Nuclear Transformations

Radioactivity

- Radioactivity is the spontaneous nuclear transformations in unstable atoms that result in the formation of new elements(Isotopes). These transformations are characterized by one of several different mechanisms, including alpha-particle emission, beta-particle and positron emission, and orbital electron capture. Each of these reactions may or may not be accompanied by gamma radiation.

- The exact mode of radioactive transformation depends on

- The energy available for the transition. The available energy, in turn, depends on two factors: on the particular type of nuclear instability, whether the neutron-to-proton ratio is too high or too low for the particular nuclide under consideration

- The mass–energy relationship among the parent nucleus, daughter nucleus, and emitted particle.

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Isotopes

- It has been found that for any particular element the number of neutrons within the nucleus is not constant. Oxygen, for example, consists of three nuclear species: one whose nucleus has 8 neutrons, one of 9 neutrons, and one of 10 neutrons. In each of these three cases, of course, the nucleus contains 8 protons. The atomic mass numbers of these three species are 16, 17, and 18, respectively. These three nuclear species of the same element are called isotopes of oxygen. Isotopes of an element are atoms that contain the same number of positive nuclear charges and have the same extra nuclear electronic structure but differ in the number of neutrons. Most elements contain several isotopes.

Radioactive Decay Law

- Let us say that in the sample of radioactive material there are **N** nuclei which have not decayed at a certain time, t. So what happens in the next brief period of time? Some nuclei will decay for sure. But how many?

- The number which will decay will depend on overall number of nuclei, N, and also on the length of the brief period of time. In other words the more nuclei there are the more will decay and the longer the time period the more nuclei will decay. Let us denote the number which will have decayed as dN and the small time interval as dt.

- So we have reasoned that the number of radioactive nuclei which will decay during the time interval from t to t+dt must be proportional to N and to dt. therefore:

$$-dN \ \alpha \ \lambda N.dt$$

turning the proportionality in this equation into an equality we can write

$$-dN = \lambda \, N \, dt$$

where the constant of proportionality, λ, is called the Decay Constant.

Dividing across by N we can rewrite this equation as:

$$-\frac{dN}{N} = \lambda \,.\, dt$$

So this equation describes the situation for any brief time interval, dt. To find out what happens for all periods of time we integrate the above equation. Expressing this more formally we can say that for the period of time from t =0 to any later time t, the number of radioactive nuclei will decrease from N_o to N_t, so that:

$$\int_{N0}^{Nt} \frac{dN}{N} = \lambda \int_{0}^{t} dt$$

$$\ln\left(\frac{N_t}{N_0}\right) = -\lambda \, t$$

$$\frac{N_t}{N_0} = \exp^{-\lambda \, t}$$

$$N(t) = N_0 \, e^{-\lambda t}$$

This final expression is known as the Radioactive Decay Law. It tells us that the number of radioactive nuclei will decrease in an exponential fashion with time with the rate of decrease being controlled by the Decay constant

$$N = \frac{6.02 \, X10^{23} \, atom/mol}{A \, g/mol} \, X \, W \cdot g$$

where A is the atomic weight and W is the weight of the sample.

$$H \cdot W - Let \quad N(t) = \frac{N_0}{2} \quad and \quad t = \frac{t}{2} \quad prove \quad that \quad t^{\frac{1}{2}} = \frac{0.693}{\lambda}$$

The term half-life ($t^{1/2}$) of a radioactive substance is defined as the time required for either the activity or the number of radioactive atoms to decay to half the initial value.

H.W- Why a negative sign is given to N?

H.W -Given that the transformation rate constant for ^{226}Ra is 4.38×10^{-4} 1/yr, calculate the half-life for radium.

Examples of the half lives of some radioisotopes are given in the following table. Notice that some of these have a relatively short half life. These tend to be the ones used for medical diagnostic purposes because they do not remain radioactive for very long following administration to a patient and hence result in a relatively low radiation dose.

