UNIVERSITY OF ANBAR

# DEPARTMENT OF CIVIL ENGINEERING

## PRESTRESSED CONCRETE


Instructor:

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# Lecture 2 :

## FLEXURAL DESIGN OF PRESTRESSED CONCRETE ELEMENTS

# INTRODUCTION

Flexural stresses are the result of external, or imposed, bending moments. In most cases, they control the selection of the geometrical dimensions of the prestressed concrete section regardless of whether it is pretensioned or post-tensioned.

The design process starts with the choice of a preliminary geometry, and by trial and adjustment it converges to a final section with geometrical details of the concrete cross section and the sizes and alignments of the prestressing strands. The section satisfies the flexural (bending) requirements of concrete stress and steel stress limitations. Thereafter, other factors such as shear and torsion capacity, deflection, and cracking are analyzed and satisfied. While the input data for the analysis of sections differ from the data needed for design, every design is essentially an analysis. One assumes the geometrical properties of the section to be prestressed and then proceeds to determine whether the section can safely carry the prestressing forces and the required external loads. Hence, a good under- standing of the fundamental principles of analysis and the alternatives presented thereby significantly simplifies the task of designing the section. As seen from the discussion in Chapter 1, the basic mechanics of materials, principles of equilibrium of internal couples, and elastic principles of superposition have to be adhered to in all stages of loading. Maryland Concert Center parking garage, Baltimore. (Courtesy, Prestressed Concrete Institute.)

# INTRODUCTION

In view of the preceding, this chapter covers the major aspects of both the service- load flexural design and the ultimate-load flexural design check. The principles and methods presented in Chapter 1 for service load computations are extended into step-by- step procedures for the design of prestressed concrete linear elements, taking into con- sideration the impact of the magnitude of prestress losses discussed in Chapter 3. Note that a logical sequence in the design process entails first the service-load design of the section required in flexure, and then the analysis of the available moment strength M, of the section for the limit state at failure. Throughout the book, a negative sign (-) is used to denote compressive stress and a positive sign (+) is used to denote tensile stress in the concrete section. A convex or hogging shape indicates negative bending moment; a con-cave or sagging shape denotes positive bending moment, as shown in Figure 4.1. Unlike the case of reinforced concrete members, the external dead load and partial live load are applied to the prestressed concrete member at varying concrete strengths at various loading stages.

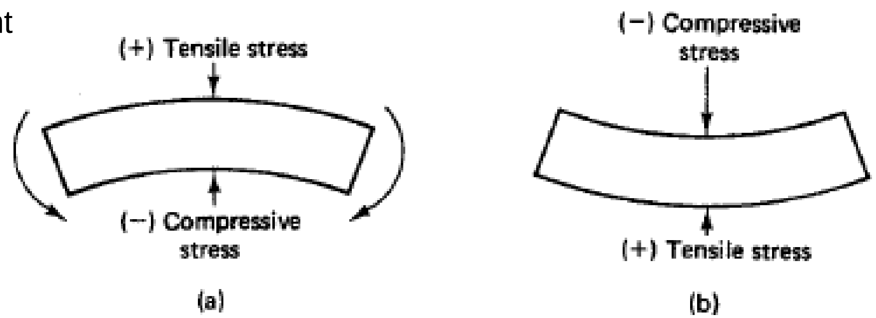
# INTRODUCTION

These loading stages can be summarized as follows:• Initial prestress force P, is applied; then, at transfer, the force is transmitted from the prestressing strands to the concrete. • The full self-weight W, acts on the member together with the initial prestressing force, provided that the member is simply supported, i.e., there is no intermediate support.

• The full superimposed dead load Wsp including topping for composite action, is ap-plied to the member.

Most short-term losses in the prestressing force occur, leading to a reduced pre-stressing force Pe.

• The member is subjected to the full-service load, with long-term losses due to creep, shrinkage, and strand relaxation taking place and leading to a net prestressing force P • Overloading of the member occurs under certain conditions up to the limit state at failure. A typical loading history and corresponding stress distribution across the depth of the critical section are shown in Figure 4.2, while a schematic plot of load versus defor-(-) Compressive stress(+) Tensile stress(-) Compressive stress(+) Tensile stress(a)(b) Figure 4.1 Sign convention for flexure stress and bending moment. (a) Negative bending moment. (b) Positive bending moment




**Sign convention for flexure stress and bending moment. (a) Negative bending moment. (b) Positive bending moment.**

The prestressing force P that satisfies the particular conditions of geometry and loading of a given element (see Figure 1.2) is determined from the principles of mechanics and of stress-strain relationships. Sometimes simplification is necessary, as when a prestressed beam is assumed to be homogeneous and elastic.

Consider, then, a simply supported rectangular beam subjected to a concentric pre- stressing force P as shown in Figure 1.2(a). The compressive stress on the beam cross section is uniform and has an intensity.

$$f = \frac{-P}{A_c} \quad \text{.............................................(1).}$$

where Ac= bh is the cross-sectional area of a beam section of width b and total depth h. A minus sign is used for compression and a plus sign for tension throughout the text. Also, bending moments are drawn on the tensile side of the member. If external transverse loads are applied to the beam, causing a maximum moment M at midspan, the resulting stress becomes

$$f^t = \frac{-P}{A} - \frac{M_C}{I_g} \quad \text{..................(2)}$$

And

$$f_b = \frac{-P}{A} + \frac{M_C}{I_g} \text{..........................................(3)}$$

where $f^t$ = stress at the top fibers

$f_b$ = stress at the bottom fibers.

C=$\frac{1}{2}$h for rectangular section

$I_g$= gross moment of inertia of the section ( $bh^3/12$)

Equation 3 indicates that the presence of prestressing-compressive stress - P/A is reducing the tensile flexural stress Mc/I to the extent intended in the design, either

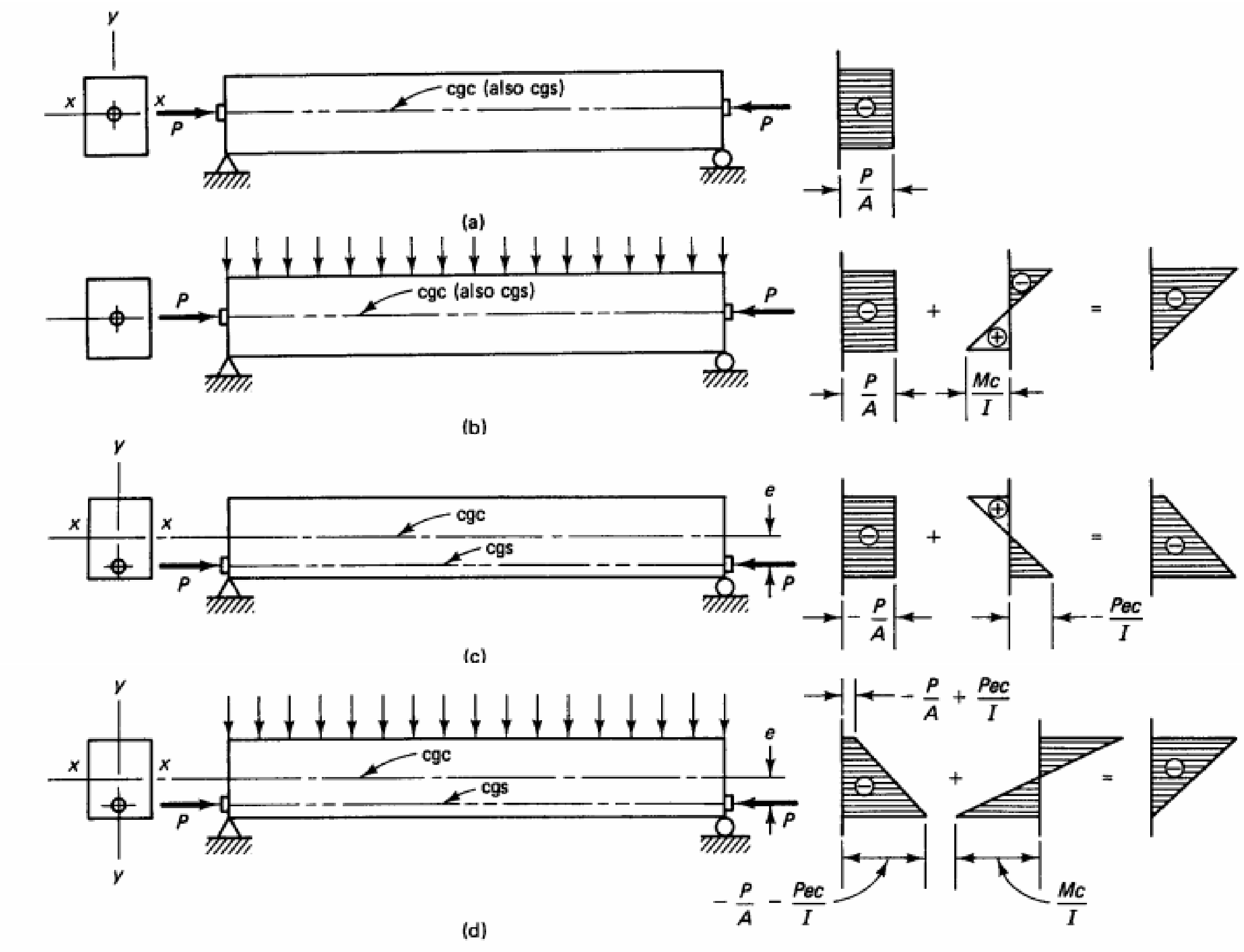
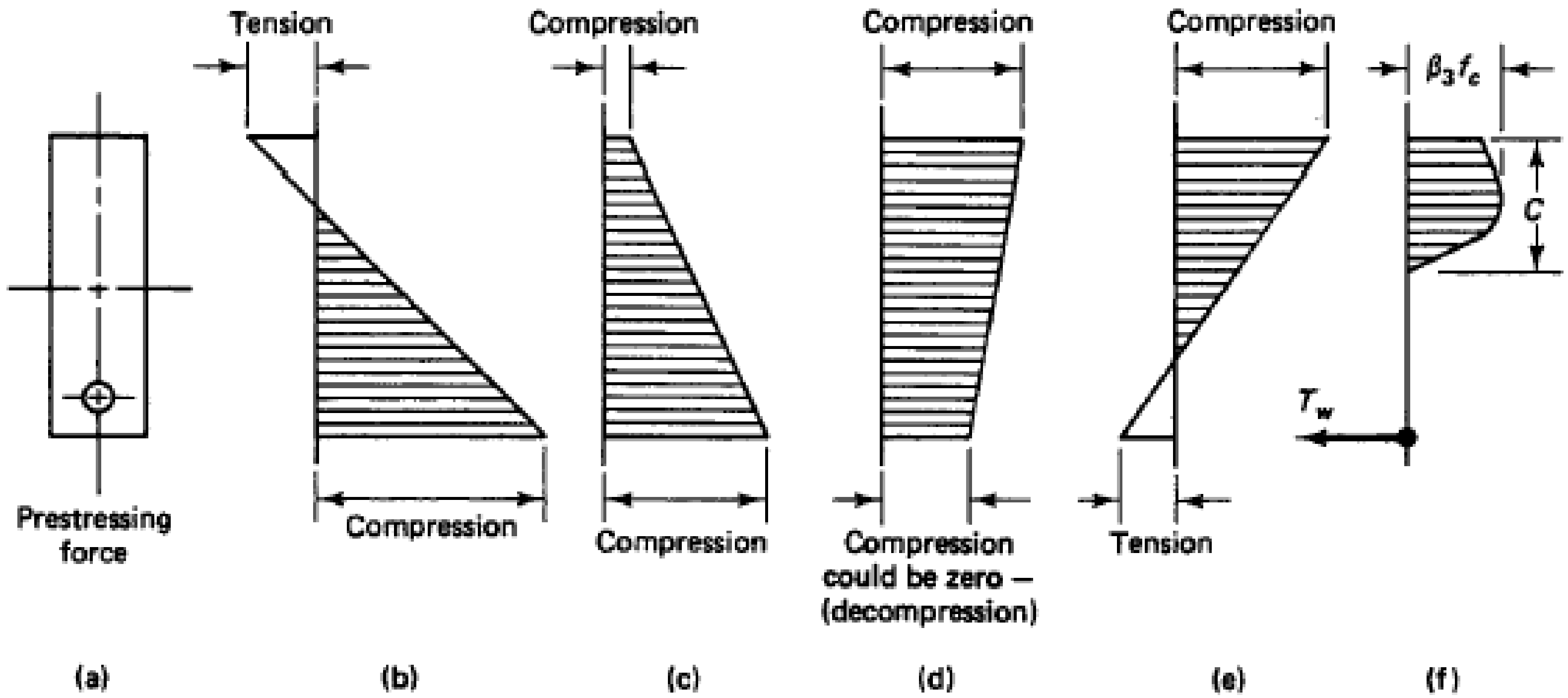



**Figure1: Concrete fiber stress distribution in a rectangular beam with straight tendon. (a) Concentric tendon, prestress only. (b) Concentric tendon, self-weight added. (c) Eccen- tric tendon, prestress only. (d) Eccentric tendon, self-weight added..**




Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full-service load plus effective prestress. (f) Limit state of stress at ultimate load for under reinforced beam.

eliminating tension totally (even inducing compression), or permitting a level of tensile stress within allowable code limits. The section is then considered uncracked and behaves elastically: the concrete's inability to withstand tensile stresses is effectively compensated for by the compressive force of the prestressing tendon. The compressive stresses in Equation 2 at the top fibers of the beam due to pre- stressing are compounded by the application of the loading stress - Mc/I, as seen in Figure 1 (b). Hence, the compressive stress capacity of the beam to take a substantial o external load is reduced by the concentric prestressing force. In order to avoid this limitation, the prestressing tendon is placed eccentrically below the neutral axis at midspan, to induce tensile stresses at the top fibers due to prestressing. [See Figure 1 (c), (d).] If the tendon is placed at eccentricity e from the center of gravity of the concrete, termed the cgc line, it creates a moment Pe, and the ensuing stresses at midspan become

$$f^t = -\frac{P}{A_c} + \frac{P_{ec}}{I_g} - \frac{M_c}{I_g} \qquad \text{.................................(4)}$$

$$f_b = -\frac{P}{A_c} - \frac{P_{ec}}{I_g} + \frac{M_c}{I_g} \qquad \text{.................................(5)}$$

