = 16.918\ MPa$$

Approximate Method

Loss of prestress:

$$f_{sL} = \frac{nQ_i}{A_g} = \frac{6(516x1040)}{60000} = 53.664\ MPa$$

Net prestress:

$$f_{sn} = f_{si} - f_{sL} = 1040 - 53.664 = 986.336\ MPa$$

$$Q_{net} = f_{sn} A_s = 986.336(516x10^{-3} = 508.949\ kN$$

Fiber stresses:

$$f = \frac{Q_{net}}{A_g} \pm \frac{Q_{net}ey}{I_c}$$

$$= \frac{508.949x10^3}{60000} \pm \frac{508.949x10^3(50)y}{\frac{200(300)^3}{12}}$$

$$= 8.48248 \pm 0.0565498\ y$$

Top fiber stress:

$$f_T = 8.48248 - 0.0565498(150) = 0.00\ MPa$$

Bottom fiber stress:

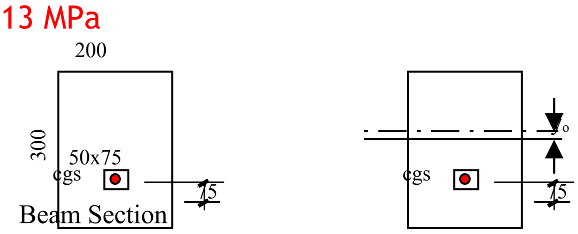
$$f_B = 8.48248 + 0.0565498(150) = 16.964\ MPa$$

Approximation introduce:

1. using approximate values of reduced prestressed
2. using the gross area of concrete

4. A post-tensioned beam has a mid span cross-section with a duct of 50mm x 75mm to house the wires. It is pretensioned with 516mm2 of steel to an initial stress of 1040 MPa. Immediately after transfer, the stress is reduced by 5% owing to anchorage loss and elastic shortening of concrete. Compute the stresses in the concrete at transfer. Ans: f_T = 4.829 MPa, f_B = +23.913 MPa

Fig. 4



Solution

Method 1: Using net section of concrete

$$A_c = A_g - A_{duct} = 200x300 - 50x75 = 56250\ mm^2$$

Locate the cg of net section:

$$y_o = \frac{A_{duct}(75)}{A_{net}} = \frac{(50x75)(75)}{56250} = 5.00\ mm$$

$$y_s = 75 + y_o = 75 + 5 = 80\ mm$$

$$c_T = 150 - y_o = 150 - 5 = 145\ mm$$

$$c_B = 150 + y_o = 150 + 5 = 155\ mm$$

Compute the moment of inertia of net section:

$$I = \frac{bh^3}{12} + bh(y_o)^2 - \frac{b'h'^3}{12} - b'h'(80)^2$$

$$= \frac{200x300^3}{12} + 60000(5)^2 - \frac{50x75^3}{12} - 3750(80)^2 = 4.527x10^8\ mm^4$$

Total prestress in steel:

$$Q = \eta(A_s f_s) = 95\%(516x1040)x10^{-3} = 509.808\ kN$$

Fiber stresses:

$$f = \frac{Q}{A_c} \pm \frac{(Qe)y}{I} = \frac{509.808x10^3}{56250} \pm \frac{509.808x10^3(80)}{4.257x10^8}y$$

$$= 9.063 \pm 0.095806\ y$$

Top fiber stress:

$$f_T = 9.063 - 0.095806(145) = -4.828\ MPa$$

Bottom fiber stress:

$$f_B = 9.063 + 0.095806(155) = 23.913\ MPa$$

Method 2: Using gross section of concrete

$$f = \frac{Q}{A_g} \pm \frac{Qec}{I} = \frac{509.808x10^3}{200x300} \pm \frac{509.808x10^3(75)(150)}{\frac{1}{12}(200x300^3)}$$

$$= 8.4968 \pm 12.7452$$

Top fiber stress:

$$f_T = -4.2484\ MPa$$

Bottom fiber stress:

$$f_B = 21.242\ MPa$$

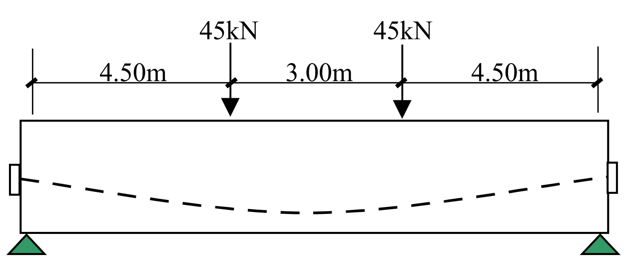
If eccentricity does not occur along one of the principal axes of the section, it is necessary to further resolved the moment into two components along the two principal axes.

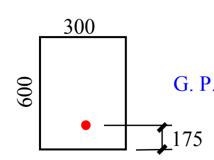
$$f = \frac{Q}{A} \pm \frac{Qe_x y}{I_x} \pm \frac{Qe_y x}{I_y}$$

5. A post-tensioned bonded concrete beam has a prestress of 1560 kN in the steel immediately after prestressing which eventually reduces to 1330 kN. The beam carries two live loads of 45 kN each in addition to its own weight of 4.40 kN/m. Compute the extreme fiber stresses at mid-span:
   a) under the initial condition with full prestress and no live load
   b) under final condition after all the losses have taken place and with full live load.