Radioisotope	Half Life (approx.)
^{81m}Kr	13 seconds
^{99m}Tc	6 hours
^{131}I	8 days
^{51}Cr	1 month
^{137}Cs	30 years
^{241}Am	462 years
^{226}Ra	1620 years
^{238}U	4.51×10^9 years

H.W. What is the logistical problem when we wish to use radioisotope with short half life for medical diagnostic purposes ؟

a) The half-life of 99mTc is 6 hours. After how much time will 1/16th of the radioisotope remain?
b) b) Verify your answer by another means.

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- For some applications, as in the case of dosimetry of internally deposited radioactive material (discussed later), it is convenient to use the average life of the radioisotope. The average life is defined simply as the sum of the lifetimes of the individual atoms divided by the total number of atoms originally present.

- The transformation rate of a quantity of radioisotope containing N atoms is λN. During the time interval between t and t + dt, the total number of transformations is λ N dt. Each of the atoms that decayed during this interval, however, had existed for a total lifetime t since the beginning of observation on them. The sum of the lifetimes, therefore, of all the atoms that were transformed during the time interval between t and t + dt, after having survived since time t = 0, is t λN dt. The average life, τ, of the radioactive species is
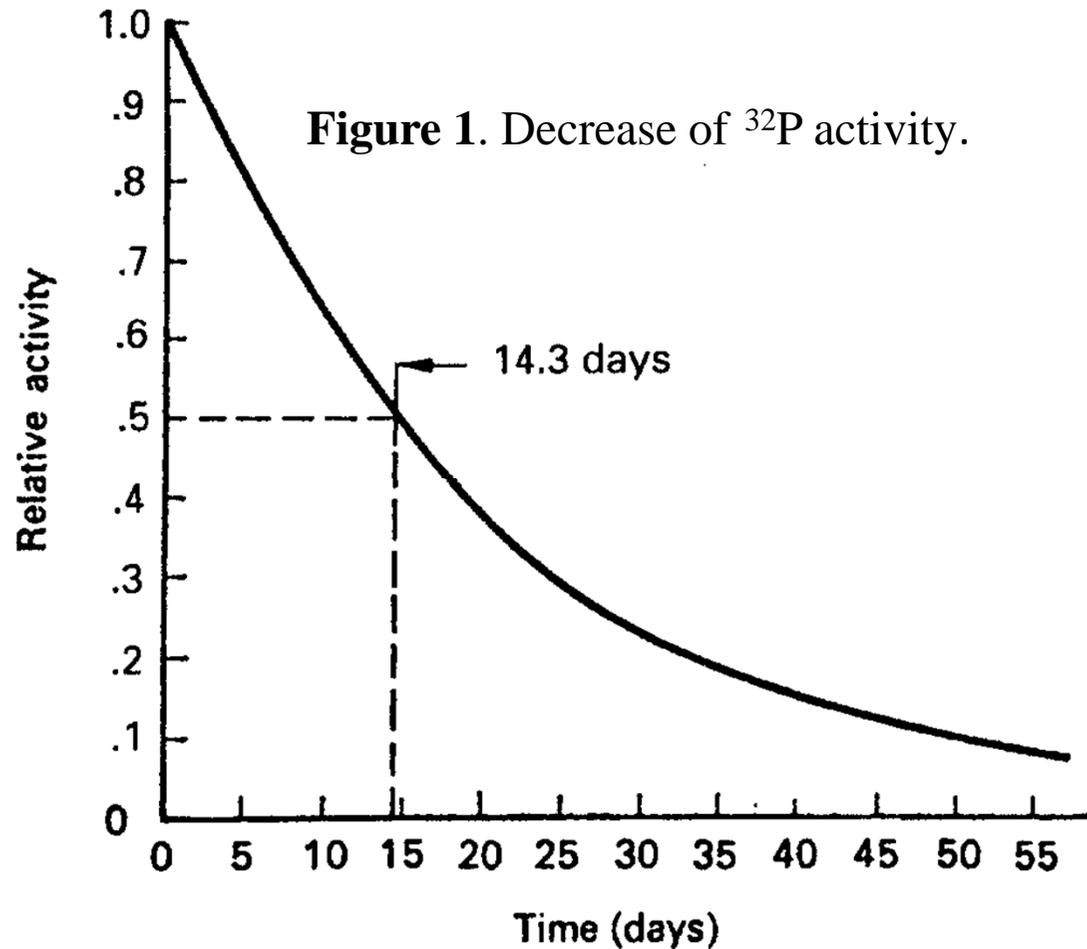
$$\tau = \frac{1}{N_0} \int_0^\infty t\lambda \ N dt$$

