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اسم المادة باللغة العربية	التحليل العقدي
اسم المادة باللغة الانكليزية	Complex Analysis
اسم التدريسي	دكتورة دنيا علاوي جروان
عنوان المحاضرة باللغة العربية	الدوال العقدية
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رقم المحاضرة	L1

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Analytic Functions

Def: Let S be a set of complex numbers. A function f defined on S is a rule which assigns to each complex number Z in S complex number ω. The number ω is called a value of f at z, and denoted by f(z). that is ω = f(z). The set S is called the domain of f (D_f).

Example: $\omega = z^2 + 2z - 7$, The domain of f is $\mathbb{C}$.

Example: $\omega = \frac{1}{z^2+4}$, $D_f = \mathbb{C} / \mp 2i$

Note: If $\omega = f(z)$, $z = x+iy$ and if $\omega = u+iv$, then each of u, v depend on the two real variables x, y. That is, $\omega = f(z)$ can be represented by the pair of real function with real variables, so $f(z) = u(x,y) + iv(x,y)$.

Example: $\omega = f(z) = z^2$

$\Rightarrow \omega = f(z) = (x+iy)^2 = x^2 - y + 2xyi$

So $u + iv = x - y + i2xy$

$\Rightarrow u = x - y$ and $v = 2xy$.

Example $f(z) = z^{-1} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$\Rightarrow u(x,y) = \frac{x}{x^2+y^2}$, $v(x,y) = \frac{-y}{x^2+y^2}$

هذا يعني ان كلا من القرارين u, v هو دالة للمتغيرين x, y اي ان كلا من الجزء الحقيقي والخيالي للدالة f عبارة عن دالة حقيقية لمتغيرين وهنا يمكن القول ا

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لدراسة دالة المتغير المركب يمكن دراسة دالتين حقيقيتين لمتغيرين

Def: The function $w = f(z)$ is called a single-valued function in region D, if $\forall z \in D$, $\exists$ only a single value of function $f(z)$

Def: The function $f(z) = w$ is called a multiple-valued function in the region D if $\forall z \in D$, $\exists$ a more than value of a function $f(z)$.

Example: $w = z^2 + z + 1$, is single valued function

Example $w = z^{\frac{1}{3}}$, it has three values w_0, w_1, w_2, since $z = r(\cos\theta + i\sin\theta)$ by demover's formula:

$$w = z^{\frac{1}{3}} = r^{\frac{1}{3}}\left(\cos\frac{\theta + 2k\pi}{3} + i\sin\frac{\theta + 2k\pi}{3}\right)$$

for $k = 0, 1, 2$.

Example 1: The function $w = z^2$ is a singular function (singular-valued function) of z. On the other hand, if $w = z^{1/2}$, then to each value of z, there are two values of w. Hence, the function

$$w = z^{1/2}$$

is a multiple-valued (in this case two-valued) function of z.

If the polar coordinates r and θ, instead of x and y, are used, then

$$u + iv = f(re^{i\theta})$$

where $w = u + iv$ and $z = re^{i\theta}$. In this case, we write

$$f(z) = u(r,\theta) + iv(r,\theta)$$

Example 2: If $f(z) = z^2$, then

$$f(x+iy) = (x+iy)^2 = x^2 - y^2 + i(2xy)$$

Hence

$$u(x,y) = x^2 - y^2 \text{ and } v(x,y) = 2xy$$

when we use polar coordinate, we have

$$u(r,\theta) = r^2\cos 2\theta \text{ and } u(r,\theta) = r^2 \sin 2\theta$$

Question: what happens when in either equations (1) and (2) the function v always has a value zero?

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