UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الاولى

أستاذ المادة : (م.د. مصطفى ابراهيم حميد) (م.م. عبدالسلام فائق طلك)

اسم المادة بالغة العربية : التفاضل والتكامل

اسم المادة باللغة الإنكليزية : **Calculus**

اسم المحاضرة السابعة باللغة العربية: (قاعدة السلسلة)

اسم المحاضرة السابعة باللغة الإنكليزية : **(Chain Rule)**

15- The Derivative of Composite functions (Chain Rule)

If y is differentiable function of (u) and (u) is differentiable function of (x). Then y is a differentiable function of (x).

That is

$$y = f(u) \Rightarrow \frac{dy}{du} \quad , \quad u = f(x) \Rightarrow \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example:- Find $\frac{dy}{dt}$, where $y = x^2 + \sqrt{x}$ and $x = 3t^2 - 2t + 1$

Solve:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \left(2x + \frac{1}{2\sqrt{x}}\right)(6t - 2)$$

Substitute

$$x = 3t^2 - 2t + 1$$

$$\Rightarrow \frac{dy}{dt} = \left(2(3t^2 - 2t + 1) + \frac{1}{2\sqrt{3t^2 - 2t + 1}}\right)(6t - 2)$$

Example:- Find $\frac{dy}{dx}$, where $x = 2t + 3$ and $y = t^2 - 1$

Solve:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2t}{2} = t$$

Substitute

$$t = \frac{x-3}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-3}{2}$$

16- Implicit Derivative

Example:- Find $\frac{dy}{dx}$, if $y^2 = x$

Solve:

$$y^2 = x$$

$$\Rightarrow 2y\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$y^2 = x \quad \Rightarrow \quad y = \pm\sqrt{x} \quad \Rightarrow \quad y_1 = +\sqrt{x} \quad and \quad y_2 = -\sqrt{x}$$

$$\frac{dy_1}{dx} = \frac{1}{2y_1} = \frac{1}{2\sqrt{x}} \quad and \quad \frac{dy_2}{dx} = \frac{1}{2y_2} = \frac{1}{2(-\sqrt{x})} = \frac{-1}{2\sqrt{2}}$$

Example:- If $y = f(x)$ for $y^3 + xy + x^2 = 2$. Find $\frac{dy}{dx}$

Solve:

$$\Rightarrow 3y^2\frac{dy}{dx} + x\frac{dy}{dx} + y + 2x = 0$$

$$\Rightarrow 3y^2\frac{dy}{dx} + x\frac{dy}{dx} = -2x - y$$

$$\Rightarrow (3y^2 + x)\frac{dy}{dx} = -2x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x - y}{3y^2 + x}$$

10-Higher Order Derivative

Definition: If $y = f(x)$ is a continuous function, then the first derivative is $y' = \frac{dy}{dx} = f'(x)$ and the second order derivative is $y'' = \frac{d^2y}{dx^2} = f''(x)$. the nth order derivative is

$$y^n = \frac{d^ny}{dx^n} = f^n(x)$$

Example:- If $y = 4x^3 + 2x + 1$, Find $\frac{d^2y}{dx^2}$

Solve:

$$\Rightarrow \; y' = \frac{dy}{dx} = 12x^2 + 2$$

$$\Rightarrow y'' = \frac{d^2y}{dx^2} = 24x$$

11- Derivative with Physical Application

1- **Velocity** $\Rightarrow$ **denoted by** v

2- **Acceleration** $\Rightarrow$ **denoted by** a

3- **Time** $\Rightarrow$ **denoted by** t

4- **Distance** $\Rightarrow$ **denoted by** x

$$v = \frac{dx}{dt} \quad , \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Example:- Find Velocity and Acceleration such that $x = t^3 + 3t^2 + t + 1$, at $t = 2$.

Solve:

$$\Rightarrow v = \frac{dx}{dt} = 3t^2 + 6t + 1$$

$$\Rightarrow v = 12 + 12 + 1 = 25m/s \quad \text{at } t = 2$$

$$\Rightarrow a = \frac{dv}{dt} = 6t + 6$$

$$\Rightarrow a = 12 + 6 = 18m/s \quad \text{at } t = 2$$