   Ans: Initial condition: f_T = 2.234 MPa, f_B = 15.10 MPa; Final condition: f_T = 13.803 MPa, f_B = 0.975 MPa

Fig. 5





Mid span section

G. P. Ancog

Solution

To be theoretically exact, net concrete section should be used up to the time of grouting, after which the transformed section should be considered.

Section Properties:

$$A_g = bh = 300x600 = 180000 \ mm^2$$

$$I_g = \frac{1}{12}bh^3 = \frac{1}{12}(300x600^3) = 5.4x10^9 \ mm^4$$

Initial condition

$$M = \frac{wL^2}{8} = \frac{4.4x12^2}{8} = 79.2 \ kN-m$$

$$f = \frac{Q_o}{A_g} \pm \frac{Q_o ey}{I_g} \pm \frac{My}{I_g} = \frac{1560 \ x10^3}{180000} \pm \frac{1560 \ x10^3(125)(300)}{5.4x10^9} \pm \frac{79.2x10^6(300)}{5.4x10^9}$$
$$= 8.667 \pm 10.833 \pm 4.40$$

Top fiber stress:

$$f_T = 8.667 - 10.833 + 4.4 = 2.234 \ MPa$$

Bottom fiber stress:

$$f_B = 8.667 + 10.833 - 4.4 = 15.10 \ MPa$$

Final condition

Live load moment at mid-span:

$$M_L = Pa = 45(4.5) = 202.5 \ kN-m$$

Dead load moment at mid-span:

$$M_D = \frac{wL^2}{8} = \frac{4.4(12^2)}{8} = 79.2 \ kN-m$$

Total moment: MT = 79.20 + 202.5 = 281.7 kN-m

Stresses:

$$f = \frac{Q}{A_g} \pm \frac{Qey}{I_g} \pm \frac{M_T y}{I_g}$$

$$f = \frac{1330 \ x10^3}{180 \ x10^3} \pm \frac{1330 \ x10^3(125)(300)}{5.4x10^9} \pm \frac{281.7x10^6(300)}{5.4x10^9}$$
$$= 7.389 \pm 9.236 \pm 15.65$$

Top fiber stress:

$$f_T = 7.389 - 9.236 + 15.65 = 13.803 \ MPa$$

Bottom fiber stress:

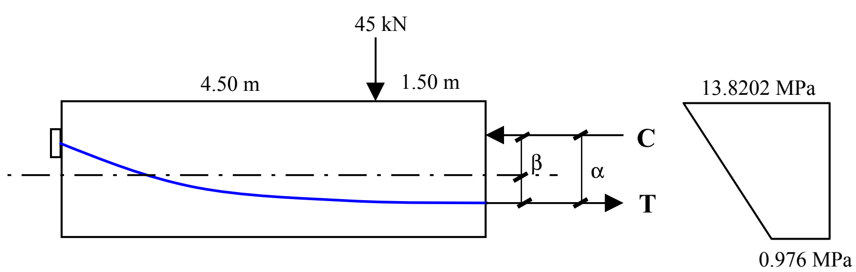
$$f_B = 7.298 + 9.236 - 15.65 = 0.975 \ MPa$$

Note:

For pre-tensioned beam, steel is always bonded to the concrete before any external moment is applied. Values of A, y and I should be computed on the basis of transformed section. For approximation, gross area of concrete can be used in calculation.

For post-tensioned and bonded beams, for any load applied after the bonding has taken place, transformed section should be used.

For post-tensioned unbonded beams, the net concrete section is the proper one for all stress calculation.



C = T, M = Cα = Tα

$$\alpha = \frac{M}{C} = \frac{M}{T} = \frac{281.7x10^6}{1330x10^3} = 211.8 \ mm$$

$$\beta = \alpha - e = 211.8 - 125 = 86.8 \ mm$$

Stresses:

$$f = \frac{C}{A} \pm \frac{C\beta y}{I}$$

$$= \frac{1330x10^3}{180x10^3} \pm \frac{1330x10^3(86.8)(300)}{5.4x10^9}$$

$$= 7.389 \pm 6.413$$

Top fiber stress:

$$f_T = 7.389 + 6.413 = 13.802 \ MPa$$

Bottom fiber stress:

$$f_B = 7.389 - 6.413 = 0.976 \ MPa$$

Computation of average strain for unbonded beams:

