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UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الاولى

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اسم المادة بالغة العربية : التفاضل والتكامل

اسم المادة باللغة الإنكليزية : **Calculus**

اسم المحاضرة العاشرة باللغة العربية: (التكامل بالأجزاء)

اسم المحاضرة العاشرة باللغة الإنكليزية : **(Integration by Parts)**

$$3 - \int \frac{1}{x^2 - 1} dx \qquad \text{H. W.}$$

19- Integration by Parts

The formula

$$\int u\, dv = uv - \int v\, du$$

Consider

$$w = u \cdot v \Rightarrow dw = u \cdot dv + v \cdot du \Rightarrow u \cdot dv = dw - vdu$$

$$\int u\, dv = \int dw - \int v\, du = w - \int v\, du$$

$$\int u\, dv = u \cdot v - \int v\, du \qquad \text{where} \quad w = u \cdot v$$

Example:- Evaluate the following integrals

$$1 - \int \ln x\, dx$$

Solve:

$$\int \ln x\, dx = u \cdot v - \int v\, du$$

$$u = \ln x \qquad \Longrightarrow \qquad du = \frac{dx}{x}$$

$$dv = dx \qquad \Longrightarrow \qquad v = x$$

$$\int \ln x\, dx = x \ln x - \int x \frac{1}{x} dx$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + c$$

$$2-\int \tan^{-1} x\, dx$$

Solve:

$$\int \tan^{-1} x\, dx = u\cdot v-\int v\, du$$

$$u=\tan^{-1} x \quad\Longrightarrow\quad du=\frac{dx}{1+x^2}$$

$$dv=dx \quad\Longrightarrow\quad v=x$$

$$\int \tan^{-1} x\, dx = x\tan^{-1} x-\int \frac{x dx}{1+x^2}$$

$$= x\tan^{-1} x-\frac{1}{2}\ln(1+x^2)+c$$

$$3-\int e^x \sin x\, dx$$

Solve:

$$\int e^x \sin x\, dx = u\cdot v-\int v\, du$$

$$u=e^x \quad\Longrightarrow\quad du=e^x dx$$

$$dv=\sin x\, dx \quad\Longrightarrow\quad v=-\cos x$$

$$\int e^x \sin x\, dx = -e^x\cos x+\int e^x \cos x\, dx$$

And

$$\int e^x \cos x\, dx = u\cdot v-\int v\, du$$

$$u=e^x \quad\Longrightarrow\quad du=e^x dx$$

$$dv=\cos x\, dx \quad\Longrightarrow\quad v=\sin x$$

$$\int e^x \cos x\, dx = e^x \sin x - \int e^x \sin x\, dx$$

Then

$$\int e^x \sin x\, dx = -e^x \cos x + \int e^x \cos x\, dx$$

$$\int e^x \sin x\, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$$

$$2\int e^x \sin x\, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\, dx = \frac{1}{2} e^x (\sin x - \cos x)$$

$4 - \int x e^x\, dx$ H.W.

20- Definite Integrals

The quantity

$$\int_a^b f(x)\, dx$$

Is called the Definite Integral of $f(x)$ from a to b. The numbers a and b are known as the lower and upper limits of the integral.

To see how to evaluate a definite integral consider the following example.

Example:- Find

$$1-\int_{1}^{4} x^2 dx$$

Solve:

$$\int_{1}^{4} x^2 dx = \left[\frac{x^3}{3}+c\right]_1^4$$

$$\left[\frac{x^3}{3}+c\right]_1^4 = (\text{evaluate at upper limit}) - (\textit{evaluate at lower limit})$$

$$\left[\frac{x^3}{3}+c\right]_1^4 = \left(\frac{(4)^3}{3}+c\right)-\left(\frac{(1)^3}{3}+c\right)$$

$$= \frac{64}{3}+c-\frac{1}{3}-c$$

$$= \frac{64}{3}+\frac{1}{3} = 21$$

$$2-\int_{0}^{\frac{\pi}{2}} \cos x\, dx$$

Solve:

$$\int_{0}^{\frac{\pi}{2}} \cos x\, dx = \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$\left[\sin x\right]_0^{\frac{\pi}{2}} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1-0 = 1$$

$$3-\int_{0}^{1} x^2 dx$$

Solve:

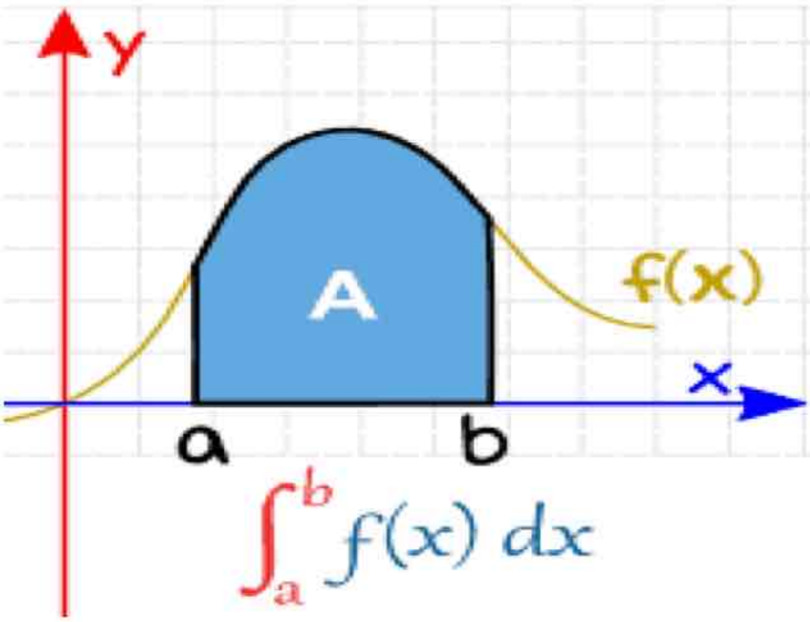
$$\int_{0}^{1} x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$4-\int_{0}^{1} e^{2x} dx$$

21- Some Properties of Definite Integral

If $a \le x \le b$, then

$$\int_{a}^{b} f(x)dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$



$$1 - \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$$

$$2 - \int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx \qquad \text{where} \quad c \in [a,b]$$

$$3 - \int_{a}^{b} cf(x)dx = c\int_{a}^{b} f(x)dx$$

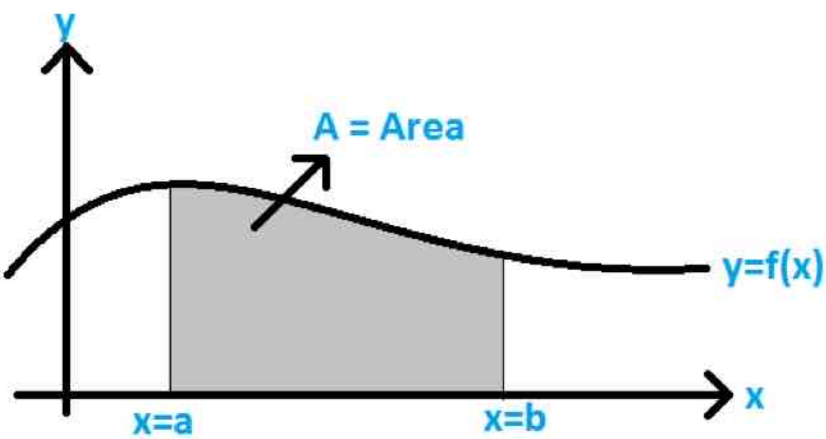
$$4 - \int_{a}^{b} [f(x) \pm g(x)]dx = \int_{a}^{b} f(x)dx \pm \int_{a}^{b} g(x)dx$$

22- Application on Integral

1- Area under the Graph

We have the law

$$A = \int_{a}^{b} f(x)dx$$



Example:- Find the Area bounded by $y = x^3$ from $x = -2$ to $x = 2$

Solve:

$$A = \int_0^2 f(x)dx + \left|\int_{-2}^0 f(x)dx\right|$$

$$= \int_0^2 x^3 dx + \left|\int_{-2}^0 x^3 dx\right|$$

$$= \left[\frac{x^4}{4}\right]_0^2 + \left|\left[\frac{x^4}{4}\right]_{-2}^0\right|$$

$$= 4 + 4 = 2$$

Example:- Find the Area bounded by $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$