Since the support section of a simply supported beam carries no moment from the external transverse load, high tensile fiber stresses at the top fibers are caused by the eccentric prestressing force. To limit such stresses, the eccentricity of the prestressing tendon pro- file, the cgs line, is made less at the support section than at the midspan section, or eliminated altogether, or else a negative eccentricity above the cgc line is used

## Basic Concept Method

In the basic concept method of designing prestressed concrete elements, the concrete fiber stresses are directly computed from the external forces applied to the concrete by longitudinal prestressing and the external transverse load. Equations 4 and 4 can be modified and simplified for use in calculating stresses at the initial prestressing stage and at service load levels. If Pi, is the initial prestressing force before stress losses, and Pe, is the effective prestressing force after losses, then

$$\gamma = \frac{P_e}{P_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

can be defined as the residual prestress factor. Substituting $r^2$ for $\frac{I_g}{A_c}$, in Equations 4, where r is the radius of gyration of the gross section, the expressions for stress can be rewritten as follows:

(a) Prestressing force only :

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

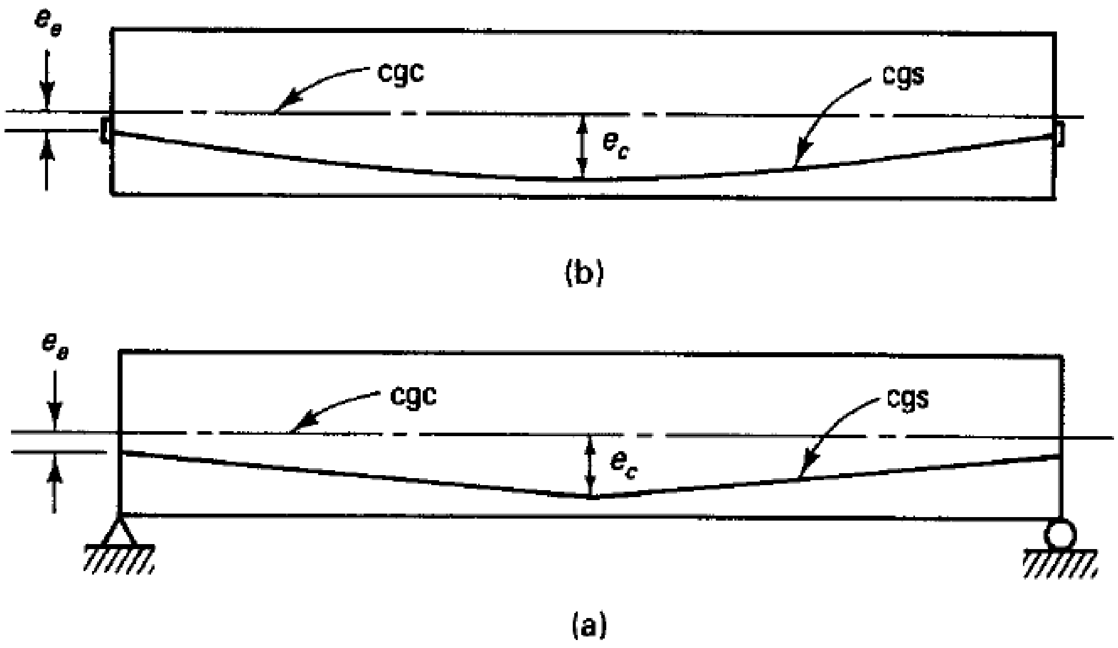



**Figure 2: Prestressing tendon profile: (a) Harped tendon; and (b) Draped tendon**

The change in eccentricity from the midspan to the support section is obtained by raising the prestressing tendon either abruptly from the midspan to the support, a process called harping, or gradually in a parabolic form, a process called draping. Figure 2(a) shows a harped profile usually used for pretensioned beams and for concentrated trans- verse loads. Figure 2(b) shows a draped tendon usually used in post-tensioning. Subsequent to erection and installation of the floor or deck, live loads act on the structure, causing a superimposed moment Ms,. The full intensity of such loads normally occurs after the building is completed and some time-dependent losses in prestress have o already taken place. Hence, the prestressing force used in the stress equations would have to be the effective prestressing force Pe. If the total moment due to gravity loads is MT, then

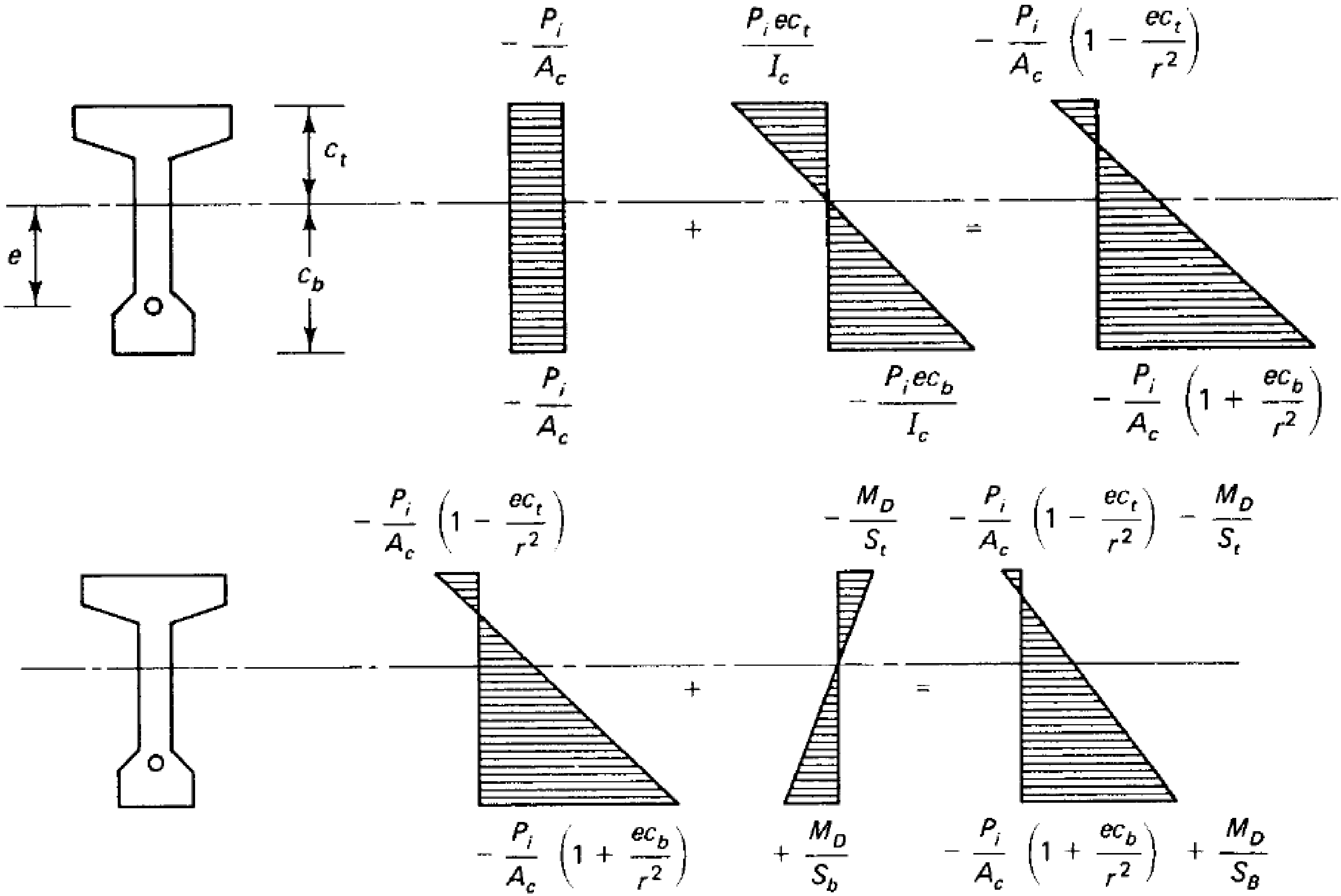
$$M_T = M_D + M_{SD} + M_L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where :

MD = moment due to self-weight

MSD = moment due to superimposed dead load, such as flooring

ML = moment due to live load, including impact and seismic loads if any


$-\frac{P_i}{A_c}$
$\frac{P_i e c_t}{I_c}$
$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)$
$c_t$
$e$
$c_b$
+
=
$-\frac{P_i}{A_c}$
$-\frac{P_i e c_b}{I_c}$
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)$
$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)$
$-\frac{M_D}{S_t}$
$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_D}{S_t}$
+
=
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)$
$+\frac{M_D}{S_b}$
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_D}{S_B}$

$P_i = initial\ prestressing\ force (N)$;
Ac: area of cross section (mm2);
I: moment of inertia of the cross section (mm4);
e: eccentricity of prestressing steel below centroid (mm);
$C_t$, $C_b$: top and bottom fiber distance from centroid (mm);
r: radius of gyration =($\sqrt{\frac{I}{A}}$) (mm);
$P_e$: $effective\ prestressing\ force\ (after\ all\ losses)(N)$;
$f$: stress (MPa)

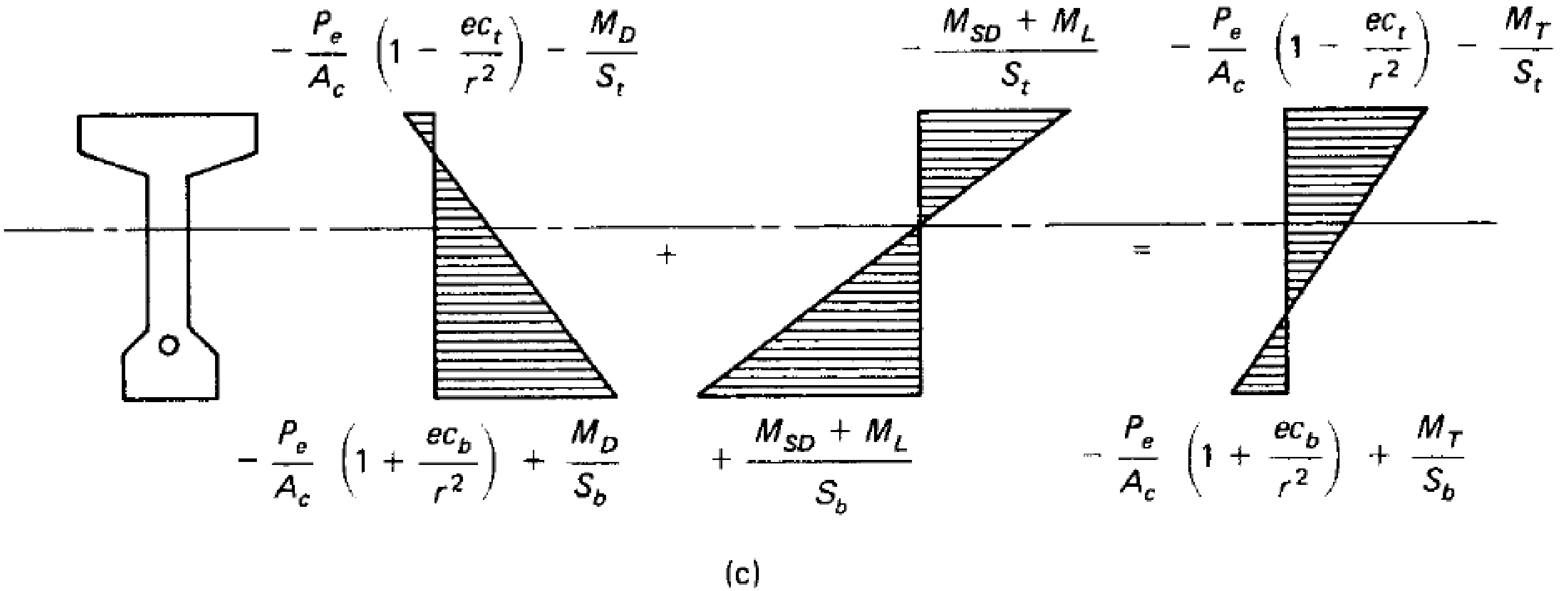



**Figure 3: Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress**.

# *Allowable stresses of concrete*

$f_{ci}$ = maximum allowable compressive stress in concrete immediately after transfer and prior to losses

$= 0.60\, f'_{ci}$

$f_{ti}$ = maximum allowable tensile stress in concrete immediately after transfer and prior to losses

$= 3\sqrt{f'_{ci}}$ (the value can be increased to $6\sqrt{f'_{ci}}$ at the supports for simply supported members)

$f_c$ = maximum allowable compressive stress in concrete after losses at service-load level

$= 0.45\, f'_c$ or $0.60\, f'_c$ when allowed by the code

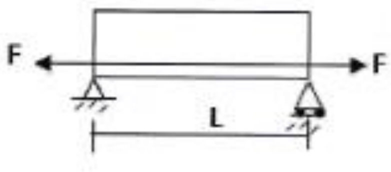
$f_t$ = maximum allowable tensile stress in concrete after losses at service load level

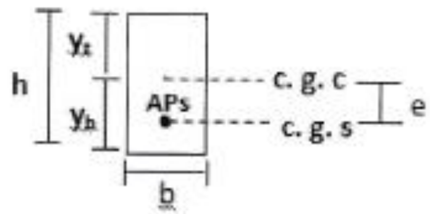
$= 6\sqrt{f'_c}$ (the value can be increased in one-way systems to $12\sqrt{f'_c}$ if long-term deflection requirements are met)