$$\varepsilon = \frac{f}{E} = \frac{My}{E_c I}$$

$$\Delta = \int \frac{My}{E_c I} dx$$

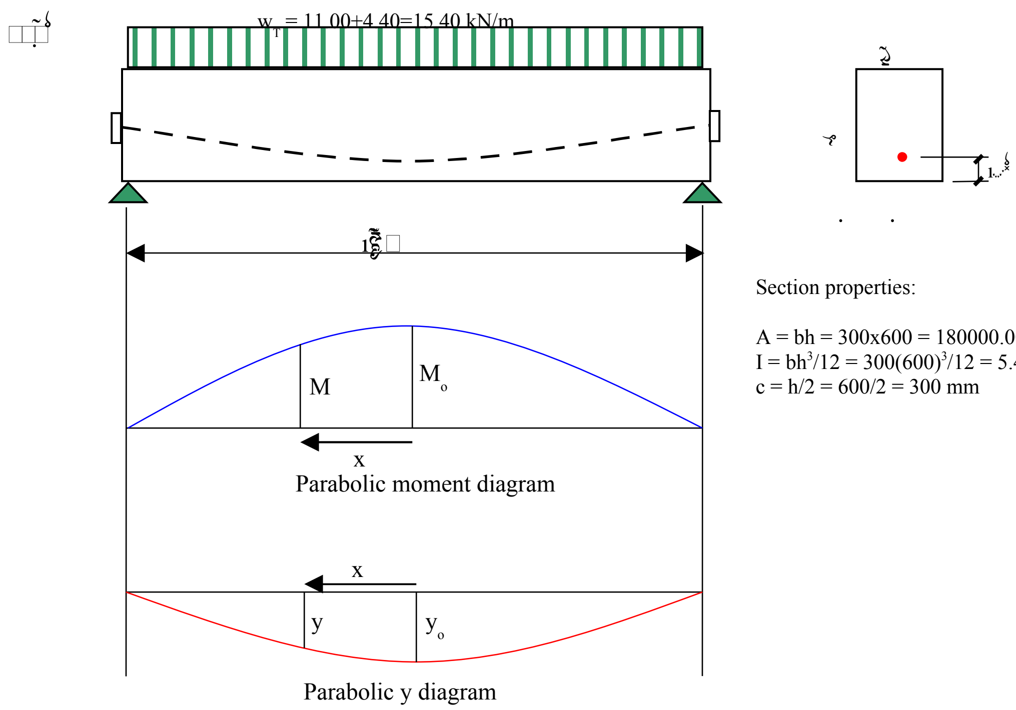
$$\varepsilon_{ave} = \frac{1}{L}\int \frac{My}{E_c I} dx$$

Average stress in steel is:

$$f_s = E_s \varepsilon_{ave} = \int \frac{MyE_s}{LE_c I} dx = \frac{n}{L}\int \frac{My}{I} dx$$

6. A post-tensioned simple beam on a span of 12 m carries a superimposed load of 11.00 kN/m in addition to its own weight of 4.40 kN/m. The initial prestress in the steel is 950 MPa, reducing to 830 MPa after deducting all loses and assuming no bending of the beam. The parabolic cable has an area of 1612.9mm2, n = 6. Compute the stresses in the steel at mid-span assuming:
   a) the steel is bonded by grouting
   b) the steel is unbonded and entirely free to slip.

   Ans: Bonded condition: f_s = 845.258 MPa, Unbonded condition: f_s = 838.137 MPa



Section properties:

A = bh = 300x600 = 180000.00 mm^2
I = bh^3/12 = 300(600)^3/12 = 5.4x109 mm^4
c = h/2 = 600/2 = 300 mm

Solution 1:

Moment at mid-span:

$$M_o = \frac{wL^2}{8} = \frac{15.4(12)^2}{8} = 277.2 kN - m$$

Moment at mid-span due to prestress:

$$M_s = Qe = (1612.9x830)(125)x10^{-6} = 167.34\ kN - m$$

Net moment at mid-span:

M_N = 277.2 – 167.34 = 109.86 kN-m

Stress in concrete at the level of steel due to bending:

Using I for gross section

$$f_c = \frac{My}{I} = \frac{109.86x10^6(125)}{5.4x10^9} = 2.543\ MPa$$

The stress in steel is increased by:

$$f_s = nf_c = 6(2.543) = 15.258\ MPa$$

Resultant stress in steel:

f_{sf} = 830 + 15.258 = 845.258 MPa

Solution 2: If the cable is unbonded and free to slip.

$$f_s = \frac{n}{L}\int \frac{My}{I}dx$$

$$M = M_o\left[1-\left(\frac{x}{L/2}\right)^2\right]$$

$$y = y_o\left[1-\left(\frac{x}{L/2}\right)^2\right]$$

$$f_s = \frac{n}{LI}\int_{-L/2}^{L/2} M_o y_o\left[1-\left(\frac{x}{L/2}\right)^2\right]^2 dx$$

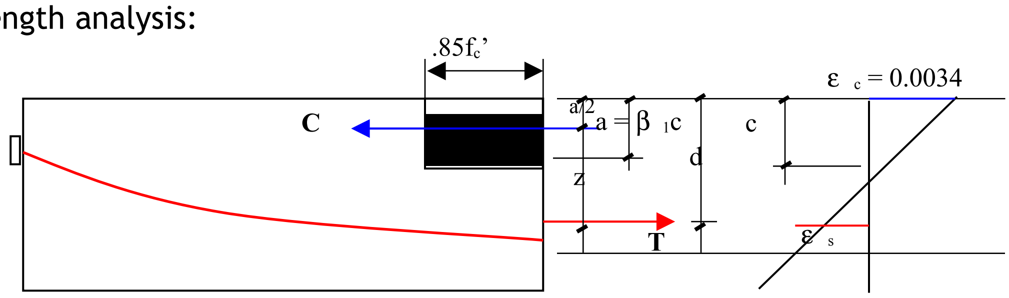
$$= \frac{nM_o y_o}{LI}\left[x - \frac{2}{3}\frac{x^3}{(L/2)^2} + \frac{x^5}{5(L/2)^2}\right]_{-L/2}^{L/2}$$

$$= \frac{8}{15}\frac{nM_o y_o}{I} = \frac{8}{15}(15.258) = 8.137\ MPa$$

Resultant stress in steel:

f_{sf} = 830 + 8.137 = 838.137 MPa

Ultimate strength analysis:



$$C = .85 f_c' ba = T = A_s f_s'$$

$$a = \frac{C}{.85 f_c' b} = \frac{A_s f_s'}{.85 f_c' b}$$

$$z = d - a/2$$

$$M = A_s f_s'(d - a/2)$$

7. A rectangular section 300mm x 600mm deep is prestressed with 937.5 mm^2 of steel wires for an initial stress of 1040 MPa. The cgs of the wires is 100mm above the bottom fiber. For the tendons, f_s' = 1650 MPa, f_c' = 34.4 MPa. Determine the ultimate resisting moment. Ans: M_u = 637.05 kN-m