$$\tau = \frac{1}{N_0} \int_0^\infty t\lambda \ N_0 e^{-\lambda t} \ dt$$

   This expression, when integrated by parts, shows the value for the mean life of a radioisotope to be

$$\tau = \frac{1}{\lambda}$$

   H.W: what is the relationship between the half-life and the mean life?

Figure 1. Decrease of ^{32}P activity.

From the definition of the half-life, it follows that the fraction of a radionuclide remaining after *n* half-lives correction factor is given by the relationship:

$A/A_0 = 1/2^n$

where A_0 is the original quantity of activity and A is the activity left after *n* half-lives

Activity

The rate of decay is referred to as the activity of a radioactive material. If ΔN/Δt is replaced by A, the symbol for activity, then:

$$A = -\lambda \ N \ dt$$

$$A = A_0 \ e^{-\lambda t}$$

Where is A the activity remaining at time t, and A_0 is the original activity equal to λN_0.
The unit of activity is the curie (Ci), defined as:

$1mCi = 3.7 \times 10^{10}$ disintegrations/sec (dps)1
The SI unit for activity is the becquerel (Bq). The becquerel is a smaller but more basic unit than the curie and is defined as:

$1mCi = 10^{-3}Ci = 3.7 \times 10^{10}$ (dps)[1]
$1Bq = dps = 2.70 \ x10^{-11}Ci$

H.W
•Calculate the number of atoms in 1 g of ^{226}Ra.
•What is the activity of 1 g of ^{226}Ra (half-life = 1,622 years)?

H.W What is the minimum mass of ^{99}mTc that can have a radioactivity of 1 MBq? Assume the half-life is 134 minutes and that Avogadro's Number is 6.023 x 10^{23}.

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Energy dose

Radioactive sources with a large lifetime t (or, equivalently, half-life $T_{1/2}$) naturally have lower activities if a given number of nuclei is considered. The activity in Bq does not say very much about possible biological effects. These are related to the deposited energy by the radioactive source in matter. The energy dose D (**absorbed energy**)

ΔW per mass unit Δm),

$$D = \frac{\Delta W}{\Delta m} = \frac{1}{\rho} \frac{\Delta W}{\Delta V} \qquad (2.13)$$

(ρ – density, ΔV – volume element^3), is measured in gray: **gray**

$$1 \text{ gray (Gy)} = 1 \text{ joule (J)} / 1 \text{ kilogram (kg)} . \qquad (2.14)$$

Gray is related to the old unit rad (radiation absorbed dose, $1 \text{ rad} = 100 \text{ erg/g}$; still in use in the US) according to:[4]

$$1 \text{ Gy} = 100 \text{ rad} . \qquad (2.15)$$

1 Gy = 1 J/1 kg
1 Gy = 100 rad

For indirectly ionizing radiation (i.e. photons and neutrons, but not electrons and other charged particles) a further quantity characterizing the energy dose, the 'kerma', is defined. Kerma is an abbreviation for "kinetic energy released per unit mass". The k erma kis defined as the sum of the initial energies of all charged particles, E, liberated in a volume element V by indirectly ionizing radiation divided by the mass m of this volume element:

$$k = \frac{\Delta E}{\Delta m} = \frac{1}{\rho} \frac{\Delta E}{\Delta V} , \qquad (2.16)$$

where ρ is the density of the absorbing material.