## Concept of P.C. ( flexure)

**Stress analysis.**

## 1- First concept (Elastic material)



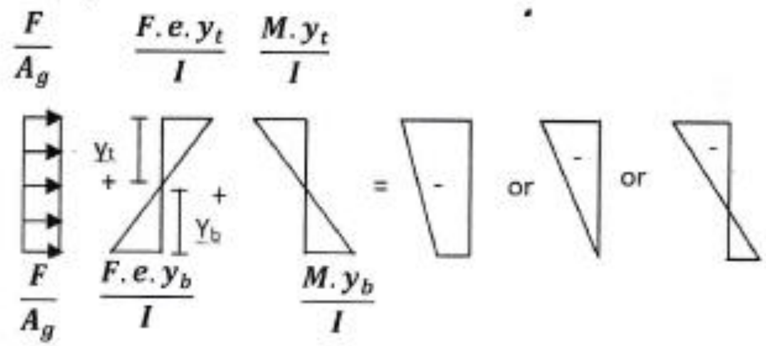




**c.g.c: centroid of concrete (gross section)**
**c.g.s.: centroid of prestresing steel (tendon)**

**e= eccentricity**

$$f_{top} = \frac{-F}{A_g} + \frac{F.e.y_t}{I} - \frac{M.y_t}{I}$$

$$f_{bot} = \frac{-F}{A_g} - \frac{F.e.y_b}{I} + \frac{M.y_b}{I}$$

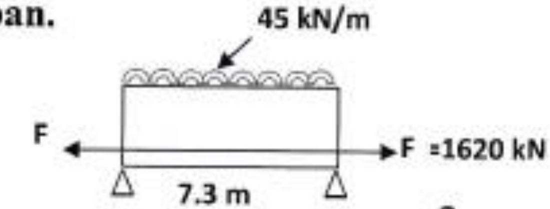



**Where :**

**F=prestress force**
**$A_g$ = gross section area**
**I= gross moment of inertia ( $I_g$)**
**I=** $\sum(I_c + Ad^2)$
**$y_t$= distance from section centriod to the top fiber**
**$y_b$= distance from section centriod to the bottom fiber**
**M=bending moment.**

**Ex1**

**For the prestressed concrete beam with a straight tendon shown in Fig below which is under the prestressing force of 1620 kN, it is required to calculate the extreme fiber stresses at the mid-span by applying concept of elastic material. The uniformly distribution load includes the self weight. Then draw the stress distribution across the section at mid-span.**

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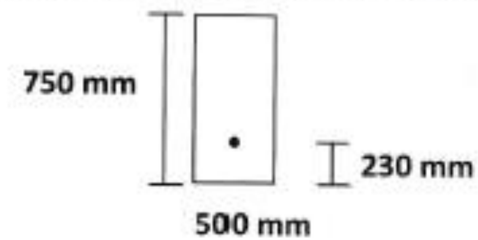



$A_g = 500*750 = 3.75*10^5\ mm^2$

$$I = \frac{500*(750)^3}{12} = 1.758*10^{10}\ mm^4$$

$$y_t = y_b = \frac{750}{2} = 375\ mm$$

e= 375-230 =145 mm

$$M = \frac{w\,l^2}{8} = \frac{45*7.3^2}{8} = 299.76\ kN.m$$

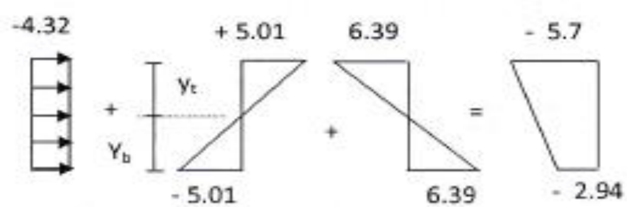
$$f_{top} = -\frac{1620*10^3}{3.75*10^5} + \frac{1620*10^3*145\,(375)}{1.758*10^{10}} - \frac{299.76*10^6*(375)}{1.758*10^{10}}$$

$= -4.32+5.01-6.39$

$= -5.7\ N/mm^2$ (comp.)

$f_{bot.} = -4.32 - 5.01 + 6.39$

$= -2.94\ N/mm^2$ (comp.)



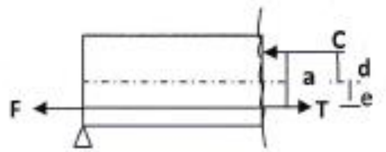
## 1- Second Concept ( Internal resisting couple):

**C: compressive internal force**

**T: tensile internal force**

**a: lever arm**

**d: distance of (C) above centriod**




**d =a – e**

**T=C**

**But**

**T=F**

**∴ C=F**

$$f_{top} = \frac{-F}{A_g} + \frac{F.e.y_t}{I} - \frac{M.y_t}{I}$$

$$\therefore f_{top} = \frac{-F}{A_g} + \frac{y_t}{I}(F.e - M)$$

$$f_{bot} = \frac{-F}{A_g} - \frac{F.e.y_b}{I} + \frac{M.y_b}{I}$$

$$\therefore f_{bot} = \frac{-F}{A_g} + \frac{y_b}{I}(M - F.e)$$

**$M_{net}$=*Mc*=*M - F.e***

**M: Applied external moment due to external load= $M_{ext}$**

**Internal moment= F.a**

**$M_{ext} = M_{int}$**

**∴M = $M_{ext}$ = F.a**

$\therefore M_c = (F.a - F.e)$

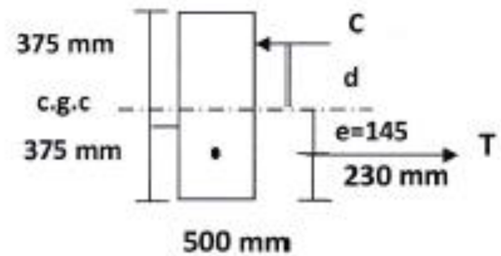
***$M_c$=F(a-e)***

***$M_c$=F.d***

$$f_{top} = \frac{-F}{A} - \frac{M_c * y_t}{I}$$

$$f_{bot} = \frac{-F}{A} + \frac{M_c * y_b}{I}$$

Ex2: Solve Ex1 by internal resisting couple?




**Solution:**

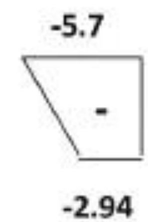
C=T=F=1620 kN
M=299.76 kN.m
Int. M=Ext. M
$T.a=299.76 *10^3$
$a= (299.76*10^3) / 1620 = 185$ mm
d =a – e
= 185-145=40 mm
Mc = C * d = (1620*40) / 1000 = 64.8 kN.m

$$f_{top} = \frac{-F}{A} - \frac{M_c * y_t}{I}$$

$$f_{bot} = \frac{-F}{A} + \frac{M_c * y_b}{I}$$

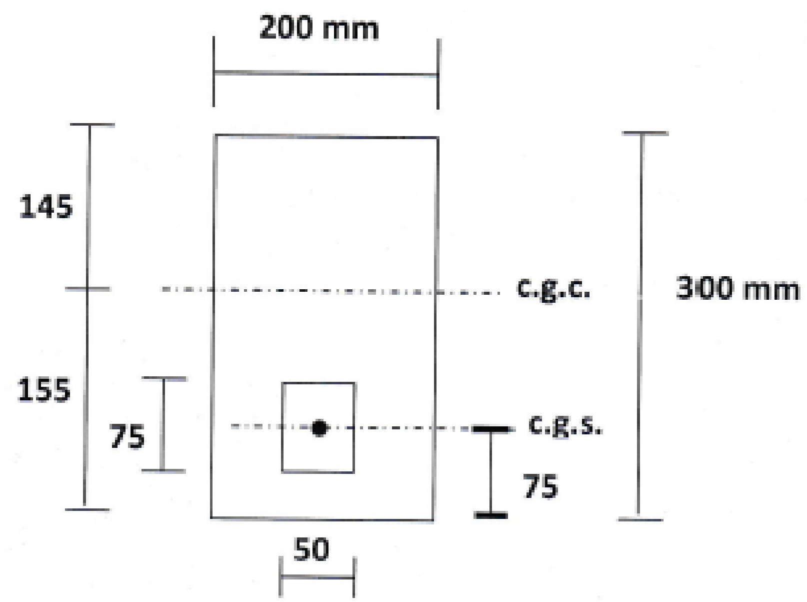
$$f_{top} = \frac{-1620 * 10^3}{375 * 10^5} - \frac{64.8 * 10^6 * 375}{1.758 * 10^{10}} = -5.7\,N/mm^2 \quad (Comp.)$$

$$f_{bot} = \frac{-1620*10^3}{375*10^5} + \frac{64.8*10^6*375}{1.758*10^{10}} = -2.94\,N/mm^2 \quad (Comp.)$$



## Example:

A post-tensioned beam has a midspan cross section of 200x300 mm with a duct of 50x75 mm to house the wires, as shown in the figure. It is prestressed with 320 $mm^2$ of steel to an stress of 1000 $N/mm^2$. The stress is reduced by 5% immediately after transfer because of the anchorage loss and elastic shortening of concrete. Compute the stresses in concrete after transfer.

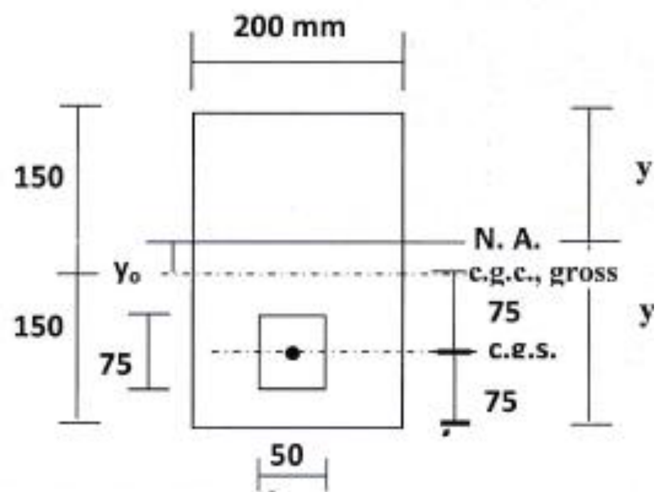


## Solution (1):

**Using the net section of concrete. The centroid and the moment of inertia of the n concrete section are computed as follows.**

**$A_c$= 200*300 – 50*75= 56250 $mm^2$**

**Moment about c.g.c.**




$$y_o = \frac{\sum A.y}{\sum A}$$

$$y_o = \frac{A_c * zero + hole * 75}{A_c - hole}$$

$$y_o = \frac{0 + 50*75*75}{200*300 - 50*75} = 5\ mm$$

**$y_t$= 150-5 = 145 mm**

**$y_{bt}$= 150+5=155 mm**

**I=200*300$^3$/12 + 200*300* (5)$^2$ – [ 50*75$^3$/12 +50*75 *(75+5)$^2$]**

**= 4.26 *10$^8$ mm$^4$**

**$F_i$=520*1000* 95/100 = 494000 N**

**Top fiber stress ($y_{top}$=145 mm)**

$$f_{top} = -\frac{F_i}{A} + \frac{F_i\, e\, y_{top}}{I}$$

$$= -\frac{494000}{56250} + \frac{494000*(75+5)*145}{4.26*10^8}$$

**= -8.78 + 13.45 = + 4.67 N/mm$^2$ (tension)**

**Bott. Fiber stress ( $y_{bott}$=155 mm)**

$$f_{bott.} = -\frac{F_i}{A} - \frac{F_i\, e\, y_{bott.}}{I}$$

$$= -\frac{494000}{56250} - \frac{494000*(75+5)*155}{4.26*10^8}$$

**= - 8.78- 14.38 = - 23.16 N/ mm$^2$ (Comp.)**

**Solution (2):**

Using gross- section of concrete

$A_g$= 200*300= 60000 $mm^2$

$I_g$= 200*300$^3$/ 12 = 4.5* $10^8$ $mm^4$

**Top fiber stress** ($y_{top}$=150 mm)

$$f_{top} = -\frac{F_i}{A_g} + \frac{F_i\, e\, y_{top}}{I_g}$$

$$= -\frac{494000}{60000} + \frac{494000 * (75) * 150}{4.5 * 10^8}$$

= -8.23 + 12.35 = + 4.12 N/$mm^2$ (tension)

$$f_{bott.} = -\frac{F_i}{A_g} - \frac{F_i\, e\, y_{bott.}}{I_g}$$

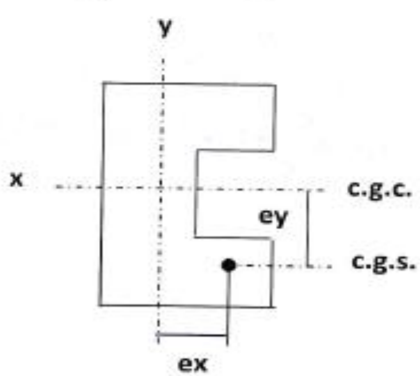
$$= -\frac{494000}{60000} - \frac{494000 * (75) * 150}{4.5 * 10^8}$$

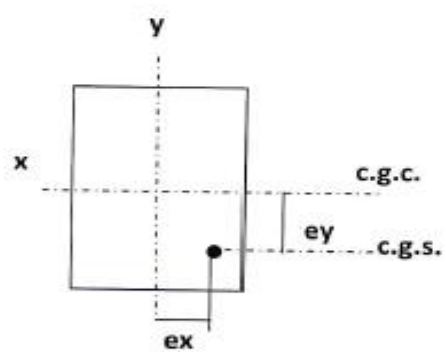
= - 8.23 – 12.35 = - 20.58 N/$mm^2$ ( Comp.)

The approximate solution using the gross concrete section would give good results in this example.