Solution

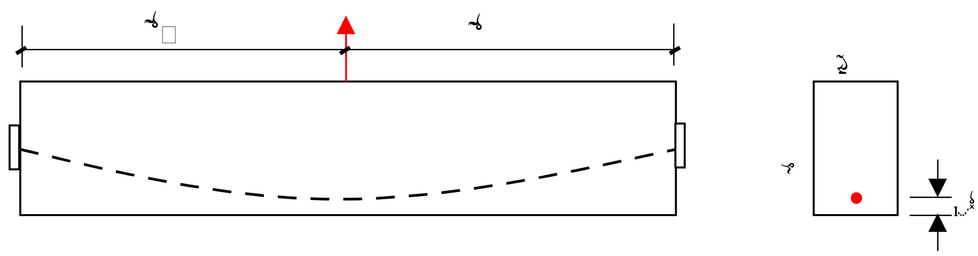
Total tension of steel at rupture

$$T = 937.5(1650)x10^{-3} = 1546.875 \ kN$$
$$C = T$$
$$.85 f_c'ba = T$$
$$a = \frac{T}{.85 f_c'b} = \frac{1546.875x10^3}{.85(34.4)(300)} = 176.34 \ mm$$

Ultimate moment

$$M_u = A_s f_{su}(d - a/2)$$
$$= 1546.875(500 - 176.34/2)x10^{-3} = 637.05 \ kN - m$$

8. A post-tensioned bonded beam with a transfer prestress of F_t = 1560 kN is being wrongly picked up at its mid-span point. Compute the critical fiber stresses. If the top fiber cracks and the concrete is assume to take no tension, compute the bottom fiber stresses. If the beam is picked up suddenly so that an impact factor of 100% is considered compute the maximum stresses. Ans: Case 1: f_T = -6.566 MPa, f_B = 23.90 MPa; Case 2: f_B = 27.905 MPa; Case 3: f_B = 35.39 MPa



Solution

Section properties:

$$A = 300x600 = 180x10^3 \ mm^2$$
$$I = \frac{1}{12}(300)(600)^3 = 5.4x10^9 \ mm^4$$
$$c = \frac{600}{2} = 300 \ mm$$

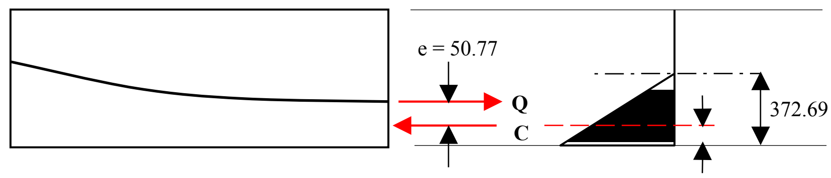
External moment at pick-up point

$$M = -\frac{wL^2}{2} = -\frac{4.4(6)^2}{2} = -79.2 \ kN - m$$

a) Fiber stress at mid-span

$$f = \frac{Q}{A} \pm \frac{Qey}{I} \pm \frac{My}{I}$$
$$= \frac{1560x10^3}{180x10^3} \pm \frac{1560x10^3(125)300}{5.4x10^9} \pm \frac{79.2x10^6(300)}{5.4x10^9}$$
$$= 8.667 \pm 10.833 \pm 4.4$$
$$f_T = 8.667 - 10.833 - 4.4 = -6.566 \ MPa$$
$$f_B = 8.667 + 10.833 + 4.4 = 23.9 \ MPa$$

b) If the fiber cracks and concrete is assume to take no tension.



Triangular Stress Block

Locate center of pressure, C:

$$M = Qe$$
$$e = \frac{M}{Q} = \frac{79.2x10^6}{1560\ x10^3} = 50.77\ mm$$

From bottom: 175-50.77 = 124.23 mm

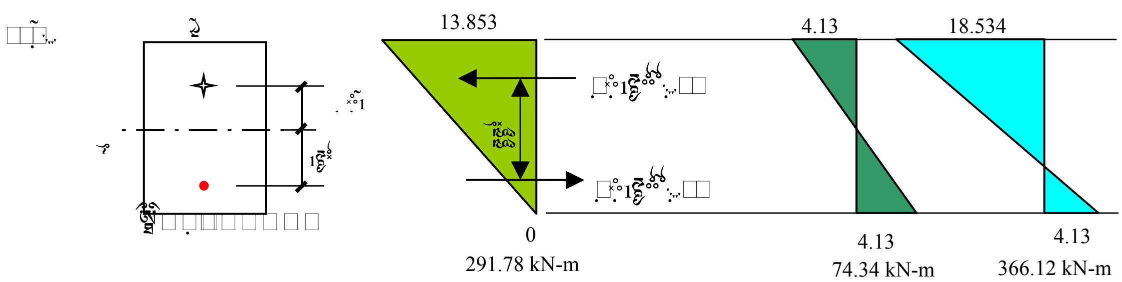
Assuming a triangular stress block, height y:

$$y = 3(124.23) = 372.69\ mm$$
$$T = C = \frac{1}{2} f_c by$$
$$f_c = \frac{2T}{by} = \frac{2(1560\ x10^3)}{300(372.69)} = 27.905\ MPa$$

