الجامعة	الانبار
الكلية	العلوم
القسم	الرياضيات
المرحلة	الرابعة
اسم المادة باللغة العربية	التحليل العقدي
اسم المادة باللغة الانكليزية	Complex Analysis
اسم التدريسي	دكتورة دنيا علاوي جروان
عنوان المحاضرة باللغة العربية	مناقشة
عنوان المحاضرة باللغة الإنكليزية	Tutorial
رقم المحاضرة	L8

Q1// Prove that $\lim\limits_{z\to i} \frac{1}{z} = -i$ by def. of lim

Sol: $\forall \epsilon > 0, \exists \delta > 0$ such that $|z-z_0|<\delta$
$\Longrightarrow |f(z)-w_0| < \epsilon$.

Let $\epsilon > 0$. To find $\delta > 0$

$\left|\frac{1}{z}+i\right| < \epsilon \quad ; \quad |z-i| < \delta$ *

$\left|\frac{1}{z}+i\right| < \epsilon \Rightarrow \left|\frac{1+iz}{z}\right| < \epsilon$

$\Rightarrow \left|\frac{iz+1}{z}\right| < \epsilon \Rightarrow \left|\frac{iz-i^2}{z}\right| < \epsilon$

$\Rightarrow \left|\frac{i(z-i)}{z}\right| < \epsilon \Rightarrow |i| \frac{|z-i|}{|z|} < \epsilon$

$\Rightarrow \frac{|z-i|}{|i|} < \epsilon \Rightarrow |z-i| < \epsilon$ **

so $\delta = \epsilon$

Q2//

$\lim\limits_{z\to z_0} \bar{z} = \bar{z}_0$

by def $\forall \epsilon > 0, \exists \delta > 0, |f(z)-w_0| < \epsilon$ where $|z-z_0| < \delta$

if $\lim\limits_{z\to z_0} f(z) = w_0$

So $\forall \epsilon > 0$, we must find $\delta > 0 \ni |\bar{z}-\bar{z}_0| < \epsilon$
when $|z-z_0| < \delta$

$\Rightarrow |\bar{z}-\bar{z}_0| = |\overline{z-z_0}| = |z-z_0| < \delta = \epsilon$

So $\delta = \epsilon$

Q3// prove that $\lim\limits_{z \to 0} \dfrac{\bar{z}^2}{z} = 0$

Sol

$\forall \epsilon > 0, \exists \delta > 0 \ni \left| \dfrac{\bar{z}^2}{z} - 0 \right| < \epsilon$, when

$|z - 0| < \delta \Rightarrow |z| < \delta$

$\Rightarrow \left| \dfrac{\bar{z}^2}{z} - 0 \right| = \left| \dfrac{\bar{z}^2}{z} \right| = \dfrac{|\bar{z}^2|}{|z|} = \dfrac{|\bar{z}|^2}{|z|} = \dfrac{|z|^2}{|z|}$

$= |z| < \delta$

$\Rightarrow \epsilon = \delta$

Q4// prove that if $\lim_{z\to z_0} f(z) = w_0$ and $\lim_{z\to z_0} g(z) = w_1$, then $\lim (f(z)+g(z)) = w_0 + w_1$

proof: $\because \lim f(z) = w_0$

So $\forall \epsilon > 0 \Rightarrow \exists\ \delta_1 > 0$ such that

if $|z - z_0| < \delta_1$ then $|f(z) - w_0| < \epsilon/2$

Also $\because \lim_{z\to z_0} g(z) = w_1$

$\Rightarrow \forall \ni \epsilon > 0, \exists\ \delta_2 > 0$ such that

if $|z - z_0| < \delta_2$ then $|g(z) - w_1| < \epsilon/2$

Let $\delta = \min\{\delta_1, \delta_2\} \Rightarrow |z - z_0| < \delta$

and

$$|f(z) + g(z) - (w_0 + w_1)| = |f(z) - w_0 + g(z) - w_1|$$

$$\leq |f(z) - w_0| + |g(z) - w_1|$$

$$< \epsilon/2 + \epsilon/2 = \epsilon$$

$$\Rightarrow \lim_{z\to z_0} (f(z) + g(z)) = w_0 + w_1$$

$\square$

Example :- $g(z) = \sqrt{r}\, e^{i\theta/2}$

so $g(z) = \sqrt{r}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$

$\Rightarrow U = \sqrt{r}\cos\frac{\theta}{2} \qquad V = \sqrt{r}\sin\frac{\theta}{2}$

$U_r = \frac{1}{2} r^{-1/2}\cos\frac{\theta}{2} \quad , U_{rr} = \frac{-1}{4} r^{-3/2}\cos\frac{\theta}{2}$

$U_\theta = -\frac{1}{2} r^{+1/2}\sin\frac{\theta}{2} \Rightarrow U_{\theta\theta} = \frac{-1}{4} r^{1/2}\cos\frac{\theta}{2}$

$\Rightarrow r^2 U_{rr} + r U_r + U_{\theta\theta} = r^2\left(\frac{-1}{4} r^{-3/2}\cos\frac{\theta}{2}\right) + r\left(\frac{1}{2} r^{-1/2}\cos\frac{\theta}{2}\right)$

$+ \frac{-1}{4} r^{1/2}\cos\frac{\theta}{2} =$

$= \frac{-1}{4} r^{1/2}\cos\frac{\theta}{2} + \frac{1}{2} r^{1/2}\cos\frac{\theta}{2} - \frac{1}{4} r^{1/2}\cos\frac{\theta}{2}$

$= \frac{-1}{2} r^{1/2}\cos\frac{\theta}{2} + \frac{1}{2} r^{1/2}\cos\frac{\theta}{2} = 0$

$\therefore$ Laplaces Equation is hold.

$\therefore U$ is hormonic.