Radioactivity Units

U^{238} and its daughter ^{234}Th each contain about the same number of atoms per gram—approximately 2.5×10^{21}. Their half-lives, however, are greatly different; ^{238}U has a half-life of 4.5×10^9 years while ^{234}Th has a half-life of 24.1 days. Thorium^{234}, therefore, is transforming 6.8×10^{10} times faster than ^{238}U. When using radioactive material, the radiations are the center of interest. In this context, therefore, 15 mg of ^{234}Th is about equivalent in activity to 1 g of ^{238}U. These examples show that when interest is centered on radioactivity, the gram is not a very useful unit of quantity. To be meaningful, the unit for quantity of must be based on the number of radioactive decays occurring within a prescribed time in the radioactive material. This quantity—the number of decays within a given time- is   called   the   activity.

- Two units for measuring the activity are used. The SI unit is called the Becquerel, symbolized by Bq, and is defined as follows:

-

  1-The becquerel is that quantity of radioactive material in which one atom is transformed per second (tps).

- 1 Bq = 1 tps = 1 dps.

-

   2-The curie, symbolized by Ci, is the unit for quantity of radioactivity that was used before the adoption of the SI units and the becquerel. The curie, which originally was defined as the activity of 1 g of ^{226}Ra, is now more explicitly defined as : the activity of that quantity of radioactive material in which 3.7×10^{10} atoms are transformed in one second.The curie is related to the becquerel by

- 1 Ci = 3.7×10^{10} Bq.

Units of Radiation Exposure

Roentgen (R)

The roentgen is a unit used to measure a quantity called exposure. This can only be used to describe an amount of gamma and X-rays, and only in air. One roentgen is equal to depositing in dry air enough energy to cause $2.58E^{-4}$ coulombs per kg.

Rad (radiation absorbed dose)

The rad is a unit used to measure a quantity called absorbed dose. This relates to the amount of energy actually absorbed in some material, and is used for any type of radiation and any material. One rad is defined as the absorption of 100 ergs per gram of material. The unit rad can be used for any type of radiation, but it does not describe the biological effects of the different radiations.

Rem (roentgen equivalent man)

The rem is a unit used to derive a quantity called equivalent dose. This relates the absorbed dose in human tissue to the effective biological damage of the radiation. Not all radiation has the same biological effect, even for the same amount of absorbed dose. Equivalent dose is often expressed in terms of thousandths of a rem, or mrem. To determine equivalent dose (rem), you multiply absorbed dose (rad) by a quality factor (Q) that is unique to the type of incident radiation.

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RAD Units

Quantity	Name	Symbol	SI Unit
Exposure	roentgen	R	air kerma (Gya)
			C/kg
Absorbed Dose	rad	rad	gray (Gy1)
Effective Dose	rem	rem	seivert (Sv)
Radioactivity	curie	Ci	becquerel (Bq)

Calculating SI units

R x 0.01 = Gya

rad x 0.01 = Gy1   rem x 0.01 = Sv

Ci x 3.7 x10 ^{10} = Bq   R = 2.58 x 10^{-4} = C/kg

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Specific Activity

- We note that the becquerel (or curie) used as a unit of quantity, does not
- imply anything about the mass or volume of the radioactive material in which the specified number of transformations occur. The concentration of radioactivity, or
- the relationship between the mass of radioactive material and the activity, is called the specific activity. Specific activity is the number of becquerels (or curies) per unit mass or volume. The specific activity of a carrier-free (pure) radioisotope-a radioisotope that is not mixed with any other isotope of the same element-may be calculated as
- follows:
- If λ is the transformation constant in units of reciprocal seconds, then the number of transformations per second and, hence, the number of becquerels in an aggregation of N atoms, is simply given by λN.
- If the radionuclide under consideration weighs 1 g, then, according to Eq below the number of atoms is given by

$$N = \frac{6.02 \, X \, 10^{23} \, atom/mol}{A \, g/mol} \, X \, W.g$$

- by using the relationship between the specific activity and weight of an isotope in terms of the isotope's half-life .
- Prove the specific activity is : $SA = \lambda N$.

$$SA = \frac{4.18 \times 10^{23}}{A \times T} \quad ------- (a)$$

Note that Eq. (a) is valid only if T *are given in time units* of seconds. A more convenient form for calculating specific activity may be derived by making use of the fact that there are 3.7×10^{10} tps in 1 g of ^{226}Ra. The specific activity, therefore, of ^{226}Ra is 3.7×10^{10} Bq/g. The ratio of the specific activity of any radionuclide, SA_i, to that of ^{226}Ra is