c) 100% impact factor

$$M_T = M + 100\%M = 2M = 2(79.2) = 158.4\ MPa$$
$$e = \frac{M_T}{Q} = \frac{158.4x10^6}{1560\ x10^3} = 101.538\ mm$$
$$From\ bottom: 175 - 101.54 = 73.46\ mm$$
$$Assuming\ a\ triangular\ stress\ block:$$
$$y = 3(73.46) = 220.38\ mm$$
$$f_c = \frac{2T}{by} = \frac{2(1560\ x10^3)}{300(220.38)} = 47.19\ MPa$$
$$Assuming\ a\ rectangular\ stress\ block:$$
$$y = 2(73.46) = 146.92\ mm$$
$$T = C = f_c by$$
$$f_c = \frac{T}{by} = \frac{1560\ x10^3}{300(146.92)} = 35.39\ MPa$$

9. Determine the total dead and live uniform load moment that can be carried by the beam with a simple span of 12m:1. for zero tensile stress in the bottom fibers. 2. for cracking in the bottom fibers at a modulus of rupture of 4.13 MPa and assuming concrete to take up tension up to that value. Ans: Case 1: w_T = 16.21 kN/m; Case 2: w_T = 20.34 kN/m



Solution

Section properties:

$$A = bh = 300(600) = 180x10^3 \ mm^2$$
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(300)(600)^3 = 5.4x10^9 \ mm^4$$
$$c = \frac{h}{2} = \frac{600}{2} = 300 \ mm$$

Prestress Q:
$$Q = A_s f_s = 1562.5(830)x10^{-3} = 1296.8 \ kN$$

1. Moment for zero tensile stress at the bottom:

$$f_B = \frac{Q}{A} + \frac{Qey}{I} - \frac{My}{I} = 0$$
$$0 = \frac{1296.8x10^3}{180x10^3} + \frac{1296.8x10^3(125)(300)}{5.4x10^9} - \frac{Mx10^6(300)}{5.4x10^9}$$
$$M = 291.78 \ kN-m$$
$$w = \frac{8M}{L^2} = \frac{8(291.78)}{12^2} = 16.21 \ kN/m$$

Top fiber stress:

$$f_T = \frac{Q}{A} - \frac{Qey}{I} + \frac{My}{I}$$
$$= \frac{1296.8x10^3}{180x10^3} - \frac{1296.8x10^3(125)(300)}{5.4x10^9} + \frac{281.78x10^6(300)}{5.4x10^9}$$
$$= 13.853 \ MPa$$

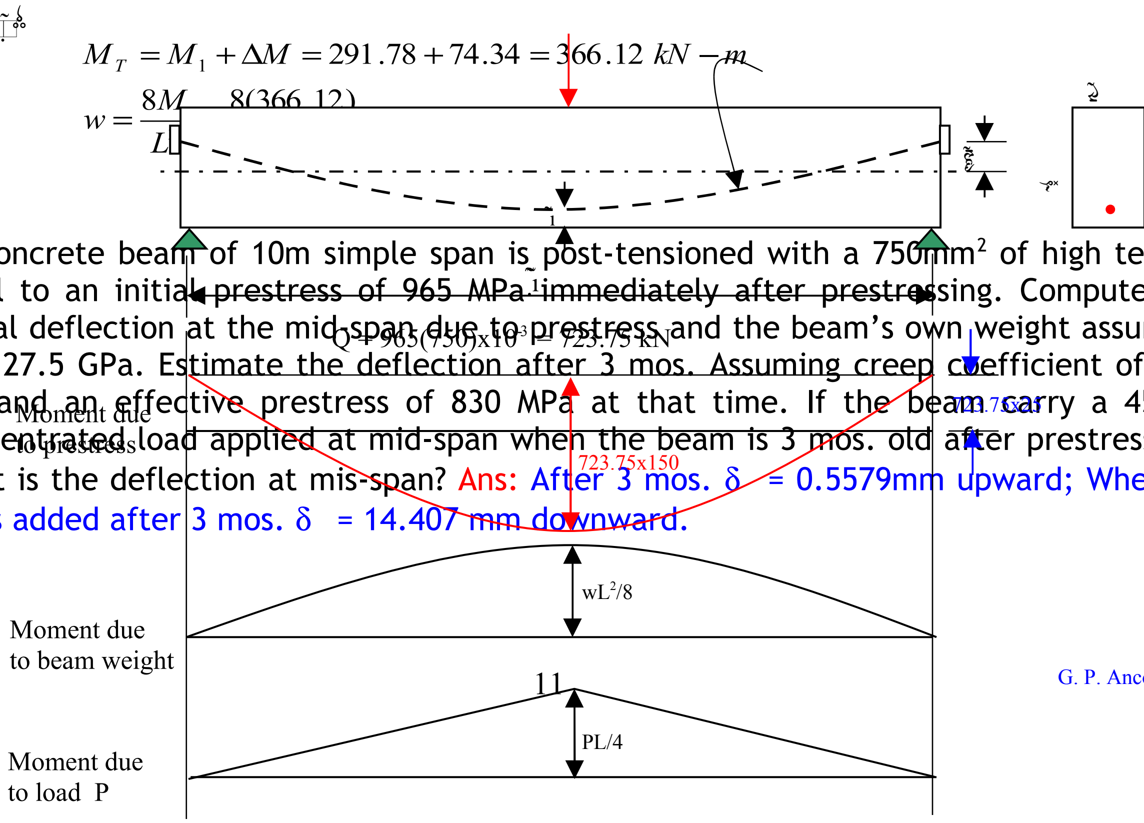
2. For cracking in the bottom fibers.

