Nuclear Transformations

Radioactivity

- Radioactivity is the spontaneous nuclear transformations in unstable atoms that result in the formation of new elements(Isotopes) . These transformations are characterized by one of several different mechanisms, including alpha-particle emission, beta-particle and positron emission, and orbital electron capture. Each of these reactions may or may not be accompanied by gamma radiation.
- The exact mode of radioactive transformation depends on
- The energy available for the transition. The available energy, in turn, depends on two factors: on the particular type of nuclear instability, whether the neutron-to-proton ratio is too high or too low for the particular nuclide under consideration

- The mass–energy relationship among the parent nucleus, daughter nucleus, and emitted particle.

Isotopes

- It has been found that for any particular element the number of neutrons within the nucleus is not constant. Oxygen, for example, consists of three nuclear species: one whose nucleus has 8 neutrons, one of 9 neutrons, and one of 10 neutrons. In each of these three cases, of course, the nucleus contains 8 protons. The atomic mass numbers of these three species are 16, 17, and 18, respectively. These three nuclear species of the same element are called isotopes of oxygen. Isotopes of an element are atoms that contain the same number of positive nuclear charges and have the same extra nuclear electronic structure but differ in the number of neutrons. Most elements contain several isotopes.

Radioactive Decay Law

- Let us say that in the sample of radioactive material there are **N** nuclei which have not decayed at a certain time, t. So what happens in the next brief period of time? Some nuclei will decay for sure. But how many?
- The number which will decay will depend on overall number of nuclei, N, and also on the length of the brief period of time. In other words the more nuclei there are the more will decay and the longer the time period the more nuclei will decay. Let us denote the number which will have decayed as dN and the small time interval as dt.
- So we have reasoned that the number of radioactive nuclei which will decay during the time interval from t to t+dt must be proportional to N and to dt. therefore:

$$-dN \;\; \alpha \;\; \lambda N.dt$$

turning the proportionality in this equation into an equality we can write

$$- dN = \lambda \ N \ dt$$

where the constant of proportionality, λ, is called the Decay Constant.

Dividing across by N we can rewrite this equation as:

$$- \frac{dN}{N} = \lambda \ .dt$$

So this equation describes the situation for any brief time interval, dt. To find out what happens for all periods of time we integrate the above equation. Expressing this more formally we can say that for the period of time from t =0 to any later time t, the number of radioactive nuclei will decrease from N_o to N_t, so that:

$$\int_{N_0}^{Nt} \frac{dN}{N} = \lambda \int_0^t dt$$

$$\ln \left(\frac{N_t}{N_0}\right) = -\lambda \ t$$

$$\frac{N_t}{N_0} = \exp^{-\lambda \ t}$$

$$N(t) = N_0\, e^{-\lambda t}$$

This final expression is known as the Radioactive Decay Law. It tells us that the number of radioactive nuclei will decrease in an exponential fashion with time with the rate of decrease being controlled by the Decay constant

$$N = \frac{6.02\ X 10^{23}\ atom/mol}{A\ g/mol}\ X\ W.g$$

where A is the atomic weight and W is the weight of the sample.

$$\text{H.W} - \ Let \quad N(t) = \frac{N_0}{2} \ and \quad t = \frac{t}{2} \quad prove \quad that \quad t^{\frac{1}{2}} = \frac{0.693}{\lambda}$$

The term half-life ($t^{1/2}$) of a radioactive substance is defined as the time required for either the activity or the number of radioactive atoms to decay to half the initial value.

H.W- Why a negative sign is given to N?

H.W -Given that the transformation rate constant for ^{226}Ra is 4.38×10^{-4} 1/yr, calculate the half-life for radium.

Examples of the half lives of some radioisotopes are given in the following table. Notice that some of these have a relatively short half life. These tend to be the ones used for medical diagnostic purposes because they do not remain radioactive for very long following administration to a patient and hence result in a relatively low radiation dose.