If the eccentricity does not occur a long one of the principal axes of the section, it is necessary to resolve the moment into two components along the two principal axes . The stress at any point is given by:

$$f = -\frac{F}{A} \pm \frac{F.e_x.x}{I_y} \pm \frac{F.e_y.y}{I_x}$$

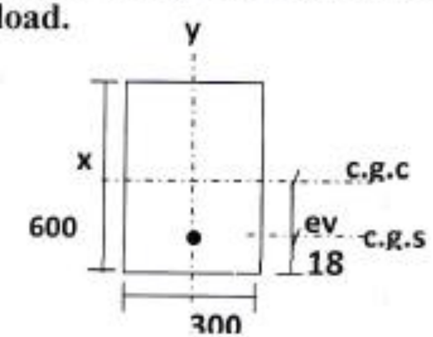


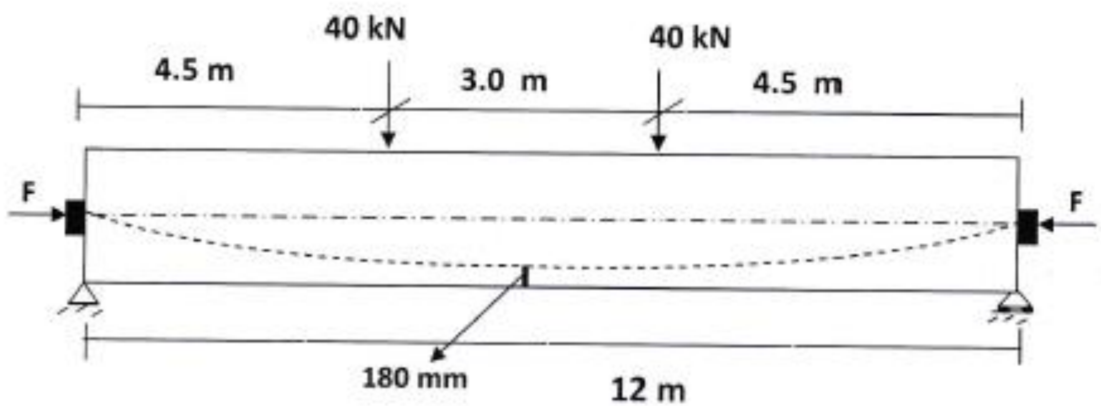



## Example:

A post- tensioned bonded concrete beam(300x600 mm), see figure below, has a prestress of 1600 kN in the steel immediately after transfer ($F_i$), which is eventually reduces to 1400 kN after losses ($F_{se}$). The beam carries two live loads of 40 kN each in addition to its own weight $w_g$= 4.3 kN/m. compute the extreme fiber stresses at midspan,

a) Under the initial condition, with full prestress and no live load, and
b) Under the final condition, after the losses have taken place, and with fully live load.



**To be theoretically exact, the net concrete section should be used up to the time of grouting, after which the transformed section should be considered. However, an approximate solution based on the gross section of concrete at all times is sufficiently exact**

$e_{midspan}$ **=300-180= 120 mm.**

$A_g = 300 * 600 = 180000\ mm^2 = 180*10^3\ mm^2$

$I_g = 300*600^3/12 = 5.4*10^9\ mm^4$

a) Initial condition

**Dead- load moment at midspan, assuming that the beam is simply supported after prestressing**

$$M_g = \frac{w_g l^2}{8} = \frac{4.3 * 12^2}{8} = 77.4\ kN.m$$

$$f = -\frac{F_i}{A} \pm \frac{F_i e\, y}{I} \mp \frac{M_g\, y}{I}$$

$$f = -\frac{1600 * 10^3}{180 * 10^3} \pm \frac{1600 * 10^3 * 120 * 300}{5.4 * 10^9} \mp \frac{77.4 * 10^6 * 300}{5.4 * 10^9}$$

$$f = -8.89 \pm 10.67 \mp 4.3$$

**$f_{top}$ = -8.89 +10.67-4.3= -2.52 N/mm$^2$ (Comp.)**

**$f_{bott}$ = -8.89 -10.67 +4.3 = -15.26 N/mm$^2$ (Comp.)**

b) Final condition

**$M_s$= p * a**
**$M_s$ =40 * 4.5 = 180 kN.m**

**$M_t$=$M_g$ + $M_s$ = 77.4 + 180 = 257.4 kN.m**

$$f = -\frac{F_{se}}{A} \pm \frac{F_{se}\, e\, y}{I} \mp \frac{M_t\, y}{I}$$

$$f = -\frac{1400 * 10^3}{180 * 10^3} \pm \frac{1400 * 10^3 * 120 * 300}{5.4 * 10^9} \mp \frac{257.4 * 10^6 * 300}{5.4 * 10^9}$$

$$f = -7.78 \pm 9.33 \mp 14.3$$

**$f_{top}$ = -7.78 +9.33 – 14.3 = -12.75 N/mm$^2$ (Comp.)**

**$f_{bot}$ = -7.78 -9.33 + 14.3 = -2.81 N/mm$^2$ (Comp.)**

Example ①: S.S. I-beam has a symmetrical X-Sec with the properties:-

$I = 5 \times 10^9\ mm^4$, $A = 114000\ mm^2$, $r^2 = 44000\ mm^2$, $h = 2C = 610^{mm}$

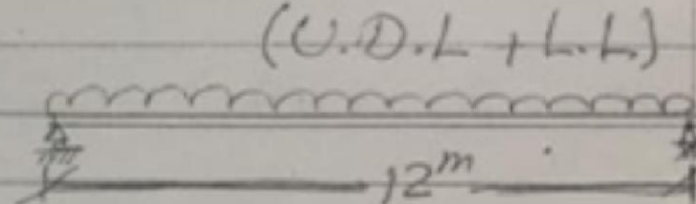
Carries u.d.l (D+L) = 8 kN/m in addition to Self wt. of 2.7 kN/m, Span = $12^m$, $e = 130^{mm}$, $P_i = 750$ kN, $P_e = 640$ kN. ① Find flexural Stresses @ mid span and ends due to intial prestress + Self wt. and due to $P_e$ + full Service load.

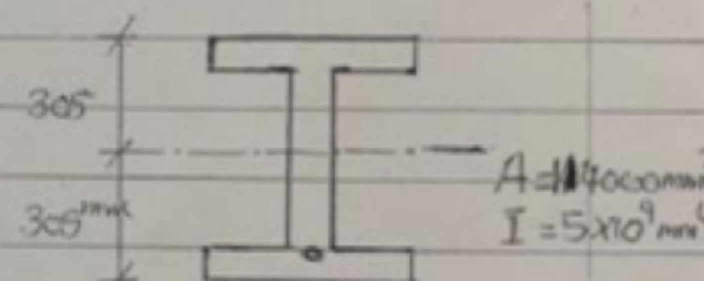
② Check with ACI permissible stresses.

Solution:

$$M_g = \frac{w_g l^2}{8} = 48.6\ kN.m$$

$$M_S = \frac{8 \times 12^2}{8} = 144\ kN.m$$



eq(2) $f_{i_{top}} = \frac{-750000}{114000}\left(1 - \frac{130 \times 305}{44000}\right) - \frac{48.6 \times 10^6 (305)}{5 \times 10^9} = \underset{\text{Trans.}}{-0.65} - 2.96$

$= -3.61 \text{ MPa}$

$f_{i,bot} = \frac{-750000}{114000}\left(1 + \frac{130 \times 305}{44000}\right) + 2.96 = \underset{\text{Trans.}}{-12.51} + 2.96$

$= -9.55 \text{ MPa}$

After losses

Eq. 3: $f_{top} = -0.65 \times \left(\frac{640}{750}\right) - 2.96 - \frac{144 \times 10^6 \times (305)}{5 \times 10^9}$ [illegible]

$= -0.55 - 2.96 - 8.78 = -12.3 \text{ Mpa.}$

$f_{bot} = -12.51(640/750) + 2.96 + 8.78 = -10.68 + 2.96 + 8.78$

$= 1.03 \text{ Mpa}$

1) Prestressed flexural members shall be classified as, class(U) , class (T) or class(C), based on $f_{ts}$, the computed extreme fiber stress in tension in the prestressed tensile zone calculated at service loads as follows.

a) Class (U): $f_{ts} \leq 0.62\sqrt{f'_c}$

b) Class (T) : $0.62\sqrt{f'_c} \leq f_{ts} \leq 1.0\sqrt{f'_c}$

c) Class (C) : $f_{ts} \geq 1.0\sqrt{f'_c}$

Where:

$f'_c$: Cylindrical compressive strength of concrete at 28 days age.

class (U) members are assumed to behave as uncracked members. .The behavior of class (T) members is assumed to be in transition between uncracked and cracked. Class (C) members are assumed to behave as cracked members.

For class (U) and class (T) flexural members stresses at service loads, shall be permitted to be calculated using the uncracked section, gross-section properties can be used instead of uncracked section for these two classes.

For class (C) flexural members, stresses at service loads shall be calculated using the cracked transformed section.
At stage of transfer (initial stage) stresses shall be permitted to be calculated using the uncracked section, and gross-section can be used for all classes of flexural members.

## Permissible (allowable) stresses:

i) Stage of transfer (initial stage).

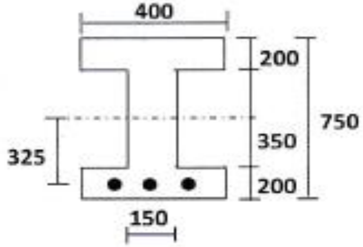
Stresses in concrete immediately after prestress transfer ( before time-dependant prestress losses shall not exceed the following:

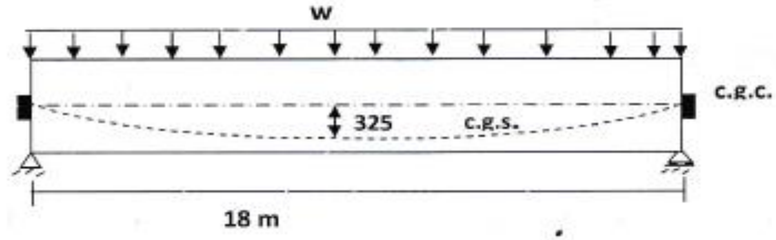
a) Extreme fiber stress in compression,
$f_{ci} = 0.6\, f'_{ci}$

b) Extreme fiber stress in tension except as permitted in (c)
$f_{ti} = 0.25\sqrt{f'_{ci}}$

c) Extreme fiber stresses in tension at ends of simply supported members
$f_{ti} = 0.5\sqrt{f'_{ci}}$

Check the simply supported beam shown in the figure with respect to the permissible concrete stresses. The following data is given : $f'_c = 36$ N/mm$^2$ , $f'_{ci} = 30$ N/mm$^2$ , $F_i =$ 1200 kN, R = 0.8, e mid-span=325 mm , span =18 m, S.D.L. + S.L.L.= 8 kN/m.




**Solution:**

**Calculation of concrete stresses at extreme tensile fiber at service stage based on gross section properties.**

**Gross-section properties**

$A_g = 2(400*200)+(350*150)=212.5\text{x}10^3 \text{ mm}^2$

$y_t = y_b = \frac{750}{2} = 375\ mm$

$$I_g = \frac{150*350^3}{12} + 2\left[\left(\frac{400*200^3}{12}\right) + (400*200)(275)^2\right]$$

$I_g = 13169.27 \text{ x } 10^6 \text{ mm}^4$

$w_g = A_g * \gamma_c = 212.5\ x10^3 * 10^{-6} * 24 = 5.1\ kN/m$

$$M_G = \frac{w_g * L^2}{8} = \frac{5.1 * 18^2}{8} = 206.55 \text{ kN.m}$$

$$M_s = \frac{w_s * L^2}{8} = \frac{8 * 18^2}{8} = 324 \text{ kN.m}$$

$M_t = M_G + M_s = 206.55 + 324 = 530.55\ kN.m$

$$f_{bot} = \frac{-F_{se}}{A} - \frac{F_{se}.e.y_b}{I} + \frac{M_t.y_b}{I}$$

$F_{se} = R * F_i = 0.8*1200=960$ kN

$$f_{bot} = \frac{-960*10^3}{212.5*10^3} - \frac{960*10^3*325*375}{13169.27*10^6} + \frac{530.55*10^6*375}{13169.27*10^6}$$

$= -4.518 - 8.884+15.108$

$f_{ts} = f_{bot} = +1.706\ N/mm^2$ (*tension*)

$0.62\sqrt{f'_c} = 0.62 * \sqrt{36} = 3.72\ N/mm^2$

$\therefore f_{ts} < 0.62\sqrt{f'_c}$

$\therefore class\ U$

$\therefore$ ***at both stages, uncracked transformed section is used***

**but gross-section properties can be used**

$\therefore A = A_g$

$I = I_g$

a) **Initial stage (stage of transfer) at midspan**

**Allowable stresses**

$f_{ti} = 0.25\sqrt{f'_{ci}} = 0.25\sqrt{30} = 1.37\ N/mm^2$ **(tension)**

$f_{ci} = 0.6\, f'_{ci} = 0.6 * 30 = 18\ N/mm^2$ **(compression)**

$$f_{top} = \frac{-F_i}{A} + \frac{F_i.e.y_t}{I} - \frac{M_G.y_t}{I} \le f_{ti}$$

$$= \frac{-1200 * 10^3}{212.5 * 10^3} + \frac{1200 * 10^3 * 325 * 375}{13169.27 * 10^6} - \frac{206.55 * 10^3 * 375}{13169.27 * 10^6}$$

**= - 5.647 +11.105-5.882**

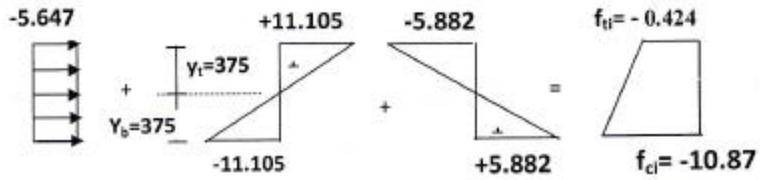
**= - 0.424 N/mm$^2$ compression (limit is 1.37 N/mm$^2$ tension) o.k.**

$$f_{bot} = \frac{-F_i}{A} - \frac{F_i.e.y_b}{I} + \frac{M_G.y_b}{I} \le f_{ci}$$

$$= \frac{-1200 * 10^3}{212.5 * 10^3} - \frac{1200 * 10^3 * 325 * 375}{13169.27 * 10^6} + \frac{206.55 * 10^3 * 375}{13169.27 * 10^6}$$

**= - 5.647 - 11.105 + 5.882**

**= - 10.87 N/mm$^2$ compression (limit is 18 N/mm$^2$ compression) o.k.**




Direct stress due to $F_i$ | Flexurel stress due to $F_i$ | Flexurel stress due to selfweight

**Stress distribution at initial stage (stage of transfer ) before time dependent losses.**

**b) Service load stage**

**Allowable stress**

$f_{cs} = 0.6\, f'_c = 0.6 * 36 = 21.6\ N/mm^2$ (compression)

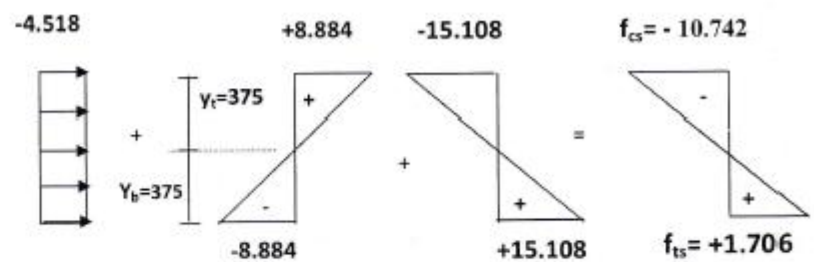
$F_{se} = R^* F_i = 0.8*1200 = 960$ kN

$$f_{top} = \frac{-F_{se}}{A} + \frac{F_{se}.e.y_t}{I} - \frac{M_t.y_t}{I} \le f_{cs}$$

$$= \frac{-960 * 10^3}{212.5 * 10^3} + \frac{960 * 10^3 * 325 * 375}{13169.27 * 10^6} - \frac{530.55 * 10^3 * 375}{13169.27 * 10^6}$$

= - 4.518 + 8.884 - 15.108

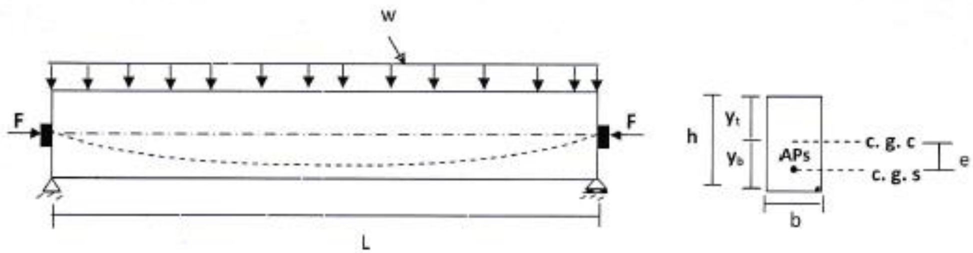
= - 10.742 N/mm$^2$ (compression) (limit is 21.6 N/mm$^2$ compression) o.k.