Solve:

$$A = \int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$= \left[\sin x\right]_0^{\frac{\pi}{2}}$$

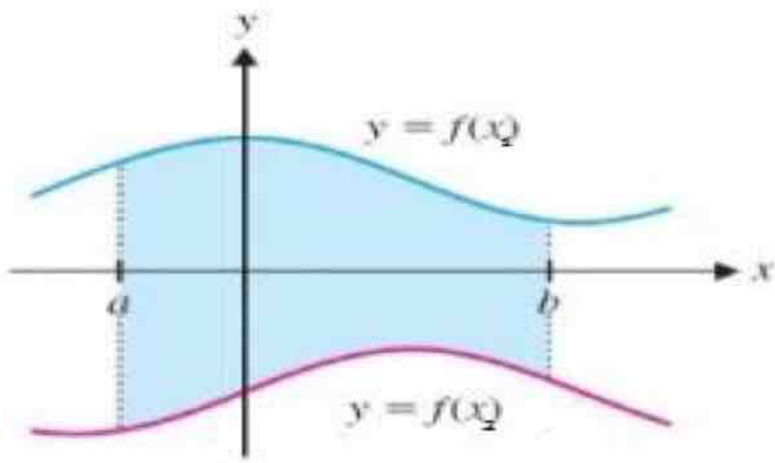
$$= \sin\frac{\pi}{2} - \sin 0 = 1$$

Example:- Find the Area bounded by $y = \frac{1}{2}x^2$ from $x = 1$ to $x = 3$ **H.W.**

2- Area between tow carvers

We have the law

$$A = \int_a^b [f(x_1) - f(x_2)]dx$$



Example:- Find the Area bounded between the carve $y = x^2$ and the line $y = x + 2$

Solve:

$$x + 2 = x^2 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = 2 \quad or \quad x = -1$$

We have the tow points $(2.4), (-1,1)$

$$A = \int_{-1}^{2} [(x+2) - x^2]dx = \int_{-1}^{2} (x + 2 - x^2)dx$$

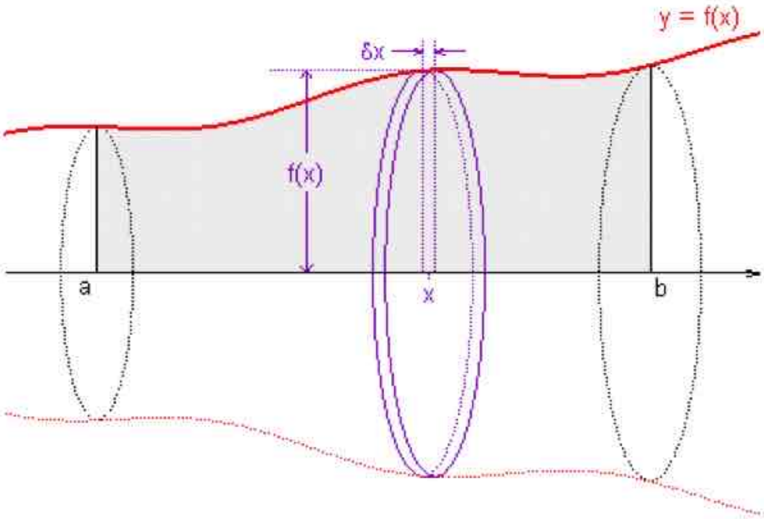
$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \left(\frac{(2)^2}{2} + 2(2) - \frac{(2)^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right)$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

3- Volumes

We have law

$$V = \int_{a}^{b} A(x)dx = \int_{a}^{b} \pi y^2 dx$$



Example:- Find the Volume generated by rotating the bounded area by $y = x^2$ and the line $x = 4$ about x-axis

Solve:

$$V = \int_a^b A(x)dx = \int_a^b \pi y^2 dx$$

$$= \int_0^4 \pi x^4 dx = \pi \int_0^4 x^4 dx$$

$$= \pi \left[\frac{x^5}{5}\right]_0^4 = \pi \frac{(4)^5}{5}$$

$$= \pi \frac{1024}{5}$$