,
$$\frac{SA_i}{3.7 \times 10^{10} \frac{Bq}{g}} = \left(\frac{4.18 \times 10^{23} \; A_i \times T_i}{4.18 \times 10^{23} \; A_{Ra} \times T_{Ra}} \right)$$

$$SA_i = 3.7 \times 10^{10} \; \frac{A_{Ra} \times T_{Ra}}{A_i \times T_i} \; \frac{Bq}{g}$$

where A_{Ra}, the atomic weight of ^{226}Ra, is 226, A_i is the atomic weight of the radioisotope whose specific activity is being calculated, and T_{Ra} and T_i are the half-lives of the radium (1600 year) and the radionuclide i.

- **H.W:** A solution of Hg(NO3)2 tagged with 203Hg has a specific activity of 1.5×10^5 Bq/mL. If the concentration of mercury in the solution is 5
- mg/mL.

- (a) what is the specific activity of the mercury Hg?

- (b) what fraction of the mercury in the $Hg(NO_3)_2$ is ^{203}Hg?

- Hint : 5.8×10^{-8}

H.W: If 2 g of carbon from a piece of wood found in an ancient temple is analyzed and found to have an activity of 10 transformations/minute/gram, what is the age of the wood if the current specific activity of ^{14}C in carbon is assumed to have been constant at 15 transformations/minute/gram?

Hint: the half-life for 14C is 5730 years

Naturally Occurring Radiation

- There are three sources for naturally occurring sources of radiation

1-The oldest source is cosmic radiation, which is believed to have originated at the birth of the universe, about 13–14 billion years ago.

2-A second source is from primordial radioactive elements that were created when the earth was born about 4.5 billion years ago.

3-A third source of naturally occurring radioactivity and radiation is cosmogenic radioactivity. The production of cosmogenic radioactivity is an ongoing process as cosmic radiation interacts with the atmosphere to produce radionuclides.

4- Another transient source of radioactivity was a naturally occurring nuclear reactor for example Oklo reactor.

Serial Transformation

- They were members of a long series of isotopes of various elements,

- Uranium, the most abundant of the radioactive elements in this mixture, consists of three different isotopes: about 99.3% of naturally occurring uranium is ^{238}U, about 0.7% is ^{235}U, and a trace quantity (about 5×10^{-3} %) is ^{234}U. The ^{238}U and ^{234}U belong to one family, the uranium series, while the ^{235}U isotope of uranium is the first member of another series called the actinium series. Uranium is ubiquitous in the natural environment and is found in the soil at average concentrations of about 3 ppm (parts per million) by weight, which corresponds to =74 mBq/g soil.

- Uranium forms extremely stable compounds with phosphorous. Phosphate-rich soil, therefore, contains uranium at concentrations much higher than average, from about 7 ppm to about 125 ppm Medium-grade uranium ore contains about 1000–5000 ppm uranium, while the uranium concentration in high-grade ore is about 10,000–40,000 ppm.

Energy dose

Radioactive sources with a large lifetime t (or, equivalently, half-life T $_{1/2}$) naturally have lower activities if a given number of nuclei is considered. The activity in Bq does not say very much about possible biological effects. These are related to the deposited energy by the radioactive source in matter. The energy dose D (**absorbed energy**)

ΔW per unit Δm

$$D = \frac{\Delta W}{\Delta m} = \frac{1}{\rho} \frac{\Delta W}{\Delta V}$$

Where ρ is density , ΔV is the volume elements [3]

1 gray= 1J/kg
or   1rad= 100erg/g where 1gray= 100rad   (radiation absorbed dose)