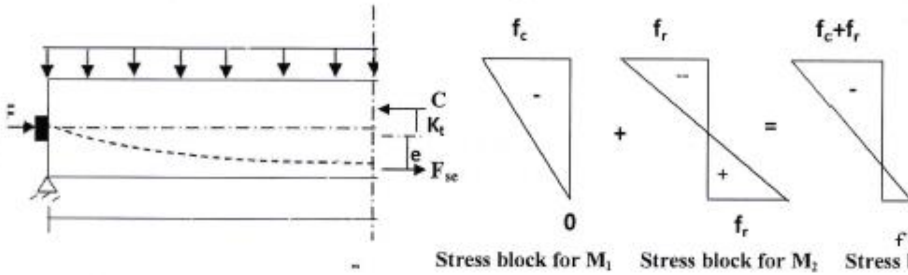



Direct stress due to $F_{se}$ | Flexurel stress due to $F_{se}$ | Flexurel stress due to load

**Stress distribution at service stage after time - dependent losses.**

- **It is the moment that causing the first hair crack in a prestressed concrete beam and is computed by elastic theory. Cracking is assumed to start when the tensile stress in the extreme fiber of concrete reaches its modulus of rupture ($f_r$).**
- **If the concrete has been previously cracked by over loading, shrinkage or other causes, cracks may reappear at a slightest tensile stress**




**Stress block for $M_1$** **Stress block for $M_2$** **Stress block for $M_1 + M_2$**

**<u>At service load stage</u>**

$$f_{bot} = \frac{-F_{se}}{A} - \frac{F_{se} * e * y_b}{I} + \frac{M_{cr} * y_b}{I} = f_r \ (tension)$$

$$M_{cr} = \frac{f_r I}{y_b} + F_{se} . e + \frac{F_{se} . I}{A \ y_b}$$

**The above expression may be derived from another approach. When the center of pressure of concrete (C) is at top kern point, the stress at the extreme bottom fiber will be equal to zero. The resisting moment ($M_1$) is given by the prestress force $F_{se}$ times its lever arm ($e+k_t$)**

$$M_1 = F_{se} \, (e + k_t)$$

$$k_t = \frac{r^2}{y_b}$$

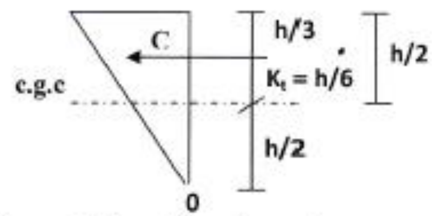
$$where: r\ is\ the\ radius\ of\ gyration = \sqrt{\frac{I}{A}}$$

**For a rectangular section**

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{b\,h^3/12}{b\,h}}$$

$$r = \sqrt{\frac{h^2}{12}}$$

$$k_t = \frac{r^2}{y_b} = \frac{h^2/12}{h/2} = \frac{h}{6}$$




**Additional moment resisted by the concrete up to its modulus of rupture is**

$$M_2 = \frac{f_r\,I}{y_b}$$

**Hence, the total moment at cracking is**

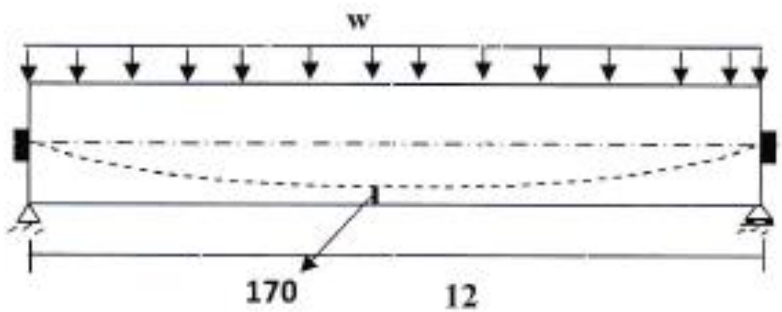
$$M_{cr} = M_1 + M_2 = F_{se}\,(e + k_t) + f_r\,\frac{I}{y_b}$$

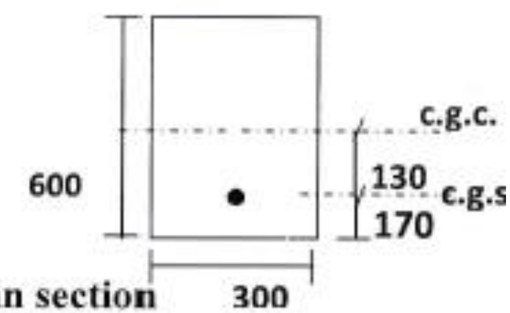
$$M_{cr} = F_{se}\left(e + \frac{r^2}{y_b}\right) + f_r\,\frac{I}{y_b}$$

Example1:

**The post-tension simple beam with a span of 12 m carries a uniformly distributed load (w) kN/m in addition to its own weight (4.4) kN/m. The prestress stress in the steel after deducting all losses (fse) is 830 N/mm$^2$. The parabolic cable has an area ($A_{ps}$) of 1600 mm$^2$. compute the uniformly distributed load (w) that can be carried by the beam,**

1- **For zero tensile stress in the bottom fiber.**
2- **For cracking in the bottom fibers at a modulus of rupture of 4.2 N/mm$^2$.**




mid-span section

**Considering the critical midspan section and use gross concrete section properties $k_t$ is computed as:**

$$k_t = \frac{r^2}{y_b} = \frac{h}{6} = \frac{600}{6} = 100\ mm \qquad above\ the\ mid\ depth$$

**(1) To obtain zero stress in the bottom fiber.**

**The center of pressure (C) must be located at the top kern point. Hence the resisting moment**

$$M_1 = F_{se}\,(e + k_t)$$
$$F_{se} = f_{se} * A_{ps}$$
$$F_{se} = 830 * 1600 * 10^{-3} = 1328\ kN$$

$$\therefore\ M_1 = 1328\,(130 + 100) * 10^{-3} = 305.4\ kN.m$$

$$M = \frac{wl^2}{8}$$
$$\therefore\ M_1 = \frac{(4.4 + w) * 12^2}{8}$$
$$305.4 = \frac{(4.4 + w) * 12^2}{8}$$
$$\therefore w = 12.57\ ^{kN}/_{m}\ (for\ zero\ stress\ at\ bottom\ fiber)$$

**(2) Additional moment carried by the section up to beginning of cracks is:**

$$M_2 = \frac{f_r\,I}{y_b} = \frac{4.2 * \frac{300 * 600^3}{12}}{300} * 10^{-6} = 75.6\ kN.m$$

**The cracking moment $M_{cr}$=$M_1 + M_2$**

**$M_{cr}$ =305.4+75.6=381 kN.m**

$$M_{cr} = \frac{(4.4 + w) * 12^2}{8}\ ,381 = \frac{(4.4 + w) * 12^2}{8}$$
$$\therefore w = 16.77\ ^{kN}/_{m}\ (for\ cracking\ stress)$$

## Ultimate Moment

- Exact analysis for the ultimate strength of a prestressed-concrete section under flexure is a complicated theoretical problem, because both steel and concrete are generally stressed beyond their elastic limits. However, for design purposes, where an accuracy of 5-10% is considered sufficient, relatively simple procedure can be developed.
- A simple method for determining ultimate flexure strength is presented here: the method is limited to the following conditions.

1- The failure is primarily a flexural failure.
2- The beams are bonded (unbounded beams posses different ultimate strength)
3- The beams are statically determinate
4- The load considered is the ultimate load obtained from short static test (impact, fatigue or long time loading are not considered).

The method is based on the simple principle of resisting couple in a prestressed beam. At ultimate load, the couple is made of two forces ( C and T ) acting with a lever arm (a). The steel supplies the tensile force (T) and the concrete supplies the compressive force ( C).

### Modes of failure of prestressed-beam sections

The failure of a section may start either in the steel or in the concrete.

1- Under-reinforced section:

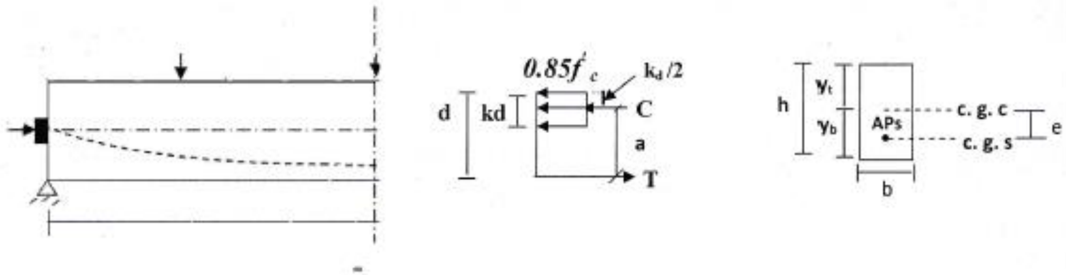
a) When the amount of steel is small and the compressive flange is restrained and possesses a higher strength, the steel yields and breaks before crushing of concrete.
b) If the amount of steel is moderate then the steel yields excessively. Failure is finalize by crushing of concrete (amount of steel 0.3-0.8%)

2- Over-reinforced section:

If the amount of steel is very high, the concrete crushes before yielding of the steel (amount of steel >1%)

- For too lightly reinforced section; failure may be occur by breaking of steel immediately following the cracking of concrete. This happens when the tensile force in the concrete is suddenly transfer to the steel whose is too small to absorb that additional tension. (amount of steel about 0.1% ).

**Under reinforced bonded beams:**




$C = T = A_{ps}\, f'_s$

$f'_s = f_{su} = f_{pu}$ = ultimate stress in steel

$M_u = T * a = C * a$

**To determine the lever arm (a) it is only necessary to locate the centriod force C. There are many plastic theories for distribution of compressive strength in concrete at failure assuming the stress block to take the shape of a rectangle, trapezoid, parabola, etc. (any of these methods is sufficiently accurate, because they would yield nearly the same lever arm (difference is about 5%)).**

**Choosing the simplest stress block, a rectangle**

$C = 0.85\, f'_c\, b\, kd$

**Where the average compressive stress at failure =** $0.85\, f'_c$

C=T

$0.85\, f'_c\, b\, kd \;=\; A_{ps}\, f'_s$

$or \quad kd = \dfrac{A_{ps}\, f'_s}{0.85\, f'_c\, b}$

**These formulas apply if the compressive flange has a uniform width (b) at failure**

**The lever arm for a the rectangular stress block is**

$a = \left(d - \dfrac{kd}{2}\right)$

**hence, the ultimate resisting moment is**

$M_u = T * a = A_{ps} f'_s \left(d - \dfrac{kd}{2}\right)$

**By substituting the expression of** $k_d$ **the above equation, we have**

$M_u = A_{ps}\, f'_s\, d \left(1 - \dfrac{A_{ps}\, f'_s}{2 * 0.85\, f'_c\, b\, d}\right)$

**Example1:**

**A rectangular section of 300x600 mm is prestressed with ($A_{ps}$ = 970 mm$^2$) of steel wires to an initial stress ($f_{si}$) of 1000N/mm$^2$. The c.g.s. of the wire is 100 mm above the bottom fiber of the beam, $f'_s$ = 1650 N/mm$^2$; $f'_c$ =34.5 N/mm$^2$. Estimate the ultimate resisting moment of the section.**