Additional moment carried by the section up to the beginning of crack.

$$\Delta M = \frac{f'I}{c} = \frac{4.13(5.4x10^9)}{300}x10^{-6} = 74.34 \ kN-m$$

Total moment capacity:

$$M_T = M_1 + \Delta M = 291.78 + 74.34 = 366.12 \ kN-m$$
$$w = \frac{8M}{L^2} = \frac{8(366.12)}{}$$



10. A concrete beam of 10m simple span is post-tensioned with a 750mm² of high tensile steel to an initial prestress of 965 MPa immediately after prestressing. Compute the initial deflection at the mid-span due to prestress and the beam's own weight assuming E_c = 27.5 GPa. Estimate the deflection after 3 mos. Assuming creep coefficient of c_c = 1.8 and an effective prestress of 830 MPa at that time. If the beam carry a 45 kN concentrated load applied at mid-span when the beam is 3 mos. old after prestressing, what is the deflection at mis-span? Ans: After 3 mos. δ = 0.5579mm upward; When 45 kN is added after 3 mos. δ = 14.407 mm downward.

Solution

Section properties:

$$A = bh = 300x450 = 135x10^3 \ mm^2$$
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(300)(450)^3 = 2.278x10^9 \ mm^4$$

The parabolic tendon with 150mm mid-ordinate is replaced by a uniform load acting along the beam.

$$Qh = \frac{w_P L^2}{8}$$
$$w_P = \frac{8Qh}{L^2} = \frac{8(723.75)(150)x10^{-3}}{10^2} = 8.685 \ kN/m$$

Moment due to eccentric load at the end of the beam

$$M = Qe' = 723.75(25)x10^{-3} = 18.093 \ kN-m$$

Dead load uniform load

$$w_D = \gamma A = 23.5(300x450)x10^{-6} = 3.17 \ kN/m$$

Net uniform load:

$$\Delta w = w_Q - w_D = 8.685 - 3.17 = 5.515 \ kN/m$$

Upward deflection at mid-span due to net uniform load:

$$\delta_U = \frac{5(\Delta w)L^4}{384EI} = \frac{5(5.515)(10^4)x10^{12}}{384(27.5x10^3)(2.278x10^9)} = 11.462 \ mm$$

Downward deflection at mid-span due to end moment:

$$\delta_D = \frac{ML^2}{8EI} = \frac{(18.093)(10^2)x10^{12}}{8(27.5x10^3)(2.278x10^9)} = 3.61\ mm$$

Initial deflection due to pretsress and beam weight:

$$\delta_{net} = \delta_U - \delta_D = 11.462 - 3.61 = 7.852\ mm,\ upward$$

Deflection due to prestress alone:

$$\delta_P = \frac{5(w_P)L^4}{384\,EI} - \frac{ML^2}{8EI} = \left[\frac{5(8.685)(10^4)}{384} - \frac{18.093(10^2)}{8}\right]\frac{10^{12}}{(27.5x10^3)(2.278x10^9)} = 14.44\ mm$$

Deflection due to dead load alone:

$$\delta_{DL} = \frac{5w_{DL}L^4}{384\,EI} = \frac{5(3.17)(10^4)x10^{12}}{384(27.5x10^3)(2.278x10^9)} = 6.59\ mm$$

The initial deflection is modified by two factors:

1. loss of prestress
2. creep effect which tend to increase deflection

Deflection after 3 months:

$$\delta_f = \delta_P\left(\frac{f_s}{f_{so}}\right) + \delta_{DL}(c_c) = 14.44\frac{830}{965} - 6.59(1.8) = 0.5579\ mm,\ upward$$

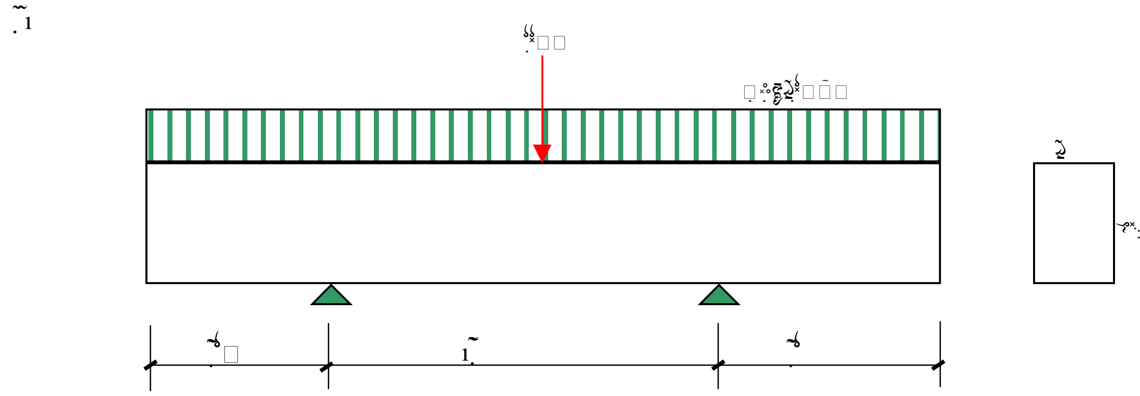
Deflection due to applied concentrated load of 45kN:

$$\delta_{LL} = \frac{PL^3}{48\,EI} = \frac{45(10^3)x10^{12}}{48(27.5x10^3)(2.278x10^9)} = 14.965\ mm,\ downward$$

