



كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الاولى

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اسم المادة باللغة العربية : التفاضل والتكامل

اسم المادة باللغة الإنكليزية : Calculus

اسم المحاضرة الثانية باللغة العربية: (الدوال)

اسم المحاضرة الثانية باللغة الإنكليزية : (The Function)

H.W.

1-  $2x - 3 < 7$

2-  $2x + 4 < x - 4$

3-  $\frac{4}{x} < \frac{3}{5}$

4-  $\left| \frac{x+3}{6-5x} \right| \leq 2$

5-  $\frac{x-2}{x+3} < \frac{x+1}{x}$

6-  $|x(x + 1)| \leq |x + 4|$

#### 4- The Functions

**Definition:-** A relation between two set  $A$  and  $B$ ,  $f: A \rightarrow B$  is called a function if and only if for each element  $x \in A$  their exist unique element  $y \in B$  such that  $y = f(x)$ .

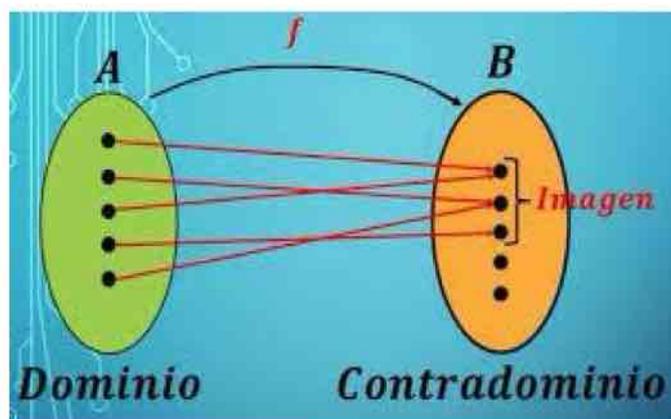
#### Notes

1-  $(x, y) \in f \Rightarrow y = f(x)$ .

2- The set  $A$  is called the domain  $D_f$ .

3- The set  $B$  is called the co-domain.

4- The set of all element of  $B$  such that  $y = f(x)$  is called the range and represented  $R_f$



**Example:- Find the Domain and the Range for each functions**

1-  $y = x^2 \Rightarrow \text{Domain} = \mathbb{R}, \text{Range} = \mathbb{R}$

2-  $y = x + 3 \Rightarrow \text{Domain} = \mathbb{R}, \text{Range} = \mathbb{R}$

3-  $y = \sqrt{x - 4}$

$$\Rightarrow x - 4 \geq 0 \Rightarrow x - 4 + 4 \geq 0 + 4 \Rightarrow x \geq 4$$

Then  $D_f = \{x: x \geq 4\}, R_f = \{y: y \geq 0\}$

4-  $y = \frac{x-3}{x+2}$

$$\Rightarrow x + 2 = 0 \Rightarrow x = -2 \Rightarrow D_f = \mathbb{R}/\{-2\}$$

$$\Rightarrow y(x + 2) = x - 3 \Rightarrow yx + 2y = x - 3$$

$$\Rightarrow yx - x = -3 - 2y \Rightarrow x(y - 1) = -3 - 2y$$

$$\Rightarrow x = \frac{-3 - 2y}{y - 1}$$

$$\Rightarrow y - 1 = 0 \Rightarrow y = 1 \Rightarrow R_f = \mathbb{R}/\{1\}$$

H.W.

Find the Domain and the Range for the functions

1-  $y = \frac{1}{x-2}$

2-  $f(x) = \frac{1}{\sqrt{x+3}}$

3-  $y = x^2 - 5x + 6$

4-  $y = \sqrt{x^2 - 9}$

5-  $y = \sqrt{x^2 - 2x - 3}$

6-  $f(x) = \frac{\sqrt{x-1}}{x^2+4}$

### Some types of Function

**Definition 1:-** Absolute Value Function is define by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Definition 2:-** A function is called even function if  $f(-x) = f(x)$ .

**Definition 3:-** A function is called odd function if  $f(-x) = -f(x) \neq f(x)$ .

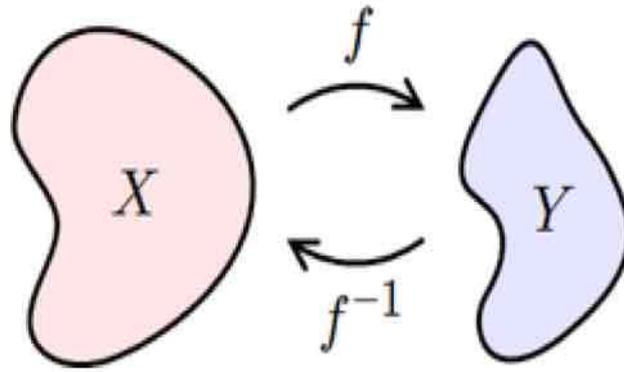
**Definition 4:-** A function is called constant function if  $f(x) = a_0$  ,  $\forall a \in \mathbb{R}$ .

**Definition 5:-** A function is called linear function if  $f(x) = a_1x + a_0$  ,  $\forall a \in \mathbb{R}$ .

**Definition 6:-** A function subjective  $f(x): X \rightarrow Y$ , we define the invers function such that  $x = f^{-1}(y): Y \rightarrow X$ .

$$f(f^{-1}(y)) = x$$

$$D_{f^{-1}} = R_f, D_f = R_{f^{-1}}$$



**Example:-**  $y = f(x) = x^3$  Find inverse function and  $D_{f^{-1}}, R_{f^{-1}}$

**Solve:**

$$y = x^3 \Rightarrow x = \sqrt[3]{y} = f^{-1}(y)$$

$$D_{f^{-1}} = \mathbb{R}^+, \quad R_{f^{-1}} = \mathbb{R}.$$

### Composite of Function

**Definition:-** If we have the two functions  $f(x), g(x)$  then we define a composite function as

$$z = f(g(x)) = f \circ g(x) \quad \text{or} \quad z = g(f(x)) = g \circ f(x)$$

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

**Example:-**  $f(x) = \sqrt{x}$ ,  $g(x) = x^2$  Find  $f \circ g(x)$  and  $g \circ f(x)$ .