**Solution:**

**Assuming that the wires will be stressed to their ultimate strength, the total force (T) at failure**

$$T = A_{ps} * f'_s = 970 * 1650 * 10^{-3} = 1600.5\ kN$$

$$kd = \frac{A_{ps} * f'_s}{0.85\, f'_c\, b}$$

$$= \frac{970 * 1650 * 10^3}{0.85 * 34.5 * 300}$$

$$= 181\ mm$$

d = h - cover

= 600 -100 = 500 mm

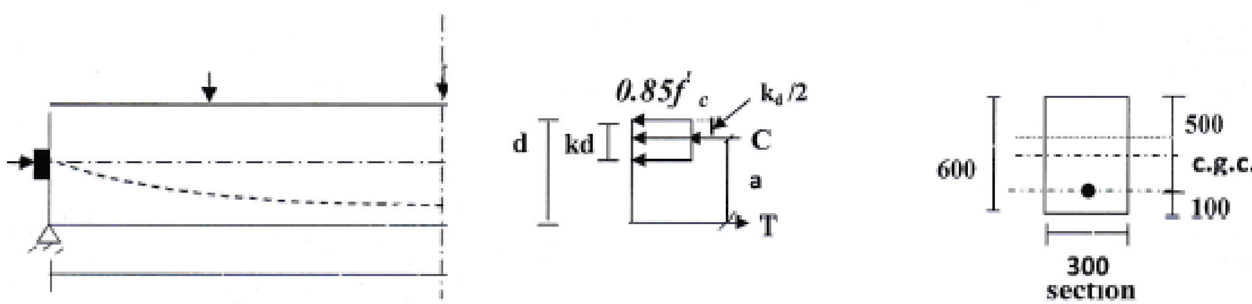
a = d – kd/2

= 500 - 181/2 = 409.5 mm

$M_u$ = T*a

= 1600.5 * 409.5*10$^{-3}$

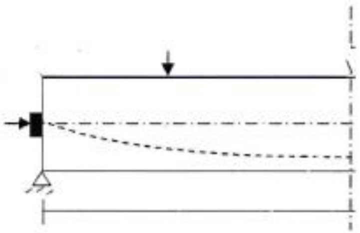
= 655.4 kN.m



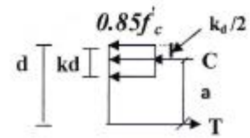

**Half elevation**

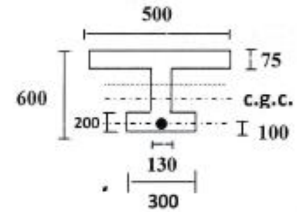
**Example2:**

**The T-section shown in the figure is reinforced with ($A_{ps}$ =970 mm$^2$ ) of wires prestressed to an initial stress ($f_{si}$) of 1000 N/mm$^2$ , $f'_s$ = 1650 N/mm$^2$ , $f'_c$= 34.5 N/mm$^2$. Estimate the ultimate resisting moment,**



**Half elevation**




**Section**

**Solution:**

**assuming that the steel is stressed to its ultimate strength hence,**

$T = A_{ps}\, f'_s = 970 * 1650 * 10^{-3} = 1600.5$ kN

**The total compressive area required for concrete**

$$A_{comp.} = \frac{C}{0.85\ f'_c} = \frac{T}{0.85\ f'_c} = \frac{1600.5 * 10^3}{0.85 * 34.5} = 54625\ mm^2$$

**The flange supplies an area of 500*75=37500 mm$^2$**

$A_{comp.} > A_{flange}$

$\therefore T - section$

$\therefore area\ to\ be\ supplied\ by\ the\ web = 54625 - 37500 = 17125\ mm^2$

**Thus the neutral axis is located**

**17125/130=132 mm below the flange**

**The center of pressure ( C ) is located at the centroid of the compressive area, thus**

$$y_o = \frac{\sum A * y}{\sum A}$$

$$y_o = \frac{(500 * 75)\left(\frac{75}{2}\right) + (132 * 130) * \left(\frac{132}{2} + 75\right)}{500 * 75 + 132 * 130}$$

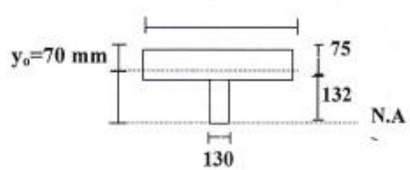
$y_o = 70\ mm$

d= 600 – 100 = 500 mm

$a = d - y_o$

$\therefore$ a=500 -70 = 430 mm

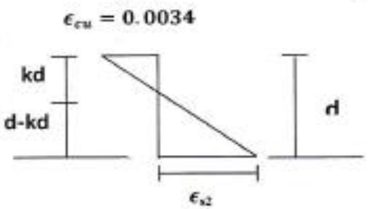
$M_u$=T*a= 1600.5*430/1000 = 688.2 kN.m



## Over-Reinforced Bonded Beams

**In an over-reinforced simply supported prestressed beam, the position of the neutral axis at rupture (failure) will be low, and compressive failure will take place in the concrete before the ultimate strength in the steel is developed. To evaluate the stress of steel at rupture of the beam ($f_s$), it is necessary to study the strain distribution in the section and relate them to the stress-strain curve of steel. The maximum strain of concrete at failure varies between (0.003-0.004) and may be taken as (0.0034). Assuming that plane section remains plane at rupture, the strain of steel at rupture of the beam is:**

$$\in_{s2} = 0.0034\,\frac{d-kd}{kd} = 0.0034\,\frac{1-k}{k}$$



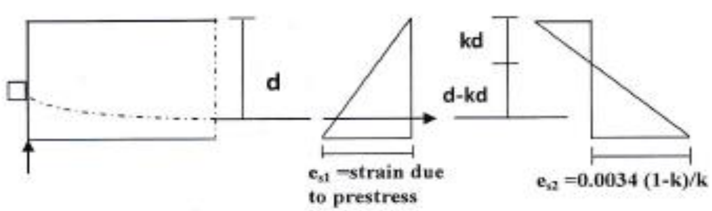

Strains due to loading

**This strain is added to the prestressed strain in the steel ($\in_{s1}$) at the time when concrete strain is zero on the top fiber. The total strain ($\in_s$) is given by**

$$\in_s = \in_{s1} + \in_{s2}$$

$where$: $\in_{s1} = \dfrac{f_{se}}{E_s}$

$f_{se}$: **effective prestress stress (after losses)**
$E_s$: **modulus of elasticity of steel= 200000 MPa**
**The corresponding stress ($f'_s$) can be obtained from the stress-strain diagram of steel.**




Beam elevation　　Strains due to prestress and beam　　Strains due to loading

**If the stress ($f_s$ ) is near the ultimate value ($f'_s$), the section is not over-reinforced and the previous method using the ultimate strength of steel is accurate enough. If (($f_s$ )is appreciably lower than ($f'_s$) the solution has to be modified. A method of trail and error can be used to obtain the actual value of ($f_s$ ) at rupture.**

**Example:-**

**For the rectangular section of example 1, compute the ultimate resisting moment using the trial and error method.**

**Solution:**

**Corresponding to the first trail in example (1) assuming a stress ( $f_s = f'_s = 1650\ N/mm^2$), the neutral axis at rupture was located at (kd=181 mm) from the top.**

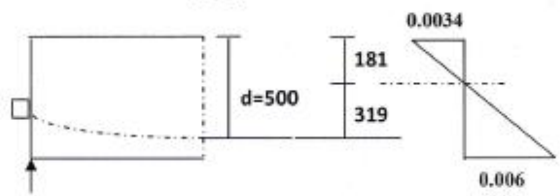
**The maximum concrete strain is assumed to be (0.0034), the strain in the steel ($\epsilon_{s2}$) can be obtained as:**

**d=h-cover=600 -100=500 mm**

**d-kd=500-181=319 mm**

$$\epsilon_{s2} = \epsilon_{cu} * \frac{d - kd}{kd}$$

$$\epsilon_{s2} = 0.0034 * \frac{319}{181} = 0.006$$

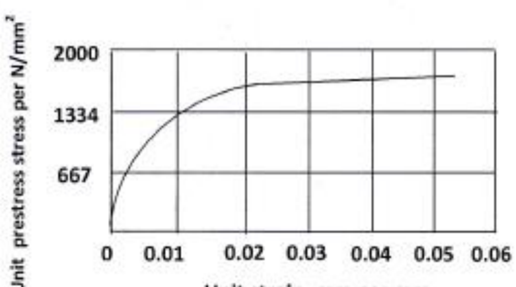



**Assume that the effective prestress stress ($f_{se} = 840\ N/mm^2$)**
**(initial stress=1000 N/mm$^2$, losses 16%, R=0.84)**
**And $E_s$=200000 N/mm$^2$ , Hence,**

$$\epsilon_{s1} = \frac{f_{se}}{E_s} = \frac{840}{200000} = 0.0042$$

**Thus the total strain at failure= 0.006 +0.0042=0.0102**

**Assume that the high-tensile wires have a stress-strain diagram as shown in the figure below.**



∴ **the stress (fs) corresponding to the total strain is $f_s$ =1400 N/mm$^2$ < $f'_s$ =1650 N/mm$^2$.**

**Next assuming a stress of $f_s$ =1400 N/mm$^2$**

$$T = A_{ps}\, f_s = \frac{970 * 1400}{1000} = 1358\; kN$$

$$kd = \frac{A_{ps} * f_s}{0.85\, f'_c\, b} = \frac{T}{0.85\, f'_c\, b} = \frac{1358 * 10^3}{0.85 * 34.5 * 300} = 154.5\; mm$$

$$\in_{s2} = 0.0034 * \frac{(500 - 154.5)}{154.5} = 0.0076$$

$$\in_s = 0.0042 + 0.0076 = 0.0118$$

**Which corresponds to $f_s$ =1460 N/mm$^2$**

**This is close enough to the assumed value ($f_s$=1400 N/mm$^2$).**

**Hence the ultimate moment is**

$$M_u = T * a = \frac{1460 * 970}{1000} * \frac{\left(500 - \frac{154.5}{2}\right)}{1000} = 598.7\; kN.m$$

$$which\ is\ about\ \left(\frac{655.4 - 598.7}{635.4} * 100 = 9\%\right)$$

**9% lower than the approximate value obtained in example 1:**

**In the above example, it is seen that $f'_s$ is [(1460/1650 )*100=89%] equal to 0.89 $f'_s$ which means that about 89% of the ultimate strength is developed at failure. Appreciable elongation of steel and considerable deflection and cracking of the beam occur before failure. This is not considered as a seriously over-reinforced beam. A beam would be really over-reinforced when the value of k was greater than 0.5**

## Ultimate Moment by ACI-code

**Moment design of prestressed flexural members may be computed using strength equations similar to those used for nonprestressed concrete members. Same approach of ACI Code provided for rectangular and flanged sections, with tension reinforcement only(singly reinforced) and with tension and compression reinforcement.**

**When part of the prestressed steel is in the compression zone, a method based on applicable conditions of equilibrium and compatibility of strain at factored load condition should be used.**

**The design moment strength $\phi Mu$ for other sections is computed by an analysis based on stress and strain compatibility, using the stress-strain properties of the prestressing steel. For prestressing steel, $f_{ps}$ shall be substituted for $f_y$ in strength computations.**

**As an alternative to a more accurate determination of $f_{ps}$ based on strain compatibility, the following values of $f_{ps}$ shall be permitted to be used if $f_{se}$ is not less than 0.5 $f_{pu}$.**

$f_{se} \geq 0.5\, f_{pu}$ ....(1-flex)

**where $f_{se}$=effective stress in tendon, after all losses**

**1)For member with bonded tendons:**

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}\left(\omega - \omega'\right)\right]\right\} \quad \text{....(2-flex)}$$

$$\text{Where } \omega = \rho \frac{f_y}{f'_c} \quad , \omega' = \rho' \frac{f_y}{f'_c} \quad \text{....(3-flex)}$$

**$f_{ps}$= stress in prestressing steel at nominal flexural strength**

$$\rho_p = A_{ps}/(b * d_p) \quad \text{....(4-flex)}$$

**$A_{ps}$=area of prestressing steel in flexural tension zone**

**b=width of compression face of member**

**$d_p$=distance from extreme comp. fiber to centroid of tendon**

$\gamma_p$ = 0.55 for $f_{py}/f_{pu} \geq 0.80$ .....(5-flex)

=0.40 for $f_{py}/f_{pu} \geq 0.85$

=0.28 for $f_{py}/f_{pu} \geq 0.90$

$\beta_1 = 0.65, \quad f'_c \geq 56\, N/mm^2$ .....(6-flex)

$\beta_1 = 0.85, \quad f'_c \leq 28\, N/mm^2$

$\beta_1 = [0.85 - (f'_c - 28)\frac{0.05}{7}], \quad 28 < f'_c < 56\, N/mm^2$

**When calculating $f_{ps}$ by above equation, if any compression reinforcement is taken into account, the term** $\left[\rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p}\left(\omega - \omega'\right)\right] \geq 0.17$ ....(7-flex)

**and $d' \leq 0.15 d_p$** ......(8-flex)

**2) For member with unbonded tendons and with a span-to-depth ratio ≤ 35 :**

$$f_{ps} = f_{se} + 70 + \frac{f'_c}{100\rho_p} \quad .....(9\text{-flex})$$

**But $f_{ps}$ in the above equation shall not be taken greater than the lesser of**

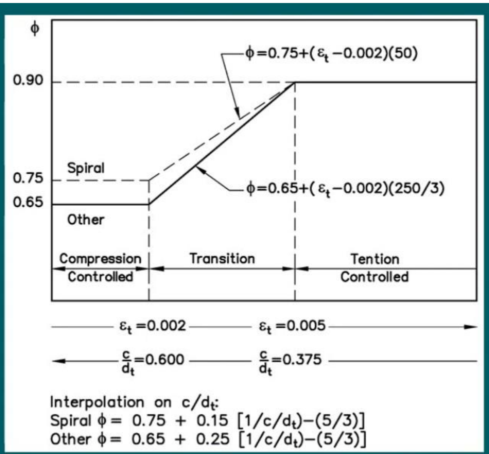
$$f_{ps} \le f_{py} \quad ......(10\text{-flex})$$
$$\le (f_{se} + 420)$$

**3)For member with unbonded tendons and with a span-to-depth ratio > 35 :**

$$f_{ps} = f_{se} + 70 + \frac{f'_c}{300\rho_p} \quad ......(11\text{-flex})$$

**But $f_{ps}$ in the above equation shall not be taken greater than the lesser of**

$$f_{ps} \le f_{py} \quad ......(12\text{-flex})$$
$$\le (f_{se} + 210)$$




The ACI 318-14 requires the strength reduction factor for tension-controlled sections to be higher that for compression-controlled sections. The strain in the tensile steel differentiates between the tension and compression controlled sections. The figure above includes a chart that relates the strength reduction factor to the tensile steel strain 13