Radioisotope	Half Life (approx.)
^{81m}Kr	13 seconds
^{99m}Tc	6 hours
^{131}I	8 days
^{51}Cr	1 month
^{137}Cs	30 years
^{241}Am	462 years
^{226}Ra	1620 years
^{238}U	4.51×10^{9} years

H.W. What is the logistical problem when we wish to use radioisotope with short half life for medical diagnostic purposes ؟

a) The half-life of 99mTc is 6 hours. After how much time will 1/16th of the radioisotope remain?
b) b) Verify your answer by another means.

- For some applications, as in the case of dosimetry of internally deposited radioactive material (discussed later), it is convenient to use the average life of the radioisotope. The average life is defined simply as the sum of the lifetimes of the individual atoms divided by the total number of atoms originally present.

- The transformation rate of a quantity of radioisotope containing N atoms is λN. During the time interval between t and t + dt, the total number of transformations is λ N dt. Each of the atoms that decayed during this interval, however, had existed for a total lifetime t since the beginning of observation on them. The sum of the lifetimes, therefore, of all the atoms that were transformed during the time interval between t and t + dt, after having survived since time t = 0, is t λN dt. The average life, τ, of the radioactive species is

$$\tau = \frac{1}{N_0}\int_0^{\infty} t\lambda \, N dt$$

$$\tau = \frac{1}{N_0}\int_0^{\infty} t\lambda \, N_0 e^{-\lambda t} \, dt$$

This expression, when integrated by parts, shows the value for the mean life of a radioisotope to be

$$\tau = \frac{1}{\lambda}$$

H.W: what is the relationship between the half-life and the mean life?