For indirectly ionizing radiation (i.e. photons and neutrons, but not electrons and other charged particles) a further quantity characterizing the energy dose, the 'kerma', is defined. Kerma is an abbreviation for "kinetic energy released per unit mass". The *k* erma k is defined as the sum of the initial energies of all charged particles, *E*, liberated in a volume element *V* by indirectly ionizing radiation divided by the mass *m* of this volume element:

$$K = \frac{\Delta E}{\Delta m} = \frac{1}{\rho} \frac{\Delta E}{\Delta V}$$

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translated into the biological effect of radiation. Electrons, for example, ionize relatively weakly while, in contrast, a rays are characterized by a high ionization density. Therefore, biological repair mechanisms cannot be very effective in the latter case. The relative biological effectiveness (RBE) is defined as the reciprocal ratio between the absorbed dose of radiation needed to cause an damage to absorbed dose of 200 Kev X-rays to cause the same damage, and its depends on the type of radiation, the radiation energy, the temporal distribution of the dose, and other quantities such as PH , tissue temperature The relative biological effectiveness' factor by which we have to multiply the energy dose D.

$$REB = \frac{D_\gamma}{D}$$

Since it is not always known in radiation protection which biological effects one has to refer to in a specific case, instead of the complicated energy-, radiation-, and dose-rate-dependent RBE factor some uses the so-called quality factor Q to assess the effect of a physical energy deposition. This leads to the dose equivalent H,

$$H \ = \ D \ \times \ Q \ \times \ F$$

Occupational dose limits

The occupational dose limits for general employees shall be controlled such that the following limits are not exceeded in a year:

• Total effective dose equivalent of 5 rems (50 mSv);
•The sum of the deep dose equivalent for external exposures and the committed dose equivalent to any organ or tissue other than the lens of the eye of 50 rems (500 mSv);
•a lens-of-the-eye dose equivalent of 15 rems (150 mSv)
•a shallow dose equivalent of 50 rems (500 mSv) to the skin or to
any extremity
The total effective dose equivalent during a year shall be determined by summing the effective dose equivalent from external exposures and the committed effective dose equivalent from intakes during the year. Determinations of the effective dose equivalent shall be made using the standard-weighting-factor values as provided by standard weighting factor the regulations The radiation and tissue weighting factors in the US regulations (see the following tables).

$$H = \sum W_i \, H_i \leq 5mSv$$

Where H effective dose equivalent, W_i tissue weighting factors , H_i tissue effective dose equivalent

Table 6.1 US quality factors ('radiation weighting factors')	radiation type	quality factor
	X rays, gamma rays, positrons, electrons	1
	neutrons, less than 10keV	3
	neutrons, over 10keV	10
	protons and singly charged particles of unknown energy with rest mass greater than one atomic mass unit	10
	alpha particles and multiply charged particles (and particles of unknown charge) of unknown energy	20

Table 6.2 US weighting factors for various organs and tissues	organs or tissues	weighting factor
	gonads	0.25
	breasts	0.15
	red bone marrow	0.12
	lungs	0.12
	thyroid gland	0.03
	periosteum, bone surface	0.03
	other organs or tissue	0.30
	whole body	1.00

H.W: calculate the total effective dose equivalent during a year for the following organs or tissues gonads, Breasts, red bone marrow, lungs, thyroid gland, bone surface? Assume that each tissue above is irradiated separately for gamm ray.

H.W :A radiation officer detects a contamination with contamination laboratory which leads to an ambient-dose rate of 1 mSv/h. He decides to seal the room and wait until the activity due to the iodine contamination has decayed to such a level that the ambient-dose rate is only 1 µSv/h. For how long has the room to be sealed ?The half-life of the ^{131}I isotope is 8 days.

Transformation Mechanisms

- All radioactive transformations fall into one of the following categories:

- Alpha emission

- Isobaric transitions (Given the atomic number of the parent nucleus is Z, that of the daughter nucleus is Z + 1, if a beta particle is emitted, or Z − 1, if a positron is emitted. The atomic mass number of the daughter is same as that of the parent.)

- ◦ Beta (negatron) emission

- ◦ Positron emission

- ◦ Orbital electron capture

- Isomeric transitions (The atomic number and the atomic mass number of the daughter is same as that of the parent.)

- ◦ Gamma ray emission

- ◦ Internal conversion