The resultant deflection:

$$\delta_R = \delta_{LL} + \delta_f = 14.965 - .5579 = 14.407\ mm,\ downward$$

11. A double cantilever beam is to be designed so that its prestress will exactly balance the total uniform load of 23.5 kN/m on the beam. Design the beam using the least amount of prestressed assuming that the cgs must have a concrete protection of 75 mm. If a concentrated load P = 65 kN is applied at the mid-span, compute the maximum top and bottom fiber stresses. Ans: F = 1410 kN; f_T = 14.934 MPa, f_B = -2.40 MPa



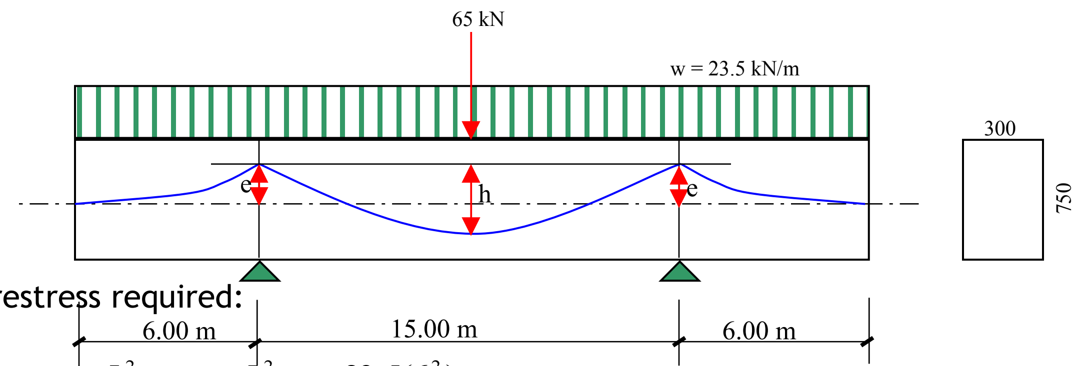
Solution

Section properties:

$$A = bh = 300\,x750 = 225\,x10^3\ mm^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(300)(750)^3 = 1.0546875\ x10^{10}\ mm^4$$

In order to balance the load on the cantilever, the cgs at the tip must coincide with the cgc with a horizontal tangent. To use the least amount of pretsress, the eccentricity over the support should be a maximum. Assume a gross cover of 75mm, e_{max} = 750/2-75 = 300 mm.



The prestress required:

$$Qe = \frac{wL^2}{2};\ Q = \frac{wL^2}{2e} = \frac{23.5(6^2)}{2(300\,x10^{-3})} = 1410\ kN$$

In order to balance the load at the mid-span, using the same prestress Q, the sag of the parabola must be:

$$Qh = \frac{wL^2}{8};\ h = \frac{wL^2}{8Q} = \frac{23.5(15^2)}{8(1410)}x10^{-3} = 468.75\ mm$$

The result will be a concordant cable and under the action of the uniform load and prestress, the beam will have no deflection any where and will only have a uniform compressive stress.

$$f_c = \frac{Q}{A} = \frac{1410\,x10^3}{225\,x10^3} = 6.267\ MPa$$

Due to concentrated load P:

$$M = \frac{PL}{4} = \frac{65(15)}{4} = 243.75\ kN - m$$

The extreme fiber stresses:

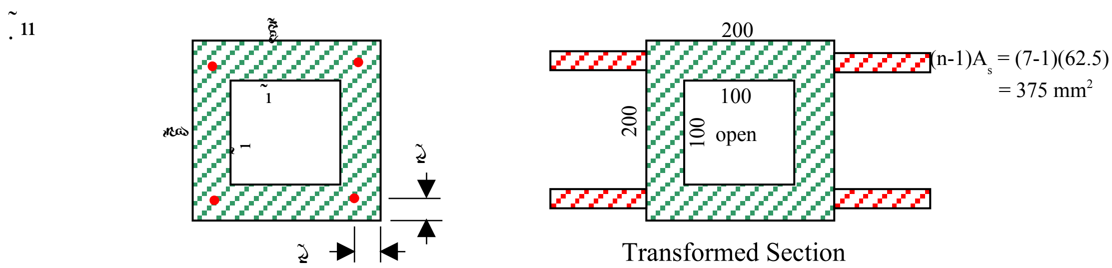
$$f = \frac{Mc}{I} = \frac{243.75\,x10^6(375)}{1.0546875\ x10^{10}} = 8.667\ MPa$$

$$f_T = \frac{Q}{A} + \frac{Mc}{I} = 6.267 + 8.667 = 14.934\ MPa$$

$$f_B = \frac{Q}{A} - \frac{Mc}{I} = 6.267 - 8.667 = -2.4\ MPa$$

12. A hollow member is reinforced with 4 wires of 62.5 mm^2 each pretensioned f_{si} = 1030 MPa. If f_c' = f_{ci} = 34.4 MPa, n = 7, determine the stresses when the wires are cut between members. Determine the moment that can be carried at a maximum tension of 0.5√(f_c') and a maximum of f_c = 0.45f_c'. If 240 MPa of the prestressed is lost (in addition

to the elastic deformation) determine this limiting moment. Ans: when the wires are cut, f_s = 936.98 MPa; Limiting moment, M_T = 9.665 kN-m



Transformed Section

Solution

Transformed section:

$$A_T = 200x200 - 100x100 + 4(n-1)A_s = 31.5x10^3 \ mm^2$$

$$I_T = \frac{1}{12}\left[200^4 - 100^4\right] + 4(n-1)A_s(70^2) = 1.3235\,x10^8 \ mm^4$$