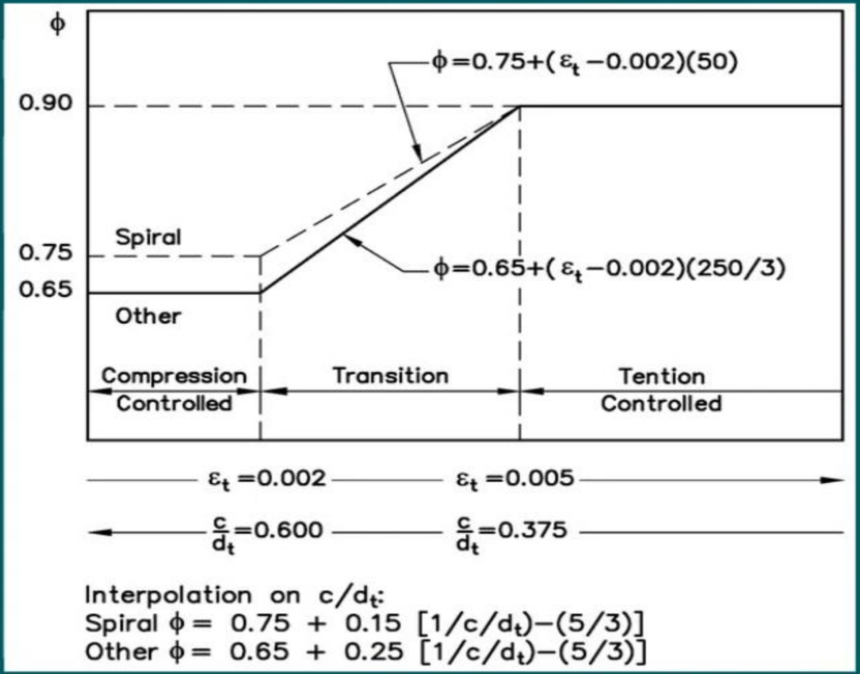
**Per ACI 318M-08Commentary Eq.(13-flex) and (14-flex) apply, based on $\in_t$ or $c/d_t$ respectively :**

$$\phi = 0.65 + (\in_t - 0.002)(250/3) \quad ......(13\text{-flex})$$

**or**

$$\phi = 0.65 + 0.25\left(\frac{1}{c/d_t} - (5/3)\right) \quad ......(14\text{-flex})$$

**Note that $d_t$ =distance from [p.21 code] extreme compression fiber to centroid of extreme layer of longitudinal tension steel.**

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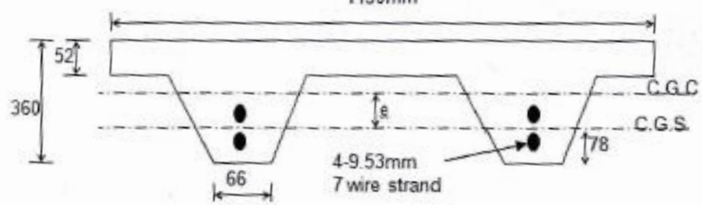
**Note that $d_t$ =distance from [p.21 code] extreme compression fiber to centroid of extreme layer of longitudinal tension steel.**

**Example 1:**

**The roof framing simply supported pretension double-tee has the following properties: $A_c=1.18*10^5$ mm², $I=1.22*10^9$ mm⁴, Yt=100mm. Yb=260mm, Aps=210 mm², f'ci=25N/mm², f'c=40N/mm², fpu=1860N/mm², constant eccentricity, simply span L=10 m, $W_G$= 2.8 kN/m , $W_D$=1.1 kN/m, $W_L$=0.8kN/m**

**Check if the ULTIMATE STRENGTH is OK per ACI 318-M08 assume $f_{py}/f_{pu}$=0.85**

**Solution:**




$$f_{ps}=f_{pu}\left\{1-\frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f'_c}+\frac{d}{d_p}(\omega-\omega')\right]\right\} \quad ......(2\text{-flex})$$

**with no rebars Eq.(2-flex)simplifies to [$\omega=\omega'=0$]:**

$$f_{ps}=f_{pu}=\left\{1-\frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f'_c}\right]\right\}$$

$\rho_p=A_{ps}/(b*d_p)$**=210/(1150*282)=0.0006475**

**Based on Eq.(5-flex) with $f_{py}/f_{pu}$=0.85;**

$\therefore \gamma_p$ **=0.40**

$$\beta_1=[0.85-(40-28)\frac{0.05}{7}]=0.7643$$

$$\therefore f_{ps}=1860\left\{1-\frac{0.4}{0.7643_1}\left[0.0006475\frac{1860}{40}\right]\right\}=1831\ \text{N/mm}^2$$

$a=A_{ps}f_{ps}/(0.85f'_c\,b)=210*1831/(0.85*40*1150)=9.832\ mm < h_f=52\ mm$

$Mn=A_{ps}f_{ps}(d_p-a/2)$

$=1.831*0.210(282-9.832/2)$

**=106.54 kN.m**

**Calculate $\epsilon_t$ :**

**c=a/ $\beta_1$ =9.832/0.7643 =12.864 mm**

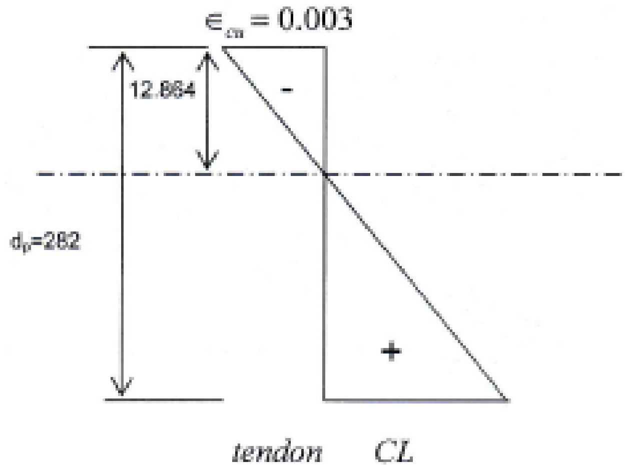
$\therefore$ **Based on CL tendon :**

$$\epsilon_t=\frac{\epsilon_{cu}}{c}*(d_t-c)$$

$\epsilon_t$ **= 0.003 * (282-12.864)/12.864**

**=0.0628 >0.005**

$\therefore$ **Tension control [see diagram P. ]**




$\therefore \phi = 0.9$ **per ACI Code**

$\therefore Mu = \phi Mn$ **= 0.9*106.54 =95.9 kN.m, the factored load resistance capacity, Mr**

**For the given loading:**

**Wu=1.2(2.8+1.1)+1.6*0.8=5.96 kN/m**

**Mu,midspan=5.96*10$^2$ /8=74.5 kN.m<Mr=95.9 kN.m Ok.**

# Reviewing

**Prestressed Beam Design**

Prestressed concrete beam design involves the selection of the beam geometrical properties, strands and material properties for the following requirements:

Design requirements:

- Flexural / bending requirements
- Shear and torsion requirements
- Deflection check
- Cracking check

Stages of design/checks:

- Prestress force transfer
- Limit state at service load
- Limit state at failure

## Load Stages

1. Prestress force transfer

- Initial prestress force, $P_i$
- Full self-weight of beam, DL

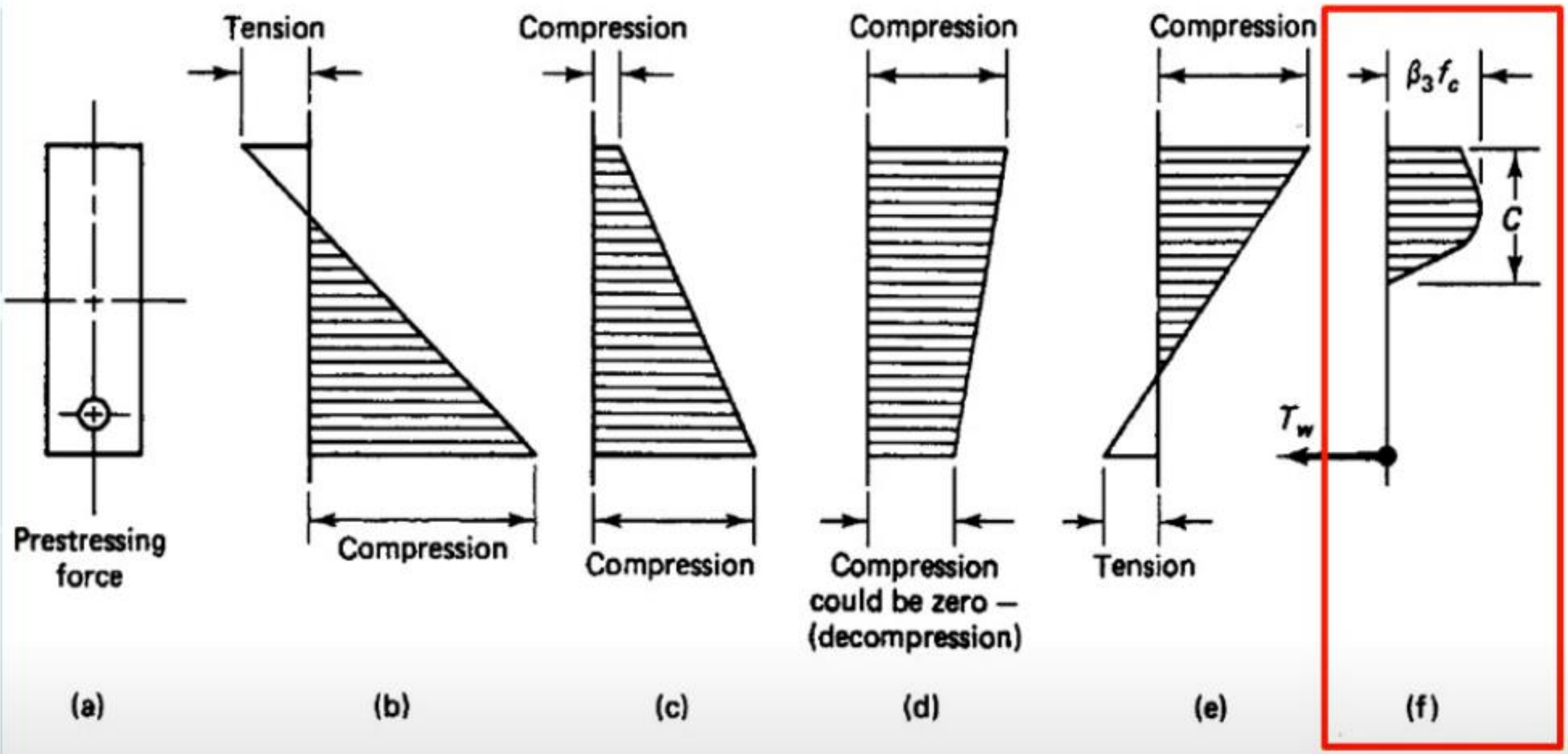
2. Limit state at service load

- Effective prestress force, $P_e$
- Full superimposed load and live load, SDL and LL

3. Limit state at failure

- Beam is overloaded to determine the nominal condition

*Typical loading history is provided in the following slide*




Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

All structural members and sections must be proportioned to meet the above criterion under the most critical load combination for all possible actions (flexural, axial load, shear, etc.):

$\phi P_n \geq P_u$ Equation 1.5

$\phi M_n \geq M_u$ Equation 1.6

$\phi V_n \geq V_u$ Equation 1.7

$\phi T_n \geq T_u$ Equation 1.8

Where:

$\phi$ = Strength reduction factor

$P_n$ = Nominal strength for axial load

$M_n$ = Nominal strength for flexural load (moment)

$V_n$ = Nominal strength for shear force

$T_n$ = Nominal strength for torsion force (moment)

## Ultimate Force (Demand)

$P_u$ = Ultimate axial load

$M_u$ = Ultimate flexural load (moment)

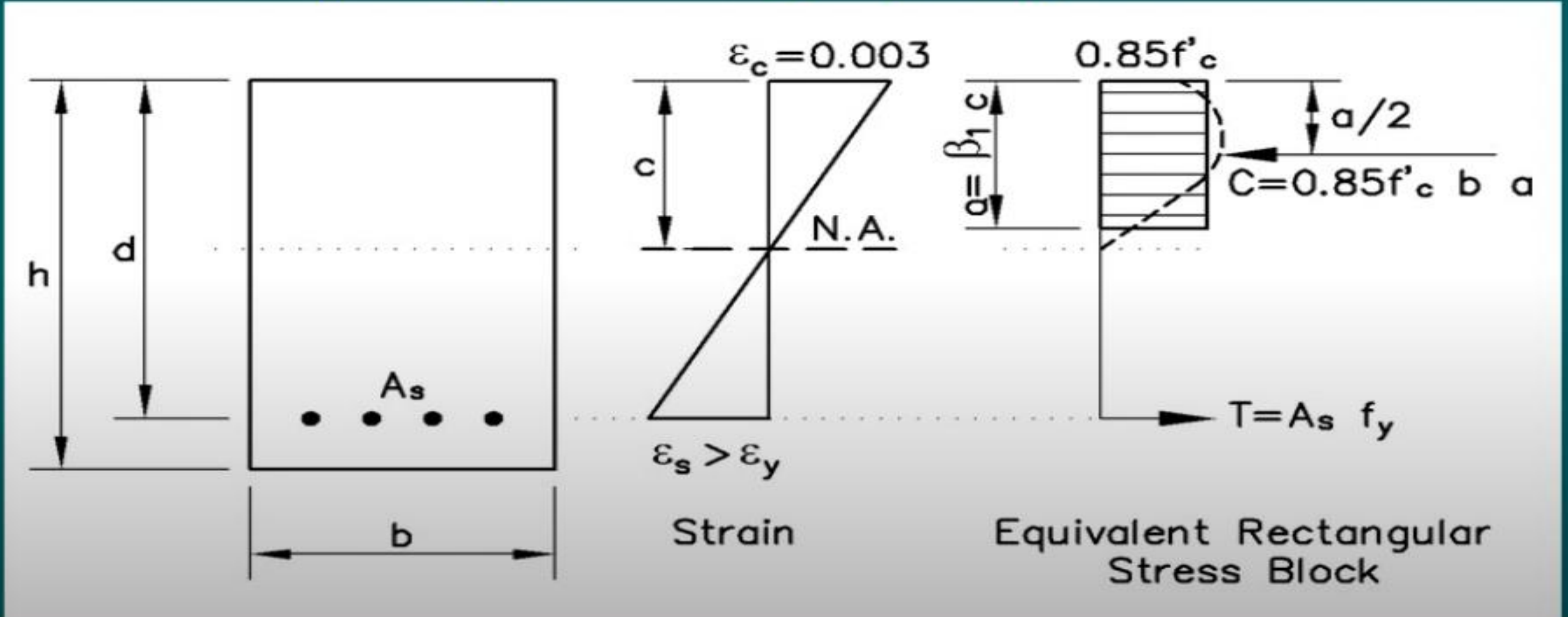
$V_u$ = Ultimate shear force

$T_u$ = Ultimate torsion force (moment)

