



كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الاولى

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اسم المادة باللغة العربية : التفاضل والتكامل

اسم المادة باللغة الإنكليزية : Calculus

اسم المحاضرة التاسعة باللغة العربية: (التكامل بالكسور الجزئية)

اسم المحاضرة التاسعة باللغة الإنكليزية : (Integration by Partial fractions)

$$11 - \int \frac{e^x}{e^x - 1} dx \quad \text{H. W.}$$

$$12 - \left(2 \int \frac{\sin(\sqrt{x})}{2\sqrt{x}} dx \right) \quad \text{H. W.}$$

18- Integration by Partial fractions

To find the Integration of a function $F(x) = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are Polynomials. If the degree of $f(x)$ is less than the degree of $g(x)$, then we need to factories $g(x)$ into linear factors.

Example:- Evaluate the following integrals

$$1 - \int \frac{5x + 5}{x^2 + 3x - 4} dx$$

Solve:

$$\begin{aligned} \int \frac{5x + 5}{x^2 + 3x - 4} dx &= \int \frac{5x + 5}{(x - 1)(x + 4)} dx \\ &= \int \frac{A}{(x - 1)} dx + \int \frac{B}{(x + 4)} dx \end{aligned}$$

Now, we will find A and B

$$\begin{aligned} \frac{5x + 5}{x^2 + 3x - 4} &= \frac{5x + 5}{(x - 1)(x + 4)} \\ &= \frac{A}{(x - 1)} + \frac{B}{(x + 4)} \end{aligned}$$

$$= \frac{A(x+4) + B(x-1)}{(x-1)(x+4)}$$

See that

$$5x + 5 = A(x+4) + B(x-1)$$

So to find A let

$$x = 1 \Rightarrow 5(1) + 5 = A(1+4) \Rightarrow 10 = 5A \Rightarrow A = 2$$

Also to find B let

$$x = -4 \Rightarrow 5(-4) + 5 = B(-4-1) \Rightarrow -15 = -5B \Rightarrow B = 3$$

$$\begin{aligned} \int \frac{5x+5}{x^2+3x-4} dx &= \int \frac{A}{(x-1)} dx + \int \frac{B}{(x+4)} dx \\ &= \int \frac{2}{(x-1)} dx + \int \frac{3}{(x+4)} dx \\ &= 2 \ln|x-1| + 3 \ln|x+4| + c \end{aligned}$$

$$2 - \int \frac{1}{x(x^2+1)^2} dx$$

Solve:

$$\int \frac{dx}{x(x^2+1)^2} = \int \frac{A}{x} dx + \int \frac{Bx+C}{x^2+1} dx + \int \frac{Dx+E}{(x^2+1)^2} dx$$

Now, we will find A, B, C, D and E

$$\frac{1}{x(x^2 + 1)^2} = \frac{A(x^2 + 1)^2 + (Bx + C)(x^2 + 1)x + (Dx + E)x}{x(x^2 + 1)^2}$$

$$1 = A(x^2 + 1)^2 + (Bx + C)(x^2 + 1)x + (Dx + E)x$$

$$1 = A(x^4 + 2x^2 + 1) + B(x^4 + x^2)C(x^3 + x) + (Dx^2 + Ex)$$

$$1 = (A + B)x^4 + Cx^3 + (2A + B + D)x^2 + (C + E)x + A$$

If we equate coefficients, we get

$$A + B = 0, \quad C = 0, \quad 2A + B + D = 0, \quad C + E = 0, \quad A = 1$$

Then

$$A = 1, \quad B = -1, \quad C = 0, \quad D = -1 \quad \text{and} \quad E = 0$$

$$\int \frac{dx}{x(x^2 + 1)^2} = \int \frac{A}{x} dx + \int \frac{Bx + C}{x^2 + 1} dx + \int \frac{Dx + E}{(x^2 + 1)^2} dx$$

$$= \int \frac{1}{x} dx + \int \frac{-x}{x^2 + 1} dx + \int \frac{-x}{(x^2 + 1)^2} dx$$

$$= \int \frac{1}{x} dx - \int \frac{x}{x^2 + 1} dx - \int \frac{x}{(x^2 + 1)^2} dx$$

$$= \ln|x| - \frac{1}{2} \ln(x^2 + 1) + \frac{1}{2(x^2 + 1)} + C$$

$$= \ln|x| - \ln \sqrt{x^2 + 1} + \frac{1}{2(x^2 + 1)} + C$$

$$= \ln \frac{|x|}{\sqrt{x^2 + 1}} + \frac{1}{2(x^2 + 1)} + C$$