Initial prestressing force, Q_i before transfer:

$$Q_i = A_{st} f_{si} = (4x62.5)(1030)x10^{-3} = 257.5 \ kN$$

$$f_c = \frac{Q_i}{A_T} = \frac{257.5x10^3}{31.5x10^3} = 8.175 \ MPa$$

$$\Delta f_s = nf_c = 7(8.175) = 57.225 \ MPa$$

Net stresses right after transfer (loss due to elastic shortening):

$$f_c = 8.175 \ MPa$$

$$f_{so} = f_{si} - nf_c = 1030 - 57.225 = 972.775 \ MPa$$

Allowable concrete stresses:

$$f_c = 0.45 f_c' = 0.45(34.4) = 15.48 \ MPa$$

$$f_t = 0.5\sqrt{f_c'} = 0.5\sqrt{34.4} = 2.93 \ MPa$$

Total moment : $M_T = M_D + M_L$

Additional concrete stress on top:

$\Delta f_{t_c} = 15.48 - 8.175 = 7.305\,MPa$ compression

Additional concrete stress on botton:

$\Delta f_b = 2.93 + 8.175 = 11.105 \ MPa$ tension

Total moment that can be carried:

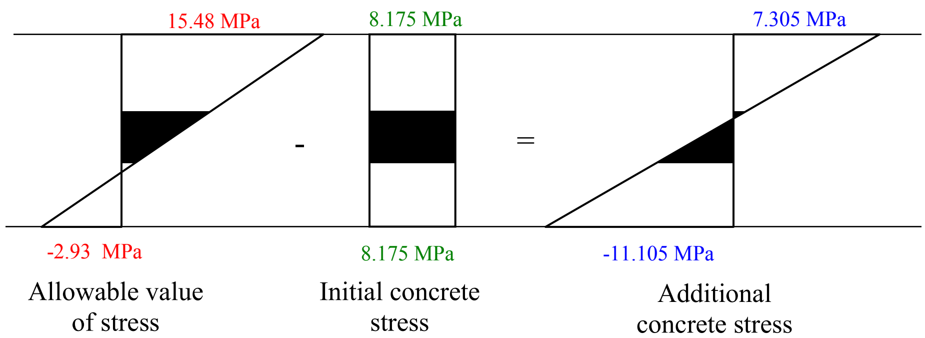
$$f = \frac{M_T c}{I};\ M_T = \frac{\Delta f_t I}{c} = \frac{7.305(1.323\,x10^8)}{100} x10^{-6} = 9.665 \ kN-m$$

Concrete stress on top reach full allowable limit:

$$f_T = f_c = 15.48\ MPa$$ compression

Concrete stress at the bottom:

$$f_B = 8.175 - \Delta f_t = 8.175 - 7.305 = 0.87\ MPa$$ compression



Net stress in steel:

concrete stress at the level of steel

$$f_{sn} = f_{so} \pm n f_{cs} = 972.775 \pm 7\left(\frac{70}{100}\right)7.305 = 972.775 \pm 35.7945.1135$$

Top steel:

$$f_{snT} = 972.775 - 35.7945 = 936.98\ MPa$$

Bottom steel:

$$f_{snB} = 972.775 + 35.7945 = 1008.5695\ MPa$$

After 240 MPa of prestress is lost (in addition to elastic deformation)

$$Q_i = f_{snet} A_{st} = (1030 - 240)(4x62.5)x10^{-3} = 197.5\ kN$$

$$f_c = \frac{Q_i}{A_T} = 8.175\left(\frac{197.5}{257.5}\right) = 6.27\ MPa$$

$$f_{se} = f_{snet} - n f_c = (1030 - 240) - 7(6.27) = 746.11\ MPa$$

Additional concrete stress on top:

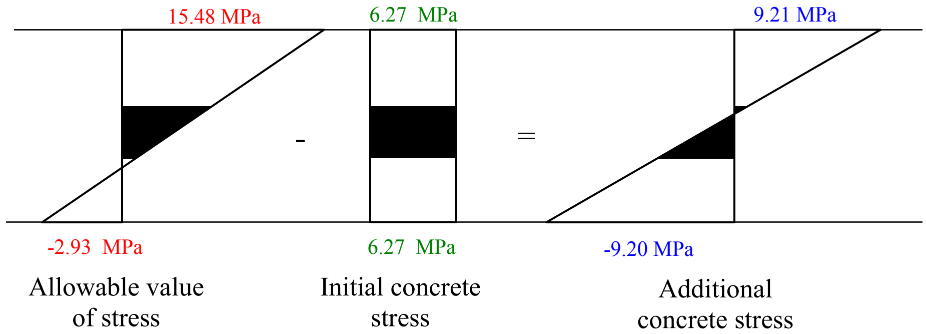
$$\Delta f_{t_c} = 15.48 - 6.27 = 9.21\, MPa$$ compression

Additional concrete stress on botton:

$$\Delta f_b = 2.93 + 6.27 = 9.2\ MPa$$ tension

Total moment that can be carried:

$$f = \frac{M_T c}{I}; M_T = \frac{\Delta f_t I}{c} = \frac{9.2(1.323x10^8)}{100} x10^{-6} = 12.17\ kN - m$$



Therefore the limiting moment:

$$M_T = [9.665, 12.17]_{\min} = 9.665 \ kN - m$$