## Analysis of Concrete Sections (Non-Prestressed)

To simplify the analysis of concrete sections, the following assumptions need to be considered:

1) Plan sections remain plan after bending
2) The compressive stress distribution is assumed to be uniform (Block)
3) Failure occurs at an ultimate concrete strain of 0.003
4) The block stress is equal to: $0.85\ f'_c$
5) The depth of the block, a, is equal to $\beta_1$ c



## Flexural Strength (Non-Prestressed)

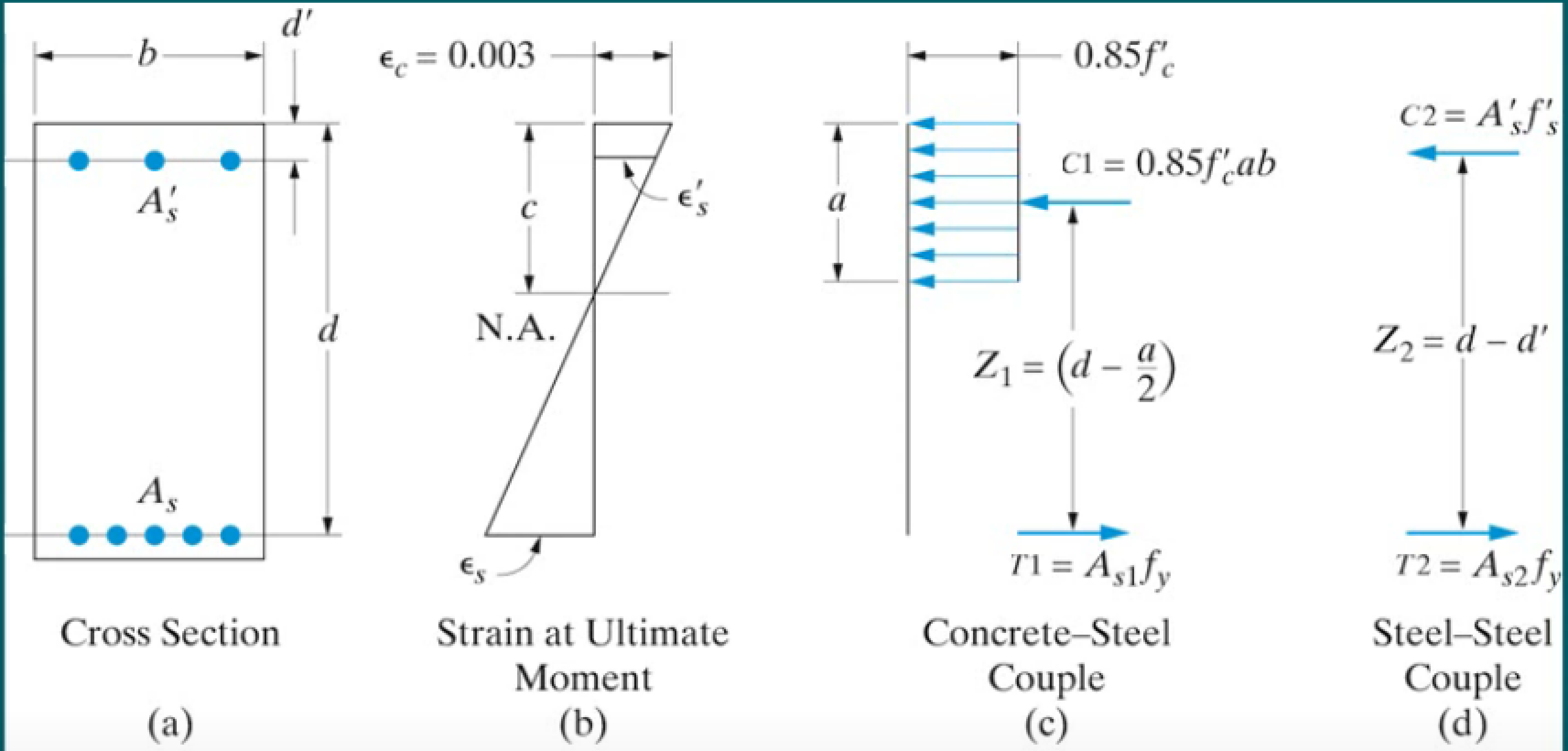
$T = A_s f_y$
$C = 0.85 f_c' b a$
$T = C$

$A_s f_y = 0.85 f'_c b a$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

The nominal flexural strength of a concrete section is equal to:

$M_n = A_s f_y (d - a / 2)$




# Beam Analysis with Compression Reinforcement

$f_{ci}$ = maximum allowable compressive stress in concrete immediately after transfer and prior to losses

$= 0.60\, f'_{ci}$

$f_{ti}$ = maximum allowable tensile stress in concrete immediately after transfer and prior to losses

$= 3\sqrt{f'_{ci}}$ (the value can be increased to $6\sqrt{f'_{ci}}$ at the supports for simply supported members)

$f_c$ = maximum allowable compressive stress in concrete after losses at service-load level

$= 0.45\, f'_c$ or $0.60\, f'_c$ when allowed by the code

$f_t$ = maximum allowable tensile stress in concrete after losses at service load level

$= 6\sqrt{f'_c}$ (the value can be increased in one-way systems to $12\sqrt{f'_c}$ if long-term deflection requirements are met)

then the *actual* extreme fiber stresses in the concrete cannot exceed the values listed.

***Stress at Transfer***

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_{ci}$$

***Effective Stresses after Losses***

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_t$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_c$$

***Service-load Final Stresses***

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \leq f_c$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \leq f_t$$

where $M_T = M_D + M_{SD} + M_L$

$P_i$ = initial prestress
$P_e$ = effective prestress after losses
$t$ denotes the top, and $b$ denotes the bottom fibers
$e$ = eccentricity of tendons from the concrete section center of gravity, cgc
$r^2$ = square of radius of gyration
$S^t/S_b$ = top/bottom section modulus value of concrete section

The *decompression stage* denotes the increase in steel strain due to the increase in load from the stage when the effective prestress $P_e$ acts *alone* to the stage when the addi-

# Notayions for Flexture

In the most general case, the uncracked cross section of a prestressed concrete beam can be characterized, for design purposes, by a number of variables and geometric properties. Geometric properties can be determined directly from the dimensions of the cross section. Referring to Fig. 4.7, the following notations and definitions will be used:

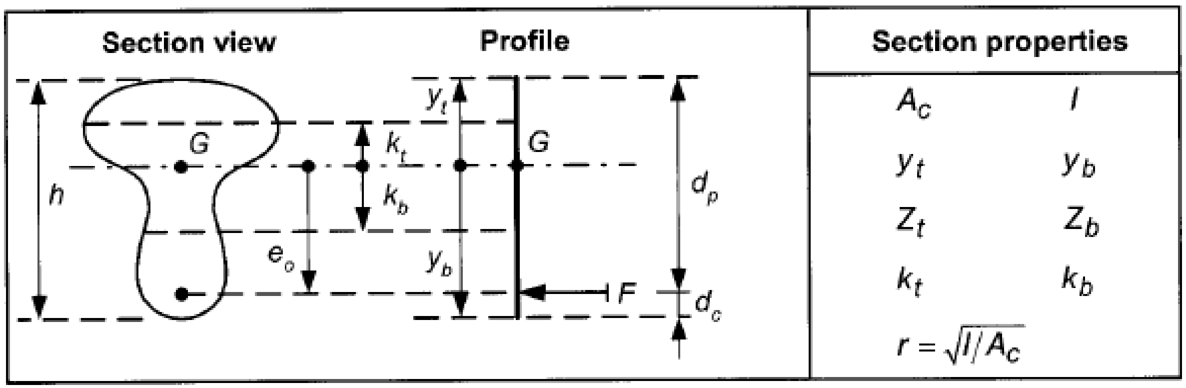



**Figure 4.7 Typical characterization of beam cross section.**

$A_c$ = area of concrete cross section (it may indicate the net or gross area depending on the problem at hand and whether preliminary or final designs are considered; practically, in pretensioned members it is taken as the gross area, while in posttensioned members it is often the net or the transformed area)

$A_{ps}$ = area of prestressing steel

$d_p$ = distance from extreme compression fiber to the centroid of prestressing steel (or force)

$d_c$ = concrete cover from the precompressed tensile fiber to the centroid of prestressing steel ($d_c$ can generally be estimated in a preliminary design and revised for the final design)

$(d_c)_{\min}$ = minimum feasible value of $d_c$

$e_o$ = eccentricity of the prestressing force (or centroid of prestressing steel) with respect to the centroid of the concrete section ($e_o$ varies along the member, thus $e_o(x)$ is used when needed)

$F$ = final prestressing force or effective prestressing force after all losses

$h$ = total height of concrete section ($h = y_t + y_b = d_p + d_c$)

$I$ = moment of inertia of the section with respect to an axis passing by its centroid (it generally implies gross sectional inertia; for a final design it may indicate the transformed inertia)

$y_t$ = distance from the centroid of the concrete section to the extreme top fiber

$y_b$ = distance from the centroid of the concrete section to the extreme bottom fiber

fiber

$\rho_p$ = $A_{ps}/bd_p$ prestressed reinforcement ratio ($b$ is the width of a rectangular section or flange width of $T$ section)

$\sigma$ = stress in concrete in general (it will be used unless a widely standard notation such as $f_c'$ applies)

$Z_t$ = $I/y_t$ = section modulus with respect to the top fiber

$Z_b$ = $I/y_b$ = section modulus with respect to the bottom fiber

$r$ = $\sqrt{I/A_c}$ = radius of gyration of the section

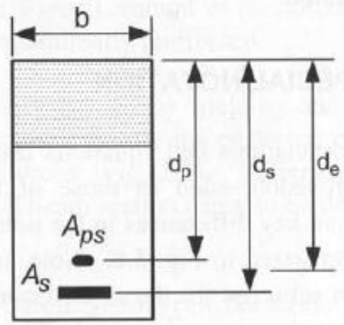
$k_t$ = $-I/A_c y_b$ = $-Z_b/A_c$ = $-r^2/y_b$ = distance from the centroid of the concrete section to the upper limit of the central kern

$k_b$ = $I/A_c y_t$ = $Z_t/A_c$ = $r^2/y_t$ = distance from the centroid of the concrete section to the lower limit of the central kern

$\gamma$ = $(-k_t + k_b)/h$ = $I/(A_c y_t y_b)$ = $r^2/(y_t y_b)$ = $Z_t/(A_c y_b)$ = $Z_b/(A_c y_t)$ = geometric efficiency of the cross section with respect to bending

## Permissible stresses in concrete in prestressed flexural members

| Condition | Class U | Class T | Class C* |
|---|---|---|---|
| a. Extreme fiber stress in compression immediately after transfer (except as in b) | $0.60f'_{ci}$ | $0.60f'_{ci}$ | $0.60f'_{ci}$ |
| b. Extreme fiber stress in compression at ends of simply supported members | $0.70f'_{ci}$ | $0.70f'_{ci}$ | $0.70f'_{ci}$ |
| c. Extreme fiber stress in tension immediately after transfer (except as in d) | $3\sqrt{f'_{ci}}$ | $3\sqrt{f'_{ci}}$ | $3\sqrt{f'_{ci}}$ |
| d. Extreme fiber stress in tension immediately after transfer at the end of simply supported members† | $6\sqrt{f'_{ci}}$ | $6\sqrt{f'_{ci}}$ | $6\sqrt{f'_{ci}}$ |
| e. Extreme fiber stress in compression due to prestress plus sustained load | $0.45f'_c$ | $0.45f'_c$ | — |
| f. Extreme fiber stress in compression due to prestress plus total load | $0.60f'_c$ | $0.60f'_c$ | — |
| g. Extreme fiber stress in tension $f_t$ in precompressed tensile zone under service load | $\leq 7.5\sqrt{f'_c}$ | $>7.5\sqrt{f'_c}$ and $\leq 12\sqrt{f'_c}$ | — |

| ACI Code | This Book (also AASHTO-LRFD) |
|---|---|
| **Notation:**<br>$d$ to nonprestressed reinforcement<br>$d_p$ to prestressed reinforcement<br>$d'$ to compression steel<br>$d_t$ to extreme layer of tensile reinforcement<br><br>**Definitions:**<br>• Reinforcement ratios: $\rho_p, \rho, \rho'$<br>$\rho = \frac{A_s}{bd}; \rho_p = \frac{A_{ps}}{bd_p}; \rho' = \frac{A'_s}{bd}$<br>• Reinforcement indices:<br>$\omega = \rho \frac{f_y}{f'_c} = \frac{A_s f_y}{bdf'_c}$<br>$\omega_p = \rho_p \frac{f_{ps}}{f'_c} = \frac{A_{ps} f_{ps}}{bd_p f'_c}$<br>$\omega' = \rho' \frac{f'_y}{f'_c}$<br>$\omega_p + \frac{d}{d_p}(\omega - \omega')$<br>$\omega_{pw}, \omega_w, \omega'_w$, reinforcement indices for flanged sections computed as for $\omega_p, \omega, \omega'$ except that $b$ shall be the web width and reinforcement area shall be that required to develop compressive strength of web only. | **Notation:**<br>$d_s$ to tensile force in nonprestressed reinforcement<br>$d_p$ to tensile force in prestressed reinforcement<br>$d_e$ to centroid of tensile force<br>$d'$ to force in compression steel<br><br>**Definitions:**<br>$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$<br>$\omega_e = \omega_p + \omega_s - \omega'$<br>$\omega_e = \frac{A_{ps} f_{ps} + A_s f_y - A'_s f'_y}{bd_e f'_c}$<br><br> |

$A_{ps}$ = area of prestressing reinforcement in the tensile zone

$f_{ps}$ = stress in the prestressing steel at nominal flexural resistance of the section (see related recommendations below)

$d_p$ = distance from extreme compression fiber to centroid of tensile force in prestressed steel

$A_s$ = area of nonprestressed tension reinforcement

$f_y$ = specified yield strength of nonprestressed tensile reinforcement

$d_s$ = distance from extreme compression fiber to centroid of tensile force in nonprestressed steel.

# Thank You