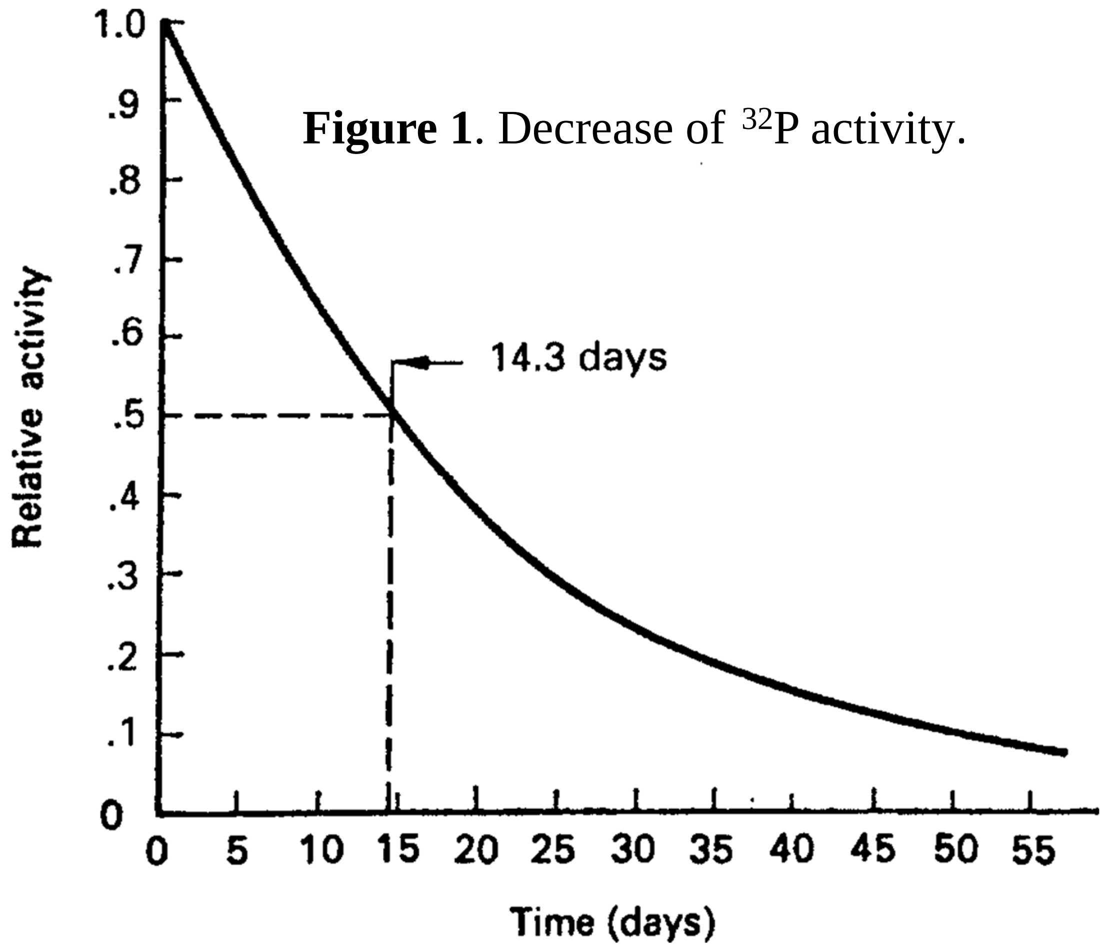

Figure 1. Decrease of ^{32}P activity.

From the definition of the half-life, it follows that the fraction of a radionuclide remaining after *n* half-lives correction factor is given by the relationship:

$A/A_0 = 1/2^n$

where A_0 is the original quantity of activity and A is the activity left after *n* half-lives

Activity

The rate of decay is referred to as the activity of a radioactive material. If ΔN/Δt is replaced by A, the symbol for activity, then:

$$A = -\lambda \; N \, dt$$

$$A = A_0 \; e^{-\lambda t}$$

Where is A the activity remaining at time t, and A_0 is the original activity equal to λN_0.

The unit of activity is the curie (Ci), defined as:

1mCi = 3.7 × 10^{10} disintegrations/sec (dps)1

The SI unit for activity is the becquerel (Bq). The becquerel is a smaller but more basic unit than the curie and is defined as:

1mCi=10^{-3}Ci= 3.7 × 10^{10} (dps)[1]

1Bq=dps=2.70 x10^{-11}Ci

H.W

•Calculate the number of atoms in 1 g of ^{226}Ra.

•What is the activity of 1 g of ^{226}Ra (half-life = 1,622 years)?

H.W What is the minimum mass of ^{99}mTc that can have a radioactivity of 1 MBq? Assume the half-life is 134 minutes and that Avogadro's Number is 6.023 x 10^{23}.

Energy dose

Radioactive sources with a large lifetime t (or, equivalently, half-life $T_{1/2}$) naturally have lower activities if a given number of nuclei is considered. The activity in Bq does not say very much about possible biological effects. These are related to the deposited energy by the radioactive source in matter. The energy dose D (**absorbed energy**)

ΔW per mass unit Δm),

$$D = \frac{\Delta W}{\Delta m} = \frac{1}{\rho}\frac{\Delta W}{\Delta V} \qquad (2.13)$$

(ρ – density, ΔV – volume element[3]), is measured in gray: **gray**

$$1 \text{ gray (Gy)} = 1 \text{ joule (J)} \; / \; 1 \text{ kilogram (kg)} \; . \qquad (2.14)$$

Gray is related to the old unit rad (radiation absorbed dose, 1 rad = 100 erg/g; still in use in the US) according to:[4]

1 Gy = 1 J/1 kg
1 Gy = 100 rad

$$1 \text{ Gy} = 100 \text{ rad} \; . \qquad (2.15)$$

For indirectly ionizing radiation (i.e. photons and neutrons, but not electrons and other charged particles) a further quantity characterizing the energy dose, the 'kerma', is defined. Kerma is an abbreviation for "kinetic energy released per unit mass". The *k* erma kis defined as the sum of the initial energies of all charged particles, *E*, liberated in a volume element *V* by indirectly ionizing radiation divided by the mass m of this volume element:

$$k = \frac{\Delta E}{\Delta m} = \frac{1}{\rho}\frac{\Delta E}{\Delta V} \; , \qquad (2.16)$$

where ρ is the density